Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.4-Cotangent/112-4.4.2.1-a+b-cot-^m-c+d-cot-^n

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [106]. This is test number [112].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (106)	0.00 (0)
Mathematica	99.06 (105)	0.94 (1)
Maple	97.17 (103)	2.83 (3)
Fricas	97.17 (103)	2.83 (3)
Mupad	97.17 (103)	2.83 (3)
Giac	2.83 (3)	97.17 (103)
Maxima	2.83 (3)	97.17 (103)
Sympy	1.89 (2)	98.11 (104)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

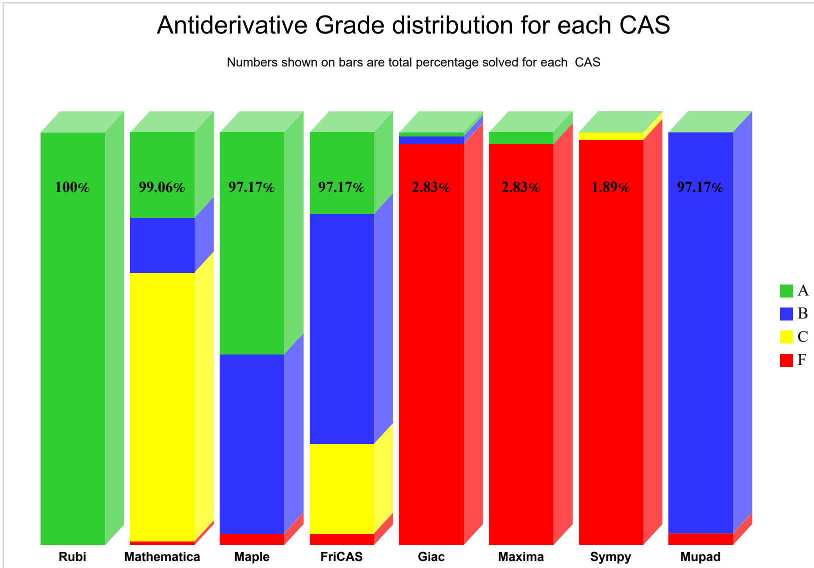
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

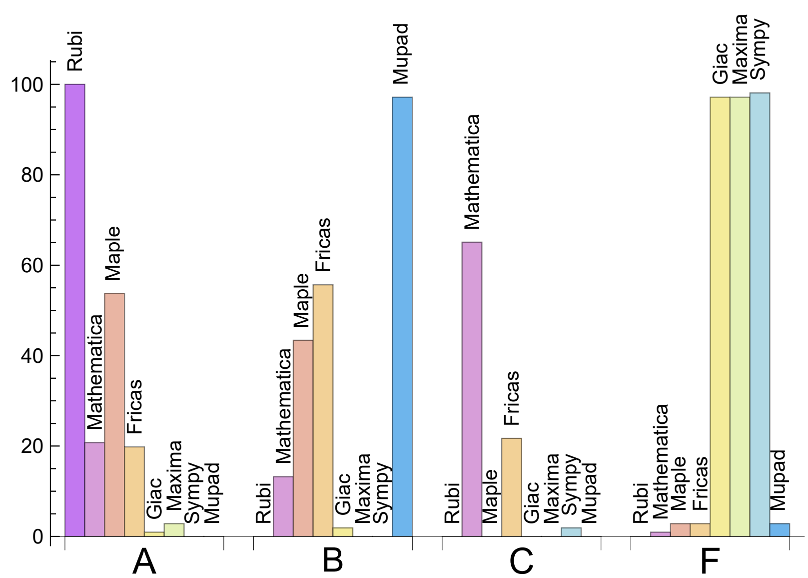
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	53.774	43.396	0.000	2.830
Mathematica	20.755	13.208	65.094	0.943
Fricas	19.811	55.660	21.698	2.830
Maxima	2.830	0.000	0.000	97.170
Giac	0.943	1.887	0.000	97.170
Mupad	0.000	97.170	0.000	2.830
Sympy	0.000	0.000	1.887	98.113

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Fricas	3	100.00	0.00	0.00
Maple	3	100.00	0.00	0.00
Mupad	3	0.00	100.00	0.00
Giac	103	95.15	4.85	0.00
Maxima	103	24.27	0.97	74.76
Sympy	104	94.23	4.81	0.96

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	0.12
Giac	0.36
Maxima	0.42
Rubi	0.94
Sympy	1.07
Fricas	1.42
Mathematica	1.99
Mupad	15.48

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	203.67	1.70	185.00	1.67
Rubi	225.97	0.99	226.00	0.97
Mathematica	236.03	1.26	203.00	0.83
Giac	249.33	2.05	241.00	2.17
Maple	530.62	3.30	333.00	1.29
Fricas	1996.96	7.20	849.00	3.87
Sympy	2244.00	22.30	2244.00	22.30
Mupad	3189.42	10.43	366.00	1.69

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

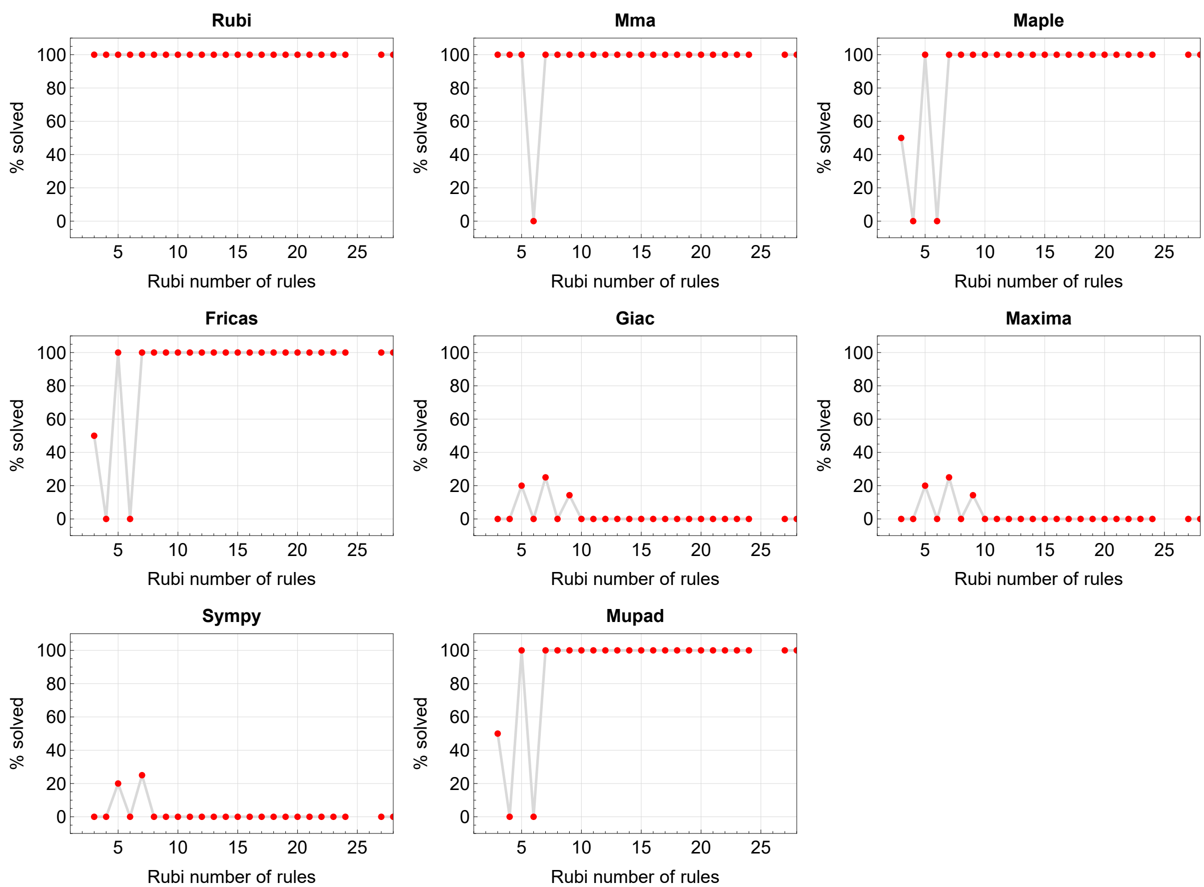


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

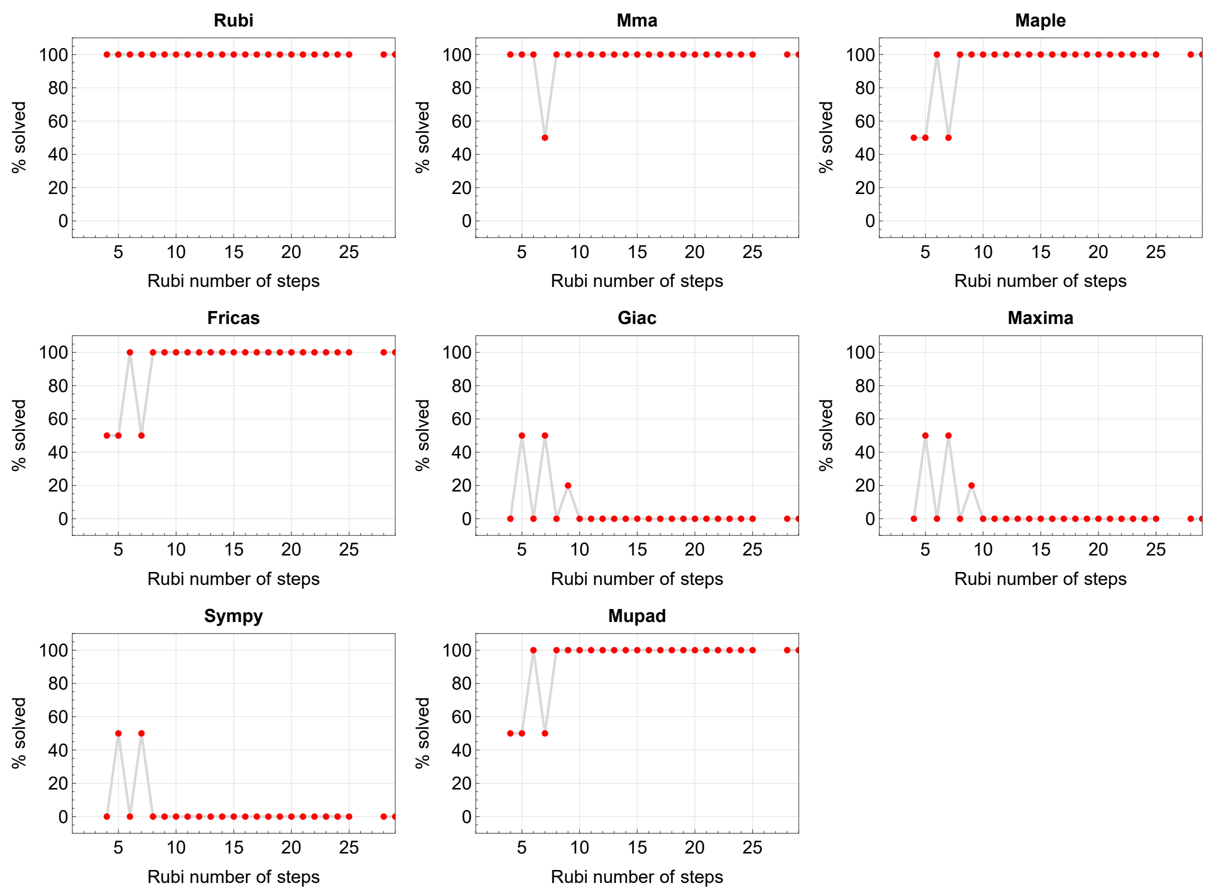


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

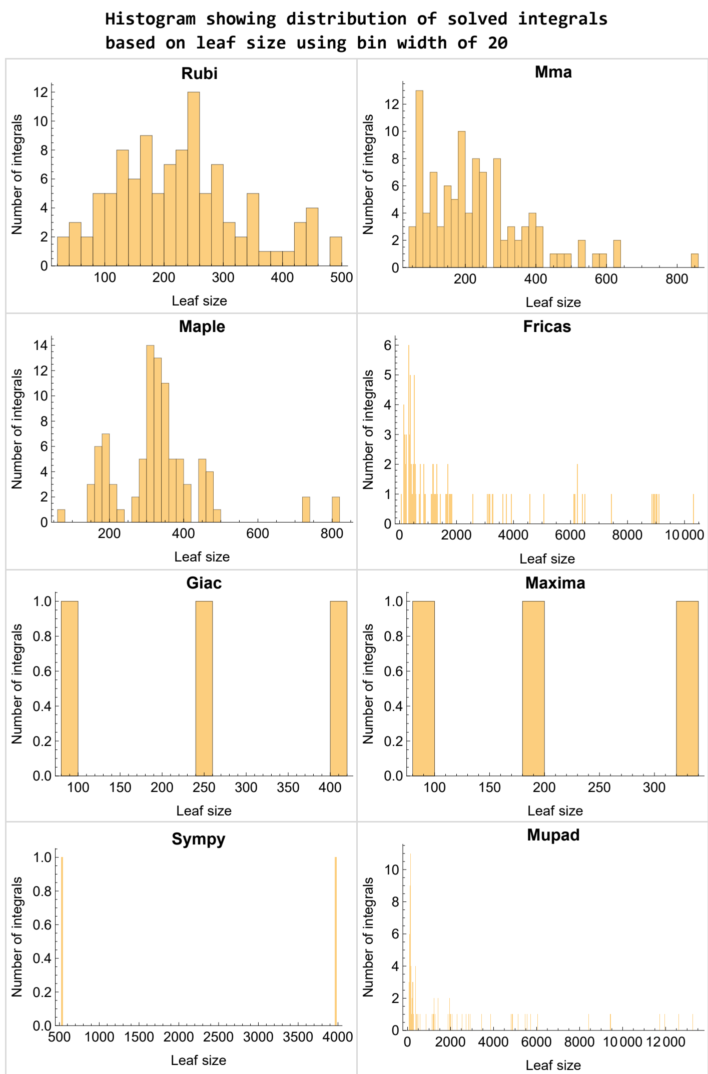


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

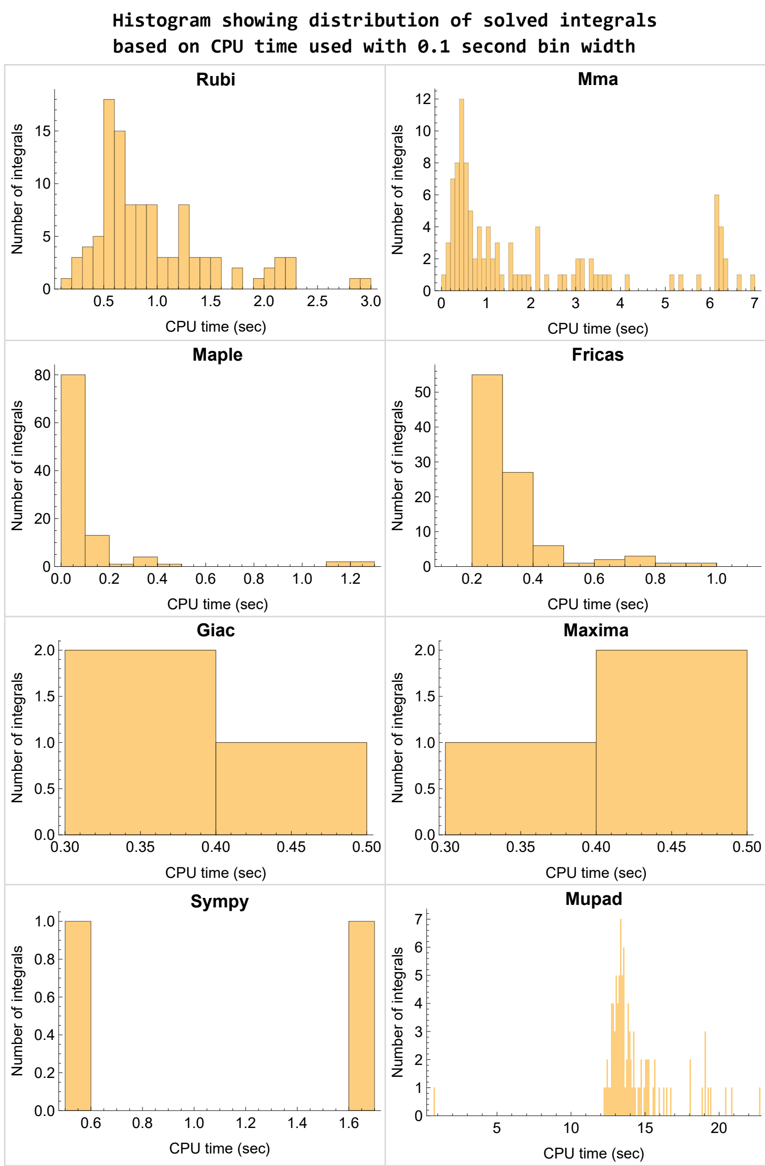


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

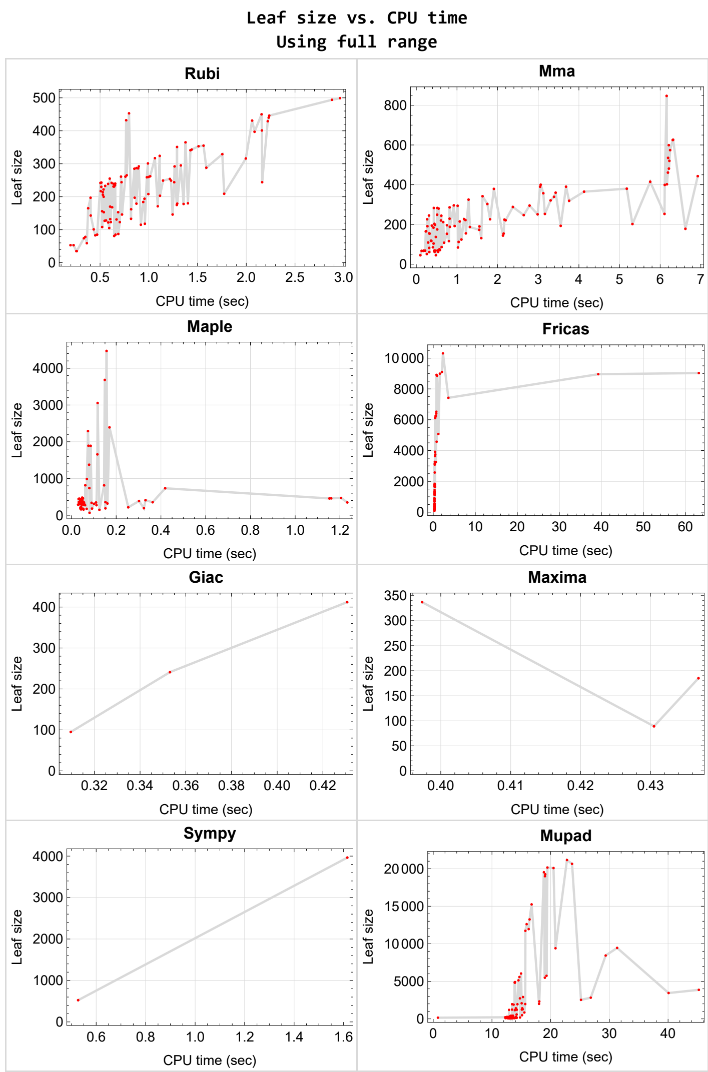


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{8, 9, 10, 11, 12, 13, 14, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106\}$

Mathematica $\{13, 16, 17, 19, 21\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

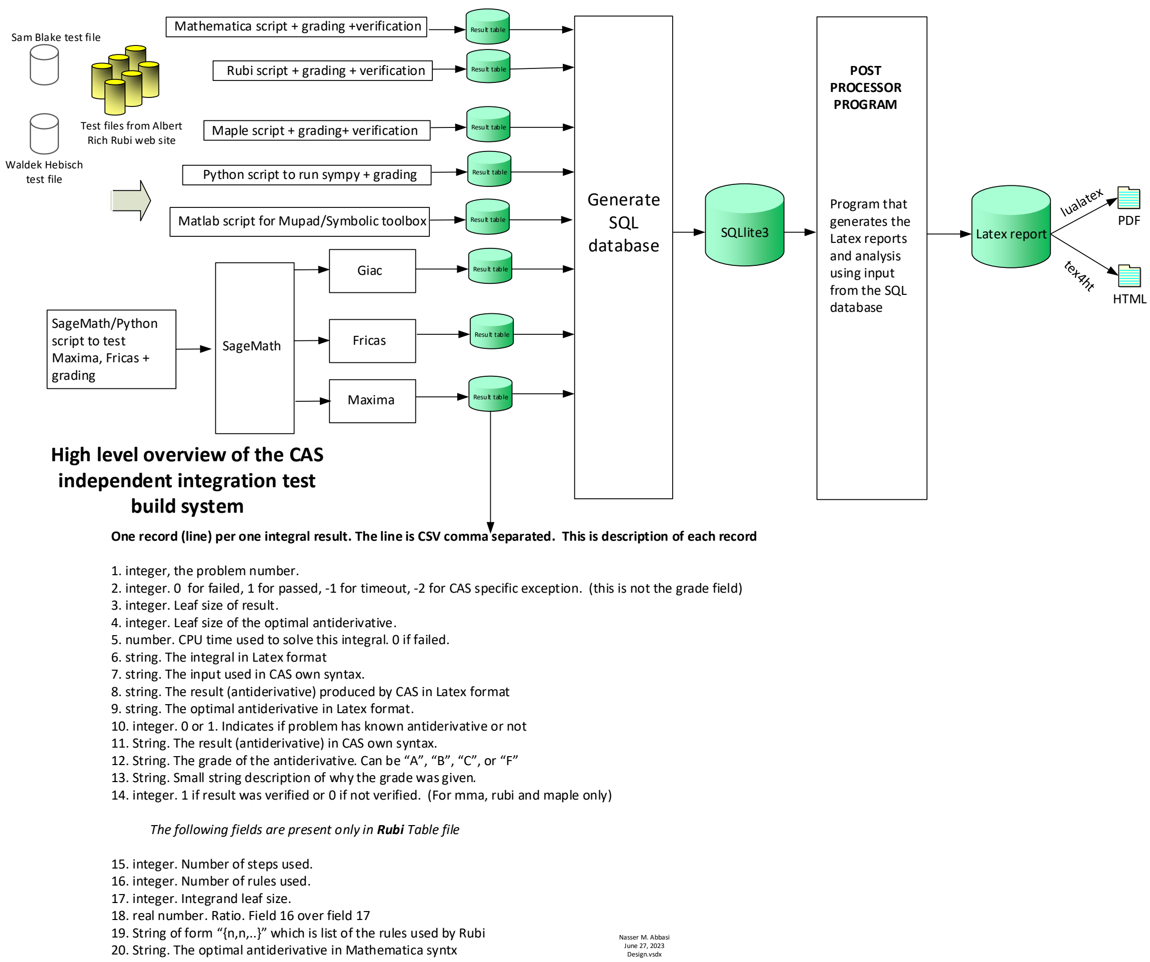
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 8, 9, 10, 11, 12, 13, 14, 31, 32, 90, 91, 92, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106 }

B grade { 1, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 95 }

C grade { 2, 3, 4, 5, 6, 7, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 93, 94, 99 }

F normal fail { 89 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 8, 9, 10, 11, 12, 13, 14, 29, 30, 31, 32, 33, 34, 42, 43, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 92, 93, 94 }

B grade { 2, 3, 4, 5, 6, 7, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 48, 50, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

C grade { }

F normal fail { 1, 88, 89 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 2, 4, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 35, 36, 37, 38, 40, 92 }

B grade { 3, 5, 6, 7, 27, 39, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

C grade { 8, 9, 10, 11, 12, 13, 14, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

F normal fail { 1, 88, 89 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 92, 93, 94 }

B grade { }

C grade { }

F normal fail { 1, 41, 42, 44, 45, 46, 47, 48, 49, 50, 88, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

F(-1) timedout fail { 43 }

F(-2) exception fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89 }

2.1.6 Giac

A grade { 92 }

B grade { 93, 94 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

F(-1) timedout fail { 21, 22, 67, 68, 80 }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 88, 89 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { }

B grade { }

C grade { 92, 93 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 80, 84, 85, 86, 87, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

F(-1) timedout fail { 75, 76, 81, 82, 83 }

F(-2) exception fail { 94 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	53	117	0	0	0	0	0	0
N.S.	1	1.08	2.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	0.370	0.000	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	123	68	319	0	377	0	0	144
N.S.	1	1.06	0.59	2.75	0.00	3.25	0.00	0.00	1.24
time (sec)	N/A	0.612	0.215	0.161	0.000	0.286	0.000	0.000	13.778

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	101	67	303	0	334	0	0	98
N.S.	1	1.07	0.71	3.22	0.00	3.55	0.00	0.00	1.04
time (sec)	N/A	0.453	0.130	0.045	0.000	0.278	0.000	0.000	13.505

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	74	154	287	0	236	0	0	128
N.S.	1	1.04	2.17	4.04	0.00	3.32	0.00	0.00	1.80
time (sec)	N/A	0.327	0.327	0.053	0.000	0.269	0.000	0.000	13.278

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	53	165	273	0	172	0	0	65
N.S.	1	1.08	3.37	5.57	0.00	3.51	0.00	0.00	1.33
time (sec)	N/A	0.236	0.233	0.112	0.000	0.277	0.000	0.000	13.062

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	78	191	294	0	321	0	0	84
N.S.	1	1.04	2.55	3.92	0.00	4.28	0.00	0.00	1.12
time (sec)	N/A	0.337	0.272	0.046	0.000	0.274	0.000	0.000	13.330

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	108	203	309	0	358	0	0	103
N.S.	1	1.09	2.05	3.12	0.00	3.62	0.00	0.00	1.04
time (sec)	N/A	0.507	0.444	0.049	0.000	0.280	0.000	0.000	13.898

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	261	187	187	0	507	0	0	125
N.S.	1	0.97	0.70	0.70	0.00	1.88	0.00	0.00	0.46
time (sec)	N/A	0.756	1.318	0.152	0.000	0.289	0.000	0.000	13.912

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	231	112	172	0	423	0	0	104
N.S.	1	0.94	0.46	0.70	0.00	1.72	0.00	0.00	0.42
time (sec)	N/A	0.627	1.041	0.041	0.000	0.279	0.000	0.000	13.366

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	235	175	170	0	372	0	0	104
N.S.	1	0.96	0.72	0.70	0.00	1.52	0.00	0.00	0.43
time (sec)	N/A	0.639	0.514	0.049	0.000	0.303	0.000	0.000	13.070

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	200	253	155	0	323	0	0	86
N.S.	1	0.90	1.14	0.70	0.00	1.45	0.00	0.00	0.39
time (sec)	N/A	0.532	6.107	0.054	0.000	0.299	0.000	0.000	12.740

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	206	184	159	0	413	0	0	86
N.S.	1	0.93	0.83	0.72	0.00	1.86	0.00	0.00	0.39
time (sec)	N/A	0.515	0.422	0.043	0.000	0.282	0.000	0.000	12.877

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	247	231	400	174	0	446	0	0	99
N.S.	1	0.94	1.62	0.70	0.00	1.81	0.00	0.00	0.40
time (sec)	N/A	0.613	3.060	0.053	0.000	0.287	0.000	0.000	13.090

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	239	173	174	0	542	0	0	99
N.S.	1	0.96	0.69	0.70	0.00	2.18	0.00	0.00	0.40
time (sec)	N/A	0.618	1.547	0.068	0.000	0.285	0.000	0.000	13.749

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	203	847	354	0	535	0	0	177
N.S.	1	1.09	4.55	1.90	0.00	2.88	0.00	0.00	0.95
time (sec)	N/A	1.096	6.159	0.363	0.000	0.282	0.000	0.000	14.778

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	160	179	380	339	0	487	0	0	143
N.S.	1	1.12	2.38	2.12	0.00	3.04	0.00	0.00	0.89
time (sec)	N/A	0.851	5.179	0.043	0.000	0.282	0.000	0.000	14.193

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	138	153	360	323	0	366	0	0	136
N.S.	1	1.11	2.61	2.34	0.00	2.65	0.00	0.00	0.99
time (sec)	N/A	0.665	3.428	0.054	0.000	0.274	0.000	0.000	13.523

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	127	342	309	0	349	0	0	100
N.S.	1	1.09	2.92	2.64	0.00	2.98	0.00	0.00	0.85
time (sec)	N/A	0.525	1.629	0.057	0.000	0.277	0.000	0.000	13.031

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	114	128	357	305	0	372	0	0	119
N.S.	1	1.12	3.13	2.68	0.00	3.26	0.00	0.00	1.04
time (sec)	N/A	0.550	3.125	0.043	0.000	0.277	0.000	0.000	12.453

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	132	599	303	0	378	0	0	101
N.S.	1	1.13	5.12	2.59	0.00	3.23	0.00	0.00	0.86
time (sec)	N/A	0.593	6.212	0.043	0.000	0.323	0.000	0.000	12.308

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F(-1)	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	141	162	415	323	0	485	0	0	126
N.S.	1	1.15	2.94	2.29	0.00	3.44	0.00	0.00	0.89
time (sec)	N/A	0.787	5.755	0.047	0.000	0.285	0.000	0.000	13.545

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	194	625	338	0	514	0	0	129
N.S.	1	1.18	3.79	2.05	0.00	3.12	0.00	0.00	0.78
time (sec)	N/A	0.933	6.309	0.090	0.000	0.293	0.000	0.000	14.208

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	115	296	312	0	400	0	0	123
N.S.	1	1.04	2.67	2.81	0.00	3.60	0.00	0.00	1.11
time (sec)	N/A	0.898	0.923	0.101	0.000	0.293	0.000	0.000	12.929

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	280	298	0	333	0	0	79
N.S.	1	1.00	3.22	3.43	0.00	3.83	0.00	0.00	0.91
time (sec)	N/A	0.655	0.549	0.046	0.000	0.275	0.000	0.000	12.784

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	85	283	304	0	331	0	0	102
N.S.	1	0.98	3.25	3.49	0.00	3.80	0.00	0.00	1.17
time (sec)	N/A	0.635	0.431	0.046	0.000	0.290	0.000	0.000	12.728

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	81	283	304	0	321	0	0	79
N.S.	1	0.98	3.41	3.66	0.00	3.87	0.00	0.00	0.95
time (sec)	N/A	0.625	0.516	0.042	0.000	0.282	0.000	0.000	12.775

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	118	229	319	0	472	0	0	123
N.S.	1	1.06	2.06	2.87	0.00	4.25	0.00	0.00	1.11
time (sec)	N/A	0.918	0.819	0.041	0.000	0.282	0.000	0.000	12.671

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	146	86	333	0	500	0	0	132
N.S.	1	1.08	0.64	2.47	0.00	3.70	0.00	0.00	0.98
time (sec)	N/A	1.224	0.622	0.042	0.000	0.279	0.000	0.000	13.312

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	254	288	191	0	1173	0	0	375
N.S.	1	0.90	1.02	0.68	0.00	4.17	0.00	0.00	1.33
time (sec)	N/A	1.198	2.376	0.324	0.000	0.297	0.000	0.000	13.298

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	244	155	197	0	1172	0	0	376
N.S.	1	0.87	0.56	0.71	0.00	4.20	0.00	0.00	1.35
time (sec)	N/A	1.216	1.220	0.040	0.000	0.296	0.000	0.000	13.435

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	249	221	200	0	1114	0	0	366
N.S.	1	0.90	0.79	0.72	0.00	4.01	0.00	0.00	1.32
time (sec)	N/A	1.127	2.197	0.041	0.000	0.330	0.000	0.000	13.129

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	249	131	197	0	1181	0	0	366
N.S.	1	0.89	0.47	0.70	0.00	4.20	0.00	0.00	1.30
time (sec)	N/A	1.212	1.598	0.045	0.000	0.301	0.000	0.000	13.137

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	288	227	212	0	1257	0	0	414
N.S.	1	0.94	0.74	0.69	0.00	4.11	0.00	0.00	1.35
time (sec)	N/A	1.572	1.174	0.042	0.000	0.315	0.000	0.000	13.241

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	316	84	227	0	1751	0	0	425
N.S.	1	0.95	0.25	0.69	0.00	5.29	0.00	0.00	1.28
time (sec)	N/A	1.949	1.025	0.043	0.000	0.326	0.000	0.000	13.570

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	179	390	349	0	567	0	0	154
N.S.	1	1.09	2.38	2.13	0.00	3.46	0.00	0.00	0.94
time (sec)	N/A	1.251	3.684	1.233	0.000	0.294	0.000	0.000	13.529

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	178	365	349	0	533	0	0	178
N.S.	1	1.09	2.23	2.13	0.00	3.25	0.00	0.00	1.09
time (sec)	N/A	1.332	4.128	0.042	0.000	0.295	0.000	0.000	13.267

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	175	443	349	0	518	0	0	151
N.S.	1	1.09	2.75	2.17	0.00	3.22	0.00	0.00	0.94
time (sec)	N/A	1.276	6.925	0.048	0.000	0.283	0.000	0.000	13.339

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	180	339	349	0	504	0	0	173
N.S.	1	1.09	2.05	2.12	0.00	3.05	0.00	0.00	1.05
time (sec)	N/A	1.338	3.388	0.056	0.000	0.286	0.000	0.000	13.412

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	209	325	364	0	697	0	0	175
N.S.	1	1.11	1.72	1.93	0.00	3.69	0.00	0.00	0.93
time (sec)	N/A	1.698	1.285	0.046	0.000	0.287	0.000	0.000	13.337

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	244	124	379	0	718	0	0	193
N.S.	1	1.13	0.58	1.76	0.00	3.34	0.00	0.00	0.90
time (sec)	N/A	2.102	1.109	0.050	0.000	0.303	0.000	0.000	13.533

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	233	69	356	0	180	0	0	119
N.S.	1	1.04	0.31	1.60	0.00	0.81	0.00	0.00	0.53
time (sec)	N/A	0.544	0.522	0.152	0.000	0.269	0.000	0.000	12.981

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	153	61	174	0	142	0	0	210
N.S.	1	1.13	0.45	1.29	0.00	1.05	0.00	0.00	1.56
time (sec)	N/A	0.531	0.319	0.043	0.000	0.265	0.000	0.000	12.260

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	168	75	197	0	202	0	0	254
N.S.	1	1.21	0.54	1.42	0.00	1.45	0.00	0.00	1.83
time (sec)	N/A	0.613	0.569	0.042	0.000	0.268	0.000	0.000	12.865

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	255	67	452	0	190	0	0	254
N.S.	1	1.15	0.30	2.05	0.00	0.86	0.00	0.00	1.15
time (sec)	N/A	0.596	0.440	0.035	0.000	0.262	0.000	0.000	12.595

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	243	67	442	0	178	0	0	238
N.S.	1	1.14	0.31	2.07	0.00	0.83	0.00	0.00	1.11
time (sec)	N/A	0.493	0.466	0.043	0.000	0.288	0.000	0.000	12.460

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	143	51	181	0	157	0	0	230
N.S.	1	1.18	0.42	1.50	0.00	1.30	0.00	0.00	1.90
time (sec)	N/A	0.399	0.258	0.092	0.000	0.253	0.000	0.000	12.872

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	156	62	173	0	198	0	0	208
N.S.	1	1.12	0.45	1.24	0.00	1.42	0.00	0.00	1.50
time (sec)	N/A	0.513	0.450	0.043	0.000	0.259	0.000	0.000	13.077

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	230	71	356	0	202	0	0	121
N.S.	1	1.02	0.31	1.58	0.00	0.89	0.00	0.00	0.54
time (sec)	N/A	0.503	0.581	0.039	0.000	0.259	0.000	0.000	12.836

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	170	62	195	0	238	0	0	242
N.S.	1	1.19	0.43	1.36	0.00	1.66	0.00	0.00	1.69
time (sec)	N/A	0.581	0.537	0.043	0.000	0.265	0.000	0.000	13.178

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	242	69	444	0	230	0	0	238
N.S.	1	1.12	0.32	2.06	0.00	1.06	0.00	0.00	1.10
time (sec)	N/A	0.496	0.452	0.032	0.000	0.275	0.000	0.000	13.103

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	240	68	303	0	843	0	0	153
N.S.	1	0.97	0.28	1.23	0.00	3.41	0.00	0.00	0.62
time (sec)	N/A	0.628	0.181	0.042	0.000	0.276	0.000	0.000	13.860

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	217	155	287	0	730	0	0	128
N.S.	1	0.96	0.69	1.27	0.00	3.23	0.00	0.00	0.57
time (sec)	N/A	0.502	0.315	0.030	0.000	0.274	0.000	0.000	13.492

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	197	166	273	0	721	0	0	118
N.S.	1	0.95	0.80	1.31	0.00	3.47	0.00	0.00	0.57
time (sec)	N/A	0.380	0.242	0.061	0.000	0.273	0.000	0.000	13.200

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	222	194	295	0	890	0	0	137
N.S.	1	0.97	0.85	1.29	0.00	3.89	0.00	0.00	0.60
time (sec)	N/A	0.518	0.382	0.032	0.000	0.289	0.000	0.000	13.314

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	244	196	311	0	905	0	0	158
N.S.	1	0.97	0.78	1.23	0.00	3.59	0.00	0.00	0.63
time (sec)	N/A	0.690	0.754	0.034	0.000	0.274	0.000	0.000	13.333

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	292	224	360	0	1313	0	0	1274
N.S.	1	0.92	0.71	1.14	0.00	4.14	0.00	0.00	4.02
time (sec)	N/A	0.854	2.168	0.033	0.000	0.298	0.000	0.000	15.244

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	261	220	321	0	1230	0	0	1157
N.S.	1	0.91	0.76	1.11	0.00	4.27	0.00	0.00	4.02
time (sec)	N/A	0.690	0.606	0.035	0.000	0.293	0.000	0.000	13.899

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	239	192	306	0	1183	0	0	1234
N.S.	1	0.90	0.72	1.15	0.00	4.43	0.00	0.00	4.62
time (sec)	N/A	0.549	0.943	0.048	0.000	0.286	0.000	0.000	13.425

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	242	188	301	0	1298	0	0	1196
N.S.	1	0.91	0.70	1.13	0.00	4.86	0.00	0.00	4.48
time (sec)	N/A	0.593	0.825	0.042	0.000	0.309	0.000	0.000	12.918

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	265	82	326	0	1318	0	0	1214
N.S.	1	0.91	0.28	1.12	0.00	4.53	0.00	0.00	4.17
time (sec)	N/A	0.746	0.316	0.042	0.000	0.315	0.000	0.000	14.232

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	301	85	347	0	1436	0	0	1227
N.S.	1	0.93	0.26	1.08	0.00	4.46	0.00	0.00	3.81
time (sec)	N/A	0.977	0.408	0.037	0.000	0.318	0.000	0.000	15.093

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	351	251	410	0	1795	0	0	2317
N.S.	1	0.94	0.67	1.10	0.00	4.83	0.00	0.00	6.23
time (sec)	N/A	1.268	2.984	0.041	0.000	0.344	0.000	0.000	18.064

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	317	247	371	0	1643	0	0	2071
N.S.	1	0.93	0.72	1.08	0.00	4.80	0.00	0.00	6.06
time (sec)	N/A	1.019	2.643	0.037	0.000	0.343	0.000	0.000	15.059

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	285	215	337	0	1633	0	0	1896
N.S.	1	0.91	0.69	1.08	0.00	5.22	0.00	0.00	6.06
time (sec)	N/A	0.822	1.068	0.039	0.000	0.414	0.000	0.000	13.685

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	287	193	332	0	1679	0	0	1951
N.S.	1	0.92	0.62	1.06	0.00	5.36	0.00	0.00	6.23
time (sec)	N/A	0.845	3.552	0.036	0.000	0.376	0.000	0.000	13.428

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	286	104	331	0	1692	0	0	1946
N.S.	1	0.91	0.33	1.06	0.00	5.41	0.00	0.00	6.22
time (sec)	N/A	0.859	0.425	0.034	0.000	0.370	0.000	0.000	14.342

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	324	108	359	0	1804	0	0	1969
N.S.	1	0.94	0.31	1.05	0.00	5.26	0.00	0.00	5.74
time (sec)	N/A	1.080	0.683	0.037	0.000	0.363	0.000	0.000	15.672

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	365	116	388	0	1839	0	0	1992
N.S.	1	0.97	0.31	1.03	0.00	4.88	0.00	0.00	5.28
time (sec)	N/A	1.365	0.802	0.043	0.000	0.341	0.000	0.000	18.021

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	292	286	347	0	3267	0	0	5579
N.S.	1	0.90	0.88	1.07	0.00	10.05	0.00	0.00	17.17
time (sec)	N/A	1.249	0.818	0.110	0.000	0.373	0.000	0.000	14.684

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	262	249	326	0	3137	0	0	5129
N.S.	1	0.87	0.82	1.08	0.00	10.39	0.00	0.00	16.98
time (sec)	N/A	0.984	0.512	0.043	0.000	0.384	0.000	0.000	14.514

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	260	226	332	0	3088	0	0	4808
N.S.	1	0.86	0.75	1.10	0.00	10.23	0.00	0.00	15.92
time (sec)	N/A	0.950	0.266	0.042	0.000	0.360	0.000	0.000	13.900

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	259	245	332	0	3160	0	0	4871
N.S.	1	0.86	0.81	1.10	0.00	10.46	0.00	0.00	16.13
time (sec)	N/A	0.958	0.312	0.048	0.000	0.381	0.000	0.000	13.881

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	295	198	354	0	3637	0	0	4899
N.S.	1	0.91	0.61	1.09	0.00	11.19	0.00	0.00	15.07
time (sec)	N/A	1.297	0.460	0.044	0.000	0.413	0.000	0.000	13.929

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	329	109	371	0	3742	0	0	6042
N.S.	1	0.94	0.31	1.06	0.00	10.66	0.00	0.00	17.21
time (sec)	N/A	1.687	0.291	0.042	0.000	0.475	0.000	0.000	14.960

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	397	461	409	0	6403	0	0	13244
N.S.	1	0.91	1.05	0.94	0.00	14.65	0.00	0.00	30.31
time (sec)	N/A	2.034	6.191	0.331	0.000	0.712	0.000	0.000	16.405

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	353	390	387	0	6258	0	0	12617
N.S.	1	0.90	0.99	0.98	0.00	15.92	0.00	0.00	32.10
time (sec)	N/A	1.476	3.046	0.302	0.000	0.586	0.000	0.000	15.910

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	343	321	391	0	6150	0	0	11953
N.S.	1	0.89	0.83	1.01	0.00	15.89	0.00	0.00	30.89
time (sec)	N/A	1.449	3.304	0.040	0.000	0.473	0.000	0.000	16.253

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	341	399	391	0	6104	0	0	11731
N.S.	1	0.88	1.03	1.01	0.00	15.81	0.00	0.00	30.39
time (sec)	N/A	1.398	6.112	0.044	0.000	0.452	0.000	0.000	15.694

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	355	295	396	0	6248	0	0	9400
N.S.	1	0.90	0.75	1.01	0.00	15.86	0.00	0.00	23.86
time (sec)	N/A	1.492	2.786	0.045	0.000	0.607	0.000	0.000	20.823

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	401	244	414	0	6519	0	0	15251
N.S.	1	0.92	0.56	0.95	0.00	14.92	0.00	0.00	34.90
time (sec)	N/A	2.089	0.615	0.048	0.000	0.723	0.000	0.000	16.753

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	494	626	471	0	9101	0	0	20651
N.S.	1	0.93	1.18	0.89	0.00	17.20	0.00	0.00	39.04
time (sec)	N/A	2.855	6.319	1.205	0.000	2.082	0.000	0.000	23.643

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	446	574	460	0	9029	0	0	20089
N.S.	1	0.94	1.21	0.97	0.00	18.97	0.00	0.00	42.20
time (sec)	N/A	2.131	6.245	1.161	0.000	63.174	0.000	0.000	20.486

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	470	440	520	457	0	8955	0	0	19256
N.S.	1	0.94	1.11	0.97	0.00	19.05	0.00	0.00	40.97
time (sec)	N/A	2.125	6.227	1.153	0.000	39.249	0.000	0.000	19.079

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	461	429	535	456	0	8913	0	0	19000
N.S.	1	0.93	1.16	0.99	0.00	19.33	0.00	0.00	41.21
time (sec)	N/A	2.122	6.195	0.051	0.000	0.776	0.000	0.000	19.029

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	431	481	460	0	8853	0	0	19534
N.S.	1	0.93	1.04	0.99	0.00	19.12	0.00	0.00	42.19
time (sec)	N/A	2.034	6.215	0.051	0.000	0.931	0.000	0.000	18.849

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	450	402	465	0	8991	0	0	20155
N.S.	1	0.95	0.84	0.98	0.00	18.89	0.00	0.00	42.34
time (sec)	N/A	2.111	6.166	0.051	0.000	1.580	0.000	0.000	19.447

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	499	303	480	0	10308	0	0	21158
N.S.	1	0.94	0.57	0.91	0.00	19.49	0.00	0.00	40.00
time (sec)	N/A	2.814	1.747	0.048	0.000	2.307	0.000	0.000	22.776

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	165	118	0	0	0	0	0	0
N.S.	1	0.99	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.368	0.360	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	193	214	0	0	0	0	0	0	0
N.S.	1	1.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.496	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	35	45	734	0	159	0	0	1410
N.S.	1	0.78	1.00	16.31	0.00	3.53	0.00	0.00	31.33
time (sec)	N/A	0.249	0.099	0.419	0.000	0.299	0.000	0.000	15.159

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	35	45	739	0	159	0	0	1410
N.S.	1	0.78	1.00	16.42	0.00	3.53	0.00	0.00	31.33
time (sec)	N/A	0.254	0.481	0.080	0.000	0.285	0.000	0.000	14.073

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	67	66	89	79	524	95	155
N.S.	1	1.00	1.14	1.12	1.51	1.34	8.88	1.61	2.63
time (sec)	N/A	0.350	0.177	0.081	0.430	0.273	0.527	0.310	0.797

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	122	144	147	185	340	3964	241	268
N.S.	1	1.10	1.30	1.32	1.67	3.06	35.71	2.17	2.41
time (sec)	N/A	0.545	2.132	0.125	0.437	0.296	1.615	0.353	14.088

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	197	202	216	337	549	0	412	481
N.S.	1	1.13	1.15	1.23	1.93	3.14	0.00	2.35	2.75
time (sec)	N/A	0.828	5.318	0.254	0.397	0.340	0.000	0.431	15.121

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	171	379	2392	0	5073	0	0	3864
N.S.	1	0.91	2.02	12.72	0.00	26.98	0.00	0.00	20.55
time (sec)	N/A	1.046	1.908	0.170	0.000	1.217	0.000	0.000	45.238

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	133	294	1657	0	3252	0	0	2823
N.S.	1	0.89	1.96	11.05	0.00	21.68	0.00	0.00	18.82
time (sec)	N/A	0.776	1.016	0.116	0.000	0.628	0.000	0.000	26.830

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	105	212	814	0	1329	0	0	843
N.S.	1	0.86	1.74	6.67	0.00	10.89	0.00	0.00	6.91
time (sec)	N/A	0.582	0.662	0.146	0.000	0.326	0.000	0.000	15.521

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	123	222	1375	0	1684	0	0	3442
N.S.	1	0.81	1.47	9.11	0.00	11.15	0.00	0.00	22.79
time (sec)	N/A	0.680	1.208	0.079	0.000	0.312	0.000	0.000	40.088

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	432	178	986	0	1148	0	0	2529
N.S.	1	1.06	0.44	2.42	0.00	2.81	0.00	0.00	6.20
time (sec)	N/A	0.714	6.621	0.069	0.000	0.290	0.000	0.000	25.166

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	453	153	816	0	849	0	0	583
N.S.	1	1.07	0.36	1.93	0.00	2.01	0.00	0.00	1.38
time (sec)	N/A	0.747	0.753	0.062	0.000	0.305	0.000	0.000	14.202

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	85	154	1890	0	1773	0	0	2909
N.S.	1	0.83	1.51	18.53	0.00	17.38	0.00	0.00	28.52
time (sec)	N/A	0.442	2.148	0.086	0.000	0.324	0.000	0.000	15.288

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	146	226	3683	0	4572	0	0	5737
N.S.	1	1.06	1.64	26.69	0.00	33.13	0.00	0.00	41.57
time (sec)	N/A	0.693	1.815	0.149	0.000	0.804	0.000	0.000	19.294

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	208	319	4472	0	7422	0	0	9453
N.S.	1	1.12	1.72	24.17	0.00	40.12	0.00	0.00	51.10
time (sec)	N/A	0.989	3.760	0.157	0.000	3.599	0.000	0.000	31.324

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	83	137	1891	0	1219	0	0	2731
N.S.	1	0.81	1.34	18.54	0.00	11.95	0.00	0.00	26.77
time (sec)	N/A	0.426	0.480	0.076	0.000	0.313	0.000	0.000	14.765

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	131	190	2291	0	2574	0	0	5475
N.S.	1	0.99	1.44	17.36	0.00	19.50	0.00	0.00	41.48
time (sec)	N/A	0.639	1.553	0.074	0.000	0.358	0.000	0.000	19.009

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	184	253	3055	0	3922	0	0	8438
N.S.	1	1.06	1.45	17.56	0.00	22.54	0.00	0.00	48.49
time (sec)	N/A	0.914	3.163	0.117	0.000	0.419	0.000	0.000	29.371

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [44] had the largest ratio of [1.363639999999996]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.08	15	0.200
2	A	10	9	1.06	23	0.391
3	A	8	7	1.07	23	0.304
4	A	6	5	1.04	23	0.217
5	A	4	3	1.08	23	0.130
6	A	6	5	1.04	23	0.217
7	A	9	8	1.09	23	0.348
8	A	20	19	0.97	25	0.760
9	A	18	17	0.94	25	0.680
10	A	18	17	0.96	25	0.680
11	A	16	15	0.90	25	0.600
12	A	15	14	0.93	25	0.560
13	A	17	16	0.94	25	0.640
14	A	17	16	0.96	25	0.640
15	A	15	14	1.09	25	0.560
16	A	13	12	1.12	25	0.480
17	A	11	10	1.11	25	0.400
18	A	9	8	1.09	25	0.320
19	A	9	8	1.12	25	0.320
20	A	10	9	1.13	25	0.360
21	A	13	12	1.15	25	0.480
22	A	15	14	1.18	25	0.560

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	13	12	1.04	25	0.480
24	A	11	10	1.00	25	0.400
25	A	10	9	0.98	25	0.360
26	A	10	9	0.98	25	0.360
27	A	13	12	1.06	25	0.480
28	A	18	17	1.08	25	0.680
29	A	22	21	0.90	25	0.840
30	A	23	22	0.87	25	0.880
31	A	22	21	0.90	25	0.840
32	A	23	22	0.89	25	0.880
33	A	25	24	0.94	25	0.960
34	A	29	28	0.95	25	1.120
35	A	16	15	1.09	25	0.600
36	A	17	16	1.09	25	0.640
37	A	17	16	1.09	25	0.640
38	A	17	16	1.09	25	0.640
39	A	20	19	1.11	25	0.760
40	A	23	22	1.13	25	0.880
41	A	14	13	1.04	13	1.000
42	A	11	10	1.13	11	0.909
43	A	15	14	1.21	13	1.077
44	A	16	15	1.15	11	1.364
45	A	13	12	1.14	13	0.923
46	A	9	8	1.18	11	0.727
47	A	11	10	1.12	13	0.769
48	A	15	14	1.02	11	1.273
49	A	15	14	1.19	13	1.077
50	A	14	13	1.12	11	1.182
51	A	16	15	0.97	23	0.652
52	A	14	13	0.96	23	0.565
53	A	12	11	0.95	23	0.478
54	A	15	14	0.97	23	0.609
55	A	18	17	0.97	23	0.739
56	A	18	17	0.92	25	0.680

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	16	15	0.91	25	0.600
58	A	14	13	0.90	25	0.520
59	A	15	14	0.91	25	0.560
60	A	18	17	0.91	25	0.680
61	A	20	19	0.93	25	0.760
62	A	21	20	0.94	25	0.800
63	A	19	18	0.93	25	0.720
64	A	17	16	0.91	25	0.640
65	A	18	17	0.92	25	0.680
66	A	19	18	0.91	25	0.720
67	A	21	20	0.94	25	0.800
68	A	24	23	0.97	25	0.920
69	A	21	20	0.90	25	0.800
70	A	19	18	0.87	25	0.720
71	A	18	17	0.86	25	0.680
72	A	17	16	0.86	25	0.640
73	A	21	20	0.91	25	0.800
74	A	24	23	0.94	25	0.920
75	A	25	24	0.91	25	0.960
76	A	22	21	0.90	25	0.840
77	A	22	21	0.89	25	0.840
78	A	22	21	0.88	25	0.840
79	A	22	21	0.90	25	0.840
80	A	25	24	0.92	25	0.960
81	A	28	27	0.93	25	1.080
82	A	25	24	0.94	25	0.960
83	A	25	24	0.94	25	0.960
84	A	25	24	0.93	25	0.960
85	A	25	24	0.93	25	0.960
86	A	25	24	0.95	25	0.960
87	A	28	27	0.94	25	1.080
88	A	5	4	0.99	12	0.333
89	A	7	6	1.11	23	0.261
90	A	6	5	0.78	27	0.185

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	6	5	0.78	27	0.185
92	A	5	5	1.00	23	0.217
93	A	7	7	1.10	23	0.304
94	A	9	9	1.13	23	0.391
95	A	14	13	0.91	25	0.520
96	A	12	11	0.89	25	0.440
97	A	10	9	0.86	25	0.360
98	A	13	12	0.81	27	0.444
99	A	14	13	1.06	27	0.481
100	A	14	13	1.07	27	0.481
101	A	8	7	0.83	25	0.280
102	A	10	9	1.06	25	0.360
103	A	12	11	1.12	25	0.440
104	A	8	7	0.81	27	0.259
105	A	11	10	0.99	27	0.370
106	A	13	12	1.06	27	0.444

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int (a + ia\cot(c + dx))^n \, dx$

3.1.1 Optimal result

Integrand size = 15, antiderivative size = 49

$$\int (a + ia\cot(c + dx))^n \, dx = \frac{i(a + ia\cot(c + dx))^n \operatorname{Hypergeometric2F1}\left(1, n, 1 + n, \frac{1}{2}(1 + i\cot(c + dx))\right)}{2dn}$$

output
```
1/2*I*(a+I*a*cot(d*x+c))^n*hypergeom([1, n],[1+n],1/2+1/2*I*cot(d*x+c))/d/
n
```

3.1.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 117 vs. 2(49) = 98.

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.39

$$\int (a + ia\cot(c + dx))^n \, dx = \frac{i(a + ia\cot(c + dx))^n \left(2(1 + n) \operatorname{Hypergeometric2F1}(1, n, 1 + n, 1 + i\cot(c + dx)) + (n + in\cot(c + dx)\right)}{}$$

input
```
Integrate[(a + I*a*Cot[c + d*x])^n,x]
```

output
```
((I/4)*(a + I*a*Cot[c + d*x])^n*(2*(1 + n)*Hypergeometric2F1[1, n, 1 + n,
1 + I*Cot[c + d*x]] + (n + I*n*Cot[c + d*x])*(Hypergeometric2F1[1, 1 + n,
2 + n, (1 + I*Cot[c + d*x])/2] - 2*Hypergeometric2F1[1, 1 + n, 2 + n, 1 +
I*Cot[c + d*x]])))/(d*n*(1 + n))
```

3.1.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3962, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\cot(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-ia\tan\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3962}$$

$$\frac{ia\int \frac{(i\cot(c+dx)a+a)^{n-1}}{a-ia\cot(c+dx)}d(ia\cot(c+dx))}{d}$$

$$\Big\downarrow \text{78}$$

$$\frac{i(a+ia\cot(c+dx))^n \text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\cot(c+dx)a+a}{2a}\right)}{2dn}$$

input
```
Int[(a + I*a*Cot[c + d*x])^n,x]
```

output
```
((I/2)*(a + I*a*Cot[c + d*x])^n*Hypergeometric2F1[1, n, 1 + n, (a + I*a*Co
t[c + d*x])/(2*a)])/(d*n)
```

3.1.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.1.4 Maple [F]

$$\int (a + ia \cot (dx + c))^n \, dx$$

input
```
int((a+I*a*cot(d*x+c))^n,x)
```

output
```
int((a+I*a*cot(d*x+c))^n,x)
```

3.1.5 Fricas [F]

$$\int (a + ia \cot(c + dx))^n \, dx = \int (i\, a \cot (dx + c) + a)^n \, dx$$

input
```
integrate((a+I*a*cot(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((-2*a/(e^(2*I*d*x + 2*I*c) - 1))^n, x)
```

3.1.6 Sympy [F]

$$\int (a+ia\cot(c+dx))^n\,dx=\int (ia\cot(c+dx)+a)^n\,dx$$

input
```
integrate((a+I*a*cot(d*x+c))**n,x)
```

output
```
Integral((I*a*cot(c + d*x) + a)**n, x)
```

3.1.7 Maxima [F]

$$\int (a+ia\cot(c+dx))^n\,dx=\int (i\,a\cot(dx+c)+a)^n\,dx$$

input
```
integrate((a+I*a*cot(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*cot(d*x + c) + a)^n, x)
```

3.1.8 Giac [F]

$$\int (a+ia\cot(c+dx))^n\,dx=\int (i\,a\cot(dx+c)+a)^n\,dx$$

input
```
integrate((a+I*a*cot(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*cot(d*x + c) + a)^n, x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int (a+ia\cot(c+dx))^n\,dx = \int (a+a\cot(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int((a + a*cot(c + d*x)*1i)^n,x)
```

output
```
int((a + a*cot(c + d*x)*1i)^n, x)
```

3.2 $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx$

3.2.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = -\frac{\sqrt{2}ae^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} + \frac{2ae^2\sqrt{e\cot(c+dx)}}{d} - \frac{2ae(e\cot(c+dx))^{3/2}}{3d} - \frac{2a(e\cot(c+dx))^{5/2}}{5d}$$

output
```
-2/3*a*e*(e*cot(d*x+c))^(3/2)/d-2/5*a*(e*cot(d*x+c))^(5/2)/d-a*e^(5/2)*arc
tanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2
)/d+2*a*e^2*(e*cot(d*x+c))^(1/2)/d
```

3.2.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.59

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = -\frac{2ae(e\cot(c+dx))^{3/2}\left(3\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\tan^2(c+dx)\right)+5\,\text{Hypergeometric}\right.}{15d}$$

input
```
Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x]),x]
```

output
```
(-2*a*e*(e*Cot[c + d*x])^(3/2)*(3*Cot[c + d*x]*Hypergeometric2F1[-5/4, 1,
-1/4, -Tan[c + d*x]^2] + 5*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2
]))/(15*d)
```

3.2.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4011, 3042, 4011, 3042, 4011, 3042, 4015, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(c+dx)+a)(e\cot(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4011}$$

$$\int (e\cot(c+dx))^{3/2}(ae\cot(c+dx)-ae)dx-\frac{2a(e\cot(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(-ae-a\tan\left(c+dx+\frac{\pi}{2}\right)e\right)dx-\frac{2a(e\cot(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{4011}$$

$$\int \sqrt{e\cot(c+dx)}\left(-ae^2-a\cot(c+dx)e^2\right)dx-\frac{2a(e\cot(c+dx))^{5/2}}{5d}-\frac{2ae(e\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(ae^2\tan\left(c+dx+\frac{\pi}{2}\right)-ae^2\right)dx-\frac{2a(e\cot(c+dx))^{5/2}}{5d}-\frac{2ae(e\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{ae^3-ae^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx+\frac{2ae^2\sqrt{e\cot(c+dx)}}{d}-\frac{2a(e\cot(c+dx))^{5/2}}{5d}-\frac{2ae(e\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{ae^3+a\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ae^2\sqrt{e\cot(c+dx)}}{d}-\frac{2a(e\cot(c+dx))^{5/2}}{5d}-\frac{2ae(e\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4015}$$

$$-\frac{2a^2e^6\int\frac{1}{2a^2e^6-(ae^3+a\cot(c+dx)e^3)^2\tan(c+dx)}d\frac{ae^3+a\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}}}{d}+\frac{2ae^2\sqrt{e\cot(c+dx)}}{d}-\frac{2ae(e\cot(c+dx))^{3/2}}{3d}-\frac{2a(e\cot(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{2}ae^{5/2}\text{arctanh}\left(\frac{ae^3\cot(c+dx)+ae^3}{\sqrt{2}ae^{5/2}\sqrt{e\cot(c+dx)}}\right)}{d}+\frac{2ae^2\sqrt{e\cot(c+dx)}}{d}-\frac{2ae(e\cot(c+dx))^{3/2}}{3d}-\frac{2a(e\cot(c+dx))^{5/2}}{5d}$$

input
```
Int[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x]),x]
```

output
```
-((Sqrt[2]*a*e^(5/2)*ArcTanh[(a*e^3 + a*e^3*Cot[c + d*x])/(Sqrt[2]*a*e^(5/
2)*Sqrt[e*Cot[c + d*x]])])/d) + (2*a*e^2*Sqrt[e*Cot[c + d*x]])/d - (2*a*e*
(e*Cot[c + d*x])^(3/2))/(3*d) - (2*a*(e*Cot[c + d*x])^(5/2))/(5*d)
```

3.2.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

3.2.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(95) = 190$.

Time = 0.16 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.75

method	result
derivativedivides	$-\frac{a\left(\frac{2(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e(e\cot(dx+c))^{\frac{3}{2}}}{3}-2\sqrt{e\cot(dx+c)}\,e^2+2e^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{\ldots}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{\ldots}}\right.\ldots}{\ldots}\right.\ldots\right)}{\ldots}$
default	$-\frac{a\left(\frac{2(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e(e\cot(dx+c))^{\frac{3}{2}}}{3}-2\sqrt{e\cot(dx+c)}\,e^2+2e^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{\ldots}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{\ldots}}\right.\ldots}{\ldots}\right.\ldots\right)}{\ldots}$
parts	$-\frac{2ae\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^2\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\ldots\right.\right.}{8(e^2)^{\frac{1}{4}}}\ldots\right)}{d}$

input
```
int((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d*a*(2/5*(e*cot(d*x+c))^(5/2)+2/3*e*(e*cot(d*x+c))^(3/2)-2*(e*cot(d*x+c
))^(1/2)*e^2+2*e^3*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1
/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*
2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2
)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arc
tan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.25

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = \left[\frac{15\sqrt{2}(ae^2\cos(2\,dx+2\,c)-ae^2)\sqrt{e}\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin\right.}{}\right.$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(2)*(a*e^2*cos(2*d*x + 2*c) - a*e^2)*sqrt(e)*log(sqrt(2)*sqr
t(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - s
in(2*d*x + 2*c) - 1) + 2*e*sin(2*d*x + 2*c) + e) + 4*(18*a*e^2*cos(2*d*x +
 2*c) + 5*a*e^2*sin(2*d*x + 2*c) - 12*a*e^2)*sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c) - d), 1/15*(15*sqrt(2)*(a*e^2*cos(
2*d*x + 2*c) - a*e^2)*sqrt(-e)*arctan(1/2*sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d
*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)
/(e*cos(2*d*x + 2*c) + e)) + 2*(18*a*e^2*cos(2*d*x + 2*c) + 5*a*e^2*sin(2*
d*x + 2*c) - 12*a*e^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d
*cos(2*d*x + 2*c) - d)]
```

3.2.6 Sympy [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = a\left(\int (e\cot(c+dx))^{\frac{5}{2}}\,dx + \int (e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)\,dx\right)$$

input
```
integrate((e*cot(d*x+c))**(5/2)*(a+a*cot(d*x+c)),x)
```

output
```
a*(Integral((e*cot(c + d*x))**(5/2), x) + Integral((e*cot(c + d*x))**(5/2)
*cot(c + d*x), x))
```

3.2.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.2.8 Giac [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = \int (a\cot(dx+c)+a)(e\cot(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)*(e*cot(d*x + c))^(5/2), x)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.24

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx = \frac{2\,a\,e^2\,\sqrt{e\cot(c+d\,x)}}{d} - \frac{2\,a\,e\,(e\cot(c+d\,x))^{3/2}}{3\,d} - \frac{2\,a\,(e\cot(c+d\,x))^{5/2}}{5\,d} + \frac{(-1)^{1/4}\,a\,e^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}\,1\mathrm{i}}{\sqrt{e}}\right)}{d} - \frac{(-1)^{1/4}\,a\,e^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d} + \frac{(-1)^{1/4}\,a\,e^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(1+1\mathrm{i})}{d}$$

input
```
int((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x)),x)
```

output
```
(2*a*e^2*(e*cot(c + d*x))^(1/2))/d - (2*a*e*(e*cot(c + d*x))^(3/2))/(3*d)
- (2*a*(e*cot(c + d*x))^(5/2))/(5*d) + ((-1)^(1/4)*a*e^(5/2)*atan(((-1)^(1
/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1 + 1i))/d + ((-1)^(1/4)*a*e^(5/2)*a
tan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2)*1i)/e^(1/2)))/d - ((-1)^(1/4)*a*e^(
5/2)*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/d
```

3.2. $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))\,dx$

3.3 $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx$

3.3.1 Optimal result

Integrand size = 23, antiderivative size = 94

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = -\frac{\sqrt{2}ae^{3/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} - \frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2a(e\cot(c+dx))^{3/2}}{3d}$$

output
```
-2/3*a*(e*cot(d*x+c))^(3/2)/d-a*e^(3/2)*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(
1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d-2*a*e*(e*cot(d*x+c))^(1/2)/d
```

3.3.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = -\frac{2ae\sqrt{e\cot(c+dx)}\left(\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\tan^2(c+dx)\right)+3\,\text{Hypergeometric2F1}\left(-\right.\right.}{3d}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x]),x]
```

output
```
(-2*a*e*Sqrt[e*Cot[c + d*x]]*(Cot[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4,
 -Tan[c + d*x]^2] + 3*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2]))/(
3*d)
```

3.3.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4011, 3042, 4011, 3042, 4015, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(c+dx)+a)(e\cot(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4011}$$

$$\int \sqrt{e\cot(c+dx)}(ae\cot(c+dx)-ae)dx-\frac{2a(e\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-ae-a\tan\left(c+dx+\frac{\pi}{2}\right)e\right)dx-\frac{2a(e\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{-ae^2-a\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx-\frac{2a(e\cot(c+dx))^{3/2}}{3d}-\frac{2ae\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{ae^2\tan\left(c+dx+\frac{\pi}{2}\right)-ae^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a(e\cot(c+dx))^{3/2}}{3d}-\frac{2ae\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{4015}$$

$$-\frac{2a^2e^4\int \frac{1}{-2a^2e^4-(ae^2-ae^2\cot(c+dx))^2\tan(c+dx)}d\left(-\frac{ae^2-ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2a(e\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\sqrt{2}ae^{3/2}\arctan\left(\frac{ae^2-ae^2\cot(c+dx)}{\sqrt{2}ae^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2a(e\cot(c+dx))^{3/2}}{3d}$$

input
```
Int[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x]),x]
```

output
```
-((Sqrt[2]*a*e^(3/2)*ArcTan[(a*e^2 - a*e^2*Cot[c + d*x])/(Sqrt[2]*a*e^(3/2
)*Sqrt[e*Cot[c + d*x]])])/d) - (2*a*e*Sqrt[e*Cot[c + d*x]])/d - (2*a*(e*Co
t[c + d*x])^(3/2))/(3*d)
```

3.3.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

3.3.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(77) = 154$.

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.22

method	result
parts	$-\frac{2ae\left(\sqrt{e\cot(dx+c)}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan \cdots}{8}\right.}{d}$
derivativedivides	$-\frac{a\left(\frac{2(e\cot(dx+c))^{\frac{3}{2}}}{3}+2e\sqrt{e\cot(dx+c)}-2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan \cdots}{8e}\right.\right.}{}$
default	$-\frac{a\left(\frac{2(e\cot(dx+c))^{\frac{3}{2}}}{3}+2e\sqrt{e\cot(dx+c)}-2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan \cdots}{8e}\right.\right.}{}$

input
```
int((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*a/d*e*((e*cot(d*x+c))^(1/2)-1/8*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)+1)))+a/d*(-2/3*(e*cot(d*x+c))^(3/2)+1/4*e^2/(e^2)^(1/4)*2^(1/2)*(ln((e
*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d
*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1
/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)+1)))
```

3.3.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.55

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = \left[\frac{3\sqrt{2}a\sqrt{-e}e\log\left(\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)-1)-2\,e\right.}{} \right.$$

$$-\frac{3\sqrt{2}ae^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)+1)}{2\,(e\cos(2\,dx+2\,c)+e)}\right)\sin(2\,dx+2\,c)+2\,(ae\cos(2\,dx+2\,c)+}{3\,d\sin(2\,dx+2\,c)}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(2)*a*sqrt(-e)*e*log(sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) - 1) - 2*e*si
n(2*d*x + 2*c) + e)*sin(2*d*x + 2*c) - 4*(a*e*cos(2*d*x + 2*c) + 3*a*e*sin
(2*d*x + 2*c) + a*e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*s
in(2*d*x + 2*c)), -1/3*(3*sqrt(2)*a*e^(3/2)*arctan(-1/2*sqrt(2)*sqrt(e)*sq
rt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*
x + 2*c) + 1)/(e*cos(2*d*x + 2*c) + e))*sin(2*d*x + 2*c) + 2*(a*e*cos(2*d*
x + 2*c) + 3*a*e*sin(2*d*x + 2*c) + a*e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c)))/(d*sin(2*d*x + 2*c))]
```

3.3.6 Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = a\left(\int (e\cot(c+dx))^{\frac{3}{2}}\,dx + \int (e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)\,dx\right)$$

input
```
integrate((e*cot(d*x+c))**(3/2)*(a+a*cot(d*x+c)),x)
```

output
```
a*(Integral((e*cot(c + d*x))**(3/2), x) + Integral((e*cot(c + d*x))**(3/2)
*cot(c + d*x), x))
```

3.3.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.3.8 Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = \int (a\cot(dx+c)+a)(e\cot(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)*(e*cot(d*x + c))^(3/2), x)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx = -\frac{2\,a\,(e\cot(c+d\,x))^{3/2}}{3\,d} - \frac{2\,a\,e\,\sqrt{e\cot(c+d\,x)}}{d} + \frac{(-1)^{1/4}\,a\,e^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(1-\mathrm{i})}{d} + \frac{(-1)^{1/4}\,a\,e^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,(-1-\mathrm{i})}{d}$$

input
```
int((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x)),x)
```

output
```
((-1)^(1/4)*a*e^(3/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1
 - 1i))/d - (2*a*e*(e*cot(c + d*x))^(1/2))/d - (2*a*(e*cot(c + d*x))^(3/2)
)/(3*d) - ((-1)^(1/4)*a*e^(3/2)*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/
e^(1/2))*(1 + 1i))/d
```

3.3. $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))\,dx$

3.4 $\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx$

3.4.1 Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx=\frac{\sqrt{2}a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2a\sqrt{e\cot(c+dx)}}{d}$$

output
```
a*arctanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2
^(1/2)*e^(1/2)/d-2*a*(e*cot(d*x+c))^(1/2)/d
```

3.4.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.17

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx = -\frac{a\sqrt{e\cot(c+dx)}\left(8\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\tan^2(c+dx)\right)+\sqrt{2}\left(2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x]),x]
```

output
```
-1/4*(a*Sqrt[e*Cot[c + d*x]]*(8*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d
*x]^2] + Sqrt[2]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c +
d*x]]))/d
```

3.4.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4011, 3042, 4015, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(c+dx)+a)\sqrt{e\cot(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{ae\cot(c+dx)-ae}{\sqrt{e\cot(c+dx)}}dx-\frac{2a\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-ae-a\tan\left(c+dx+\frac{\pi}{2}\right)e}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{4015}$$

$$-\frac{2a^2e^2\int \frac{1}{2a^2e^2-(ae+a\cot(c+dx)e)^2\tan(c+dx)}d\left(-\frac{ae+a\cot(c+dx)e}{\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2a\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{2}a\sqrt{e}\text{arctanh}\left(\frac{ae\cot(c+dx)+ae}{\sqrt{2}a\sqrt{e}\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2a\sqrt{e\cot(c+dx)}}{d}$$

input
```
Int[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x]),x]
```

output
```
(Sqrt[2]*a*Sqrt[e]*ArcTanh[(a*e + a*e*Cot[c + d*x])/(Sqrt[2]*a*Sqrt[e]*Sqr
t[e*Cot[c + d*x]])])/d - (2*a*Sqrt[e*Cot[c + d*x]])/d
```

3.4.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

3.4.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(58) = 116$.

Time = 0.05 (sec) , antiderivative size = 287, normalized size of antiderivative = 4.04

method	result
parts	$-\frac{ae\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right.\right.}{4d\left(e^2\right)^{\frac{1}{4}}}$
derivativedivides	$-\frac{a\left(2\sqrt{e\cot(dx+c)}-2e\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arc}\cdots\right.}{8e}\right.\right.}{\cdots}$
default	$-\frac{a\left(2\sqrt{e\cot(dx+c)}-2e\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arc}\cdots\right.}{8e}\right.\right.}{\cdots}$

input
```
int((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/d*e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+a/d*(-2*(e*cot(d
*x+c))^(1/2)+1/4*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

3.4.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(58) = 116$.

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.32

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx$$
$$= \left[\frac{\sqrt{2}a\sqrt{e}\log\left(-\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)-1)+2\,e\sin(2\,dx+2\,c)+e\right)-}{2\,d}\right.$$
$$\left.-\frac{\sqrt{2}a\sqrt{-e}\arctan\left(\frac{\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)+1)}{2\,(e\cos(2\,dx+2\,c)+e)}\right)+2\,a\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{d}\right]$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*a*sqrt(e)*log(-sqrt(2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1) + 2*e*sin(2*d
*x + 2*c) + e) - 4*a*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/d, -
(sqrt(2)*a*sqrt(-e)*arctan(1/2*sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) +
 e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(e*cos(2*d
*x + 2*c) + e)) + 2*a*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/d]
```

3.4.6 Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx = a\left(\int \sqrt{e\cot{(c+dx)}}\,dx + \int \sqrt{e\cot{(c+dx)}}\cot{(c+dx)}\,dx\right)$$

input
```
integrate((e*cot(d*x+c))**(1/2)*(a+a*cot(d*x+c)),x)
```

output
```
a*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(sqrt(e*cot(c + d*x))*cot(c
 + d*x), x))
```

3.4.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.4.8 Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx = \int (a\cot(dx+c)+a)\sqrt{e\cot(dx+c)}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)*sqrt(e*cot(d*x + c)), x)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.80

$$\begin{aligned}
&\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx \\
&= -\frac{2a\sqrt{e\cot(c+d\,x)}}{d} \\
&\quad - \frac{(-1)^{1/4}\,a\sqrt{e}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\right)}{d} \\
&\quad - \frac{(-1)^{1/4}\,a\sqrt{e}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d} - \frac{(-1)^{1/4}\,a\sqrt{e}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d}
\end{aligned}$$

input
```
int((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x)),x)
```

output
```
- (2*a*(e*cot(c + d*x))^(1/2))/d - ((-1)^(1/4)*a*e^(1/2)*atan(((-1)^(1/4)*
(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*a*e^(1/2)*atanh(((-1)
^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*a*e^(1/2)*(ata
n(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)) - atanh(((-1)^(1/4)*(e*cot(
c + d*x))^(1/2))/e^(1/2))))/d
```

3.4. $\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))\,dx$

3.5 $\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

3.5.1 Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx=\frac{\sqrt{2}a\arctan\left(\frac{\sqrt{e}(1-\cot(c+dx))}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}}$$

output
```
a*arctan(1/2*(1-cot(d*x+c))*e^(1/2)*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/
d/e^(1/2)
```

3.5.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.37

$$\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$$
$$=\frac{a\left(3\sqrt{2}\left(-2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\tan(c+\right.\right.\right.}{12d}$$

input
```
Integrate[(a + a*Cot[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

output
```
(a*(3*Sqrt[2]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d
*x]] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) + 8*Hypergeomet
ric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(3/2)))/(12*d*Sqrt[e*Cot
[c + d*x]]*Sqrt[Tan[c + d*x]])
```

3.5.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4015, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cot(c+dx)+a}{\sqrt{e\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-a\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4015}$$

$$-\frac{2a^2\int \frac{1}{-2a^2-(a-a\cot(c+dx))^2\tan(c+dx)}d\frac{a-a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{2}a\arctan\left(\frac{\sqrt{e}(a-a\cot(c+dx))}{\sqrt{2}a\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}}$$

input
```
Int[(a + a*Cot[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

output
```
(Sqrt[2]*a*ArcTan[(Sqrt[e]*(a - a*Cot[c + d*x]))/(Sqrt[2]*a*Sqrt[e*Cot[c +
 d*x]])])/(d*Sqrt[e])
```

3.5.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

3.5.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(40) = 80$.

Time = 0.11 (sec) , antiderivative size = 273, normalized size of antiderivative = 5.57

method	result
derivativedivides	$-\frac{a\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
default	$-\frac{a\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
parts	$-\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4de}$

input
```
int((a+a*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*a*(1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/4/(e^2)^(1/
4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2
)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2
)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)
/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(38) = 76$.

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.51

$$\int \frac{a + a\cot(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx$$
$$= \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{e}}\log\left(-\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sqrt{-\frac{1}{e}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)-1)-2\,\sin(2\,dx+2\,c)+\right.}{2\,d}\right.$$

input
```
integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*a*sqrt(-1/e)*log(-sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2
*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) - 1) - 2*sin(
2*d*x + 2*c) + 1)/d, sqrt(2)*a*arctan(-1/2*sqrt(2)*sqrt((e*cos(2*d*x + 2*c
) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(sqrt(e
)*(cos(2*d*x + 2*c) + 1)))/(d*sqrt(e))]
```

3.5.6 Sympy [F]

$$\int \frac{a + a\cot(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx = a\left(\int \frac{1}{\sqrt{e\cot(c + dx)}}\,dx + \int \frac{\cot(c + dx)}{\sqrt{e\cot(c + dx)}}\,dx\right)$$

input
```
integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

output
```
a*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(cot(c + d*x)/sqrt(e*cot(
c + d*x)), x))
```

3.5.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.5.8 Giac [F]

$$\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{a\cot(dx+c)+a}{\sqrt{e\cot(dx+c)}}\,dx$$

input
```
integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)/sqrt(e*cot(d*x + c)), x)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 13.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)(-1+1\mathrm{i})}{d\sqrt{e}} + \frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)(1+1\mathrm{i})}{d\sqrt{e}}$$

input
```
int((a + a*cot(c + d*x))/(e*cot(c + d*x))^(1/2),x)
```

output
```
((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1 + 1i))
/(d*e^(1/2)) - ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1
/2))*(1 - 1i))/(d*e^(1/2))
```

3.5. $\int \frac{a+a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

3.6 $\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

3.6.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = -\frac{\sqrt{2}a\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}}+\frac{2a}{de\sqrt{e\cot(c+dx)}}$$

output
```
-a*arctanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*
2^(1/2)/d/e^(3/2)+2*a/d/e/(e*cot(d*x+c))^(1/2)
```

3.6.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.55

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{a\left(6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-6\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\right.}{}$$

input
```
Integrate[(a + a*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

output
```
(a*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*ArcTan[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]]] + 3*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] - 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]] + 24*Sqrt[Tan[c + d*x]] + 8*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c
+ d*x]^2]*Tan[c + d*x]^(3/2)))/(12*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(
3/2))
```

3.6.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4012, 3042, 4015, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cot(c+dx)+a}{(e\cot(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-a\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int \frac{ae-ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{2a}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{ae+a\tan\left(c+dx+\frac{\pi}{2}\right)e}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}+\frac{2a}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{4015}$$

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{2a^2\int \frac{1}{2a^2e^2-(ae+a\cot(c+dx)e)^2\tan(c+dx)}d\frac{ae+a\cot(c+dx)e}{\sqrt{e\cot(c+dx)}}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{\sqrt{2}a\text{arctanh}\left(\frac{ae\cot(c+dx)+ae}{\sqrt{2}a\sqrt{e}\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}}$$

input
```
Int[(a + a*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

output
```
-((Sqrt[2]*a*ArcTanh[(a*e + a*e*Cot[c + d*x])/(Sqrt[2]*a*Sqrt[e]*Sqrt[e*Co
t[c + d*x]])])/(d*e^(3/2))) + (2*a)/(d*e*Sqrt[e*Cot[c + d*x]])
```

3.6.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

3.6.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(62) = 124$.

Time = 0.05 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.92

method	result
derivativedivides	$-\frac{a\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
default	$-\frac{a\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
parts	$-\frac{2ae\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+\right.\right.}{8e^2(e^2)^{\frac{1}{4}}}\right.}{d}$

input
```
int((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*a*(2/e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e^2
)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)
+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)
^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^
(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-2/e/(e*cot(d*x+c))^(1/2))
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(62) = 124$.

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 4.28

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \left[\frac{4\,a\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sin\left(2\,dx+2\,c\right)+\frac{\sqrt{2}(ae\cos(2\,dx+2\,c)+ae)\log\left(\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\ldots}{\sqrt{e}}\ldots\right.}{\sqrt{e}}\ldots}{2\left(de^2\cos\left(2\,dx+2\,c\right)+de^2\right)}\ldots\right.$$

input
```
integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(4*a*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)
 + sqrt(2)*(a*e*cos(2*d*x + 2*c) + a*e)*log(sqrt(2)*sqrt((e*cos(2*d*x + 2*
c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1)/sqrt(e
) + 2*sin(2*d*x + 2*c) + 1)/sqrt(e))/(d*e^2*cos(2*d*x + 2*c) + d*e^2), (sq
rt(2)*(a*e*cos(2*d*x + 2*c) + a*e)*sqrt(-1/e)*arctan(1/2*sqrt(2)*sqrt((e*c
os(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(
2*d*x + 2*c) + 1)/(cos(2*d*x + 2*c) + 1)) + 2*a*sqrt((e*cos(2*d*x + 2*c) +
 e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c))/(d*e^2*cos(2*d*x + 2*c) + d*e^2)]
```

3.6.6 Sympy [F]

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = a\left(\int \frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{\cot\left(c+dx\right)}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))**(3/2),x)
```

output
```
a*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(cot(c + d*x)/(e*cot(c
+ d*x))**(3/2), x))
```

3.6.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.6.8 Giac [F]

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{a\cot(dx+c)+a}{(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)/(e*cot(d*x + c))^(3/2), x)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2a}{d\,e\,\sqrt{e\cot(c+d\,x)}} + \frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)(1+1\mathrm{i})}{d\,e^{3/2}} + \frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)(-1+1\mathrm{i})}{d\,e^{3/2}}$$

input
```
int((a + a*cot(c + d*x))/(e*cot(c + d*x))^(3/2),x)
```

output
```
(2*a)/(d*e*(e*cot(c + d*x))^(1/2)) + ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot
(c + d*x))^(1/2))/e^(1/2))*(1 + 1i))/(d*e^(3/2)) - ((-1)^(1/4)*a*atanh(((-
1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1 - 1i))/(d*e^(3/2))
```

3.6. $\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

3.7 $\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx$

3.7.1 Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx=-\frac{\sqrt{2}a\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}+\frac{2a}{de^2\sqrt{e\cot(c+dx)}}$$

output
```
2/3*a/d/e/(e*cot(d*x+c))^(3/2)-a*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2
^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d/e^(5/2)+2*a/d/e^2/(e*cot(d*x+c))^(1
/2)
```

3.7.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.05

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx=\frac{a\left(6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-6\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\ldots\right.}{\ldots}$$

input
```
Integrate[(a + a*Cot[c + d*x])/(e*Cot[c + d*x])^(5/2),x]
```

output
```
(a*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 6*Sqrt[2]*ArcTan[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]]] + 3*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] - 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]] + 24*Sqrt[Tan[c + d*x]] + 8*Tan[c + d*x]^(3/2) - 8*Hypergeometric2
F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(3/2)))/(12*d*(e*Cot[c + d*x
])^(5/2)*Tan[c + d*x]^(5/2))
```

3.7.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4012, 3042, 4012, 25, 3042, 4015, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cot(c+dx)+a}{(e\cot(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-a\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int \frac{ae-ae\cot(c+dx)}{(e\cot(c+dx))^{3/2}}dx}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{ae+a\tan\left(c+dx+\frac{\pi}{2}\right)e}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\int -\frac{ae^2+a\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{2a}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2a}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{ae^2+a\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx}{e^2}}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{2a}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{ae^2-ae^2\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \ 4015$$

$$\frac{\frac{2a^2e^2\int \frac{1}{-2a^2e^4-\left(ae^2-ae^2\cot(c+dx)\right)^2\tan(c+dx)}d\frac{ae^2-ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}}{d}+\frac{2a}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{2a}{d\sqrt{e\cot(c+dx)}}-\frac{\sqrt{2}a\arctan\left(\frac{ae^2-ae^2\cot(c+dx)}{\sqrt{2}ae^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}}}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

input
```
Int[(a + a*Cot[c + d*x])/(e*Cot[c + d*x])^(5/2),x]
```

output
```
(2*a)/(3*d*e*(e*Cot[c + d*x])^(3/2)) + (-((Sqrt[2]*a*ArcTan[(a*e^2 - a*e^2
*Cot[c + d*x])/(Sqrt[2]*a*e^(3/2)*Sqrt[e*Cot[c + d*x]])])/(d*Sqrt[e])) + (
2*a)/(d*Sqrt[e*Cot[c + d*x]]))/e^2
```

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

3.7.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(82) = 164$.

Time = 0.05 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.12

method	result
derivativedivides	$-\frac{a\left(-\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right.\right.}{4e}\cdots\right)}{\cdots}$
default	$-\frac{a\left(-\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right.\right.}{4e}\cdots\right)}{\cdots}$
parts	$-\frac{2ae\left(-\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots)}}{\left(e^{2}\right)^{\frac{1}{4}}}\cdots\right.\right.}{8e^{4}}\cdots\right)}{d}\cdots$

input
```
int((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*a*(2/e^2*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(
e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(
-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-2/e^2/(e*cot(d*x+c))^(1/2)-
2/3/e/(e*cot(d*x+c))^(3/2))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.62

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = \left[\frac{3\sqrt{2}(ae\cos(2dx+2c)+ae)\sqrt{-\frac{1}{e}}\log\left(\sqrt{2}\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}\sqrt{-\frac{1}{e}}(\cos(2dx+2\right.}{}\right.$$

$$-\frac{\frac{3\sqrt{2}(ae\cos(2dx+2c)+ae)\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}(\cos(2dx+2c)-\sin(2dx+2c)+1)}{2\sqrt{e}(\cos(2dx+2c)+1)}\right)}{\sqrt{e}}+2(a\cos(2dx+2c)-3a\sin(2dx+}{3(de^3\cos(2dx+2c)+de^3)}$$

input
```
integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(2)*(a*e*cos(2*d*x + 2*c) + a*e)*sqrt(-1/e)*log(sqrt(2)*sqrt((
e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + s
in(2*d*x + 2*c) - 1) - 2*sin(2*d*x + 2*c) + 1) - 4*(a*cos(2*d*x + 2*c) - 3
*a*sin(2*d*x + 2*c) - a)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/
(d*e^3*cos(2*d*x + 2*c) + d*e^3), -1/3*(3*sqrt(2)*(a*e*cos(2*d*x + 2*c) +
a*e)*arctan(-1/2*sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(
cos(2*d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(sqrt(e)*(cos(2*d*x + 2*c) + 1)))
/sqrt(e) + 2*(a*cos(2*d*x + 2*c) - 3*a*sin(2*d*x + 2*c) - a)*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^3*cos(2*d*x + 2*c) + d*e^3)]
```

3.7.6 Sympy [F]

$$\int \frac{a + a\cot(c + dx)}{(e\cot(c + dx))^{5/2}}\,dx = a\left(\int \frac{1}{(e\cot(c + dx))^{\frac{5}{2}}}\,dx + \int \frac{\cot(c + dx)}{(e\cot(c + dx))^{\frac{5}{2}}}\,dx\right)$$

input
```
integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))**(5/2),x)
```

output
```
a*(Integral((e*cot(c + d*x))**(-5/2), x) + Integral(cot(c + d*x)/(e*cot(c
+ d*x))**(5/2), x))
```

3.7.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + a\cot(c + dx)}{(e\cot(c + dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.7.8 Giac [F]

$$\int \frac{a + a\cot(c + dx)}{(e\cot(c + dx))^{5/2}}\,dx = \int \frac{a\cot(dx + c) + a}{(e\cot(dx + c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)/(e*cot(d*x + c))^(5/2), x)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int \frac{a+a\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = \frac{2\,a}{d\,e^2\,\sqrt{e\cot\left(c+d\,x\right)}} + \frac{2\,a}{3\,d\,e\,{\left(e\cot\left(c+d\,x\right)\right)}^{3/2}} + \frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,\left(1-\mathrm{i}\right)}{d\,e^{5/2}} + \frac{{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,\left(-1-\mathrm{i}\right)}{d\,e^{5/2}}$$

input
```
int((a + a*cot(c + d*x))/(e*cot(c + d*x))^(5/2),x)
```

output
```
(2*a)/(d*e^2*(e*cot(c + d*x))^(1/2)) + (2*a)/(3*d*e*(e*cot(c + d*x))^(3/2)
) + ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*(1 - 1
i))/(d*e^(5/2)) - ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/
e^(1/2))*(1 + 1i))/(d*e^(5/2))
```

3.8 $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx$

3.8.1 Optimal result

Integrand size = 25, antiderivative size = 269

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx = \frac{\sqrt{2}a^2e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{\sqrt{2}a^2e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{4a^2e^2\sqrt{e\cot(c+dx)}}{d} - \frac{4a^2(e\cot(c+dx))^{5/2}}{5d} - \frac{2a^2(e\cot(c+dx))^{7/2}}{7de} + \frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{a^2e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}$$

output
```
-4/5*a^2*(e*cot(d*x+c))^(5/2)/d-2/7*a^2*(e*cot(d*x+c))^(7/2)/d/e+1/2*a^2*e
^(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/
2)-1/2*a^2*e^(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1
/2))/d*2^(1/2)+a^2*e^(5/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*
2^(1/2)/d-a^2*e^(5/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/
2)/d+4*a^2*e^2*(e*cot(d*x+c))^(1/2)/d
```

3.8.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.70

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx =$$
$$-\frac{a^2(e\cot(c+dx))^{5/2}\left(-70\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)+70\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)-28\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2,x]
```

output
```
-1/70*(a^2*(e*Cot[c + d*x])^(5/2)*(-70*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot
[c + d*x]]] + 70*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] - 280*Sqrt
[Cot[c + d*x]] + 56*Cot[c + d*x]^(5/2) + 20*Cot[c + d*x]^(7/2) - 35*Sqrt[2
]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] + 35*Sqrt[2]*Log[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(d*Cot[c + d*x]^(5/2))
```

3.8.3 Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.97, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4026, 27, 2030, 3042, 3954, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(c+dx)+a)^2(e\cot(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4026}$$

$$\int 2a^2\cot(c+dx)(e\cot(c+dx))^{5/2}dx-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{27}$$

$$2a^2\int \cot(c+dx)(e\cot(c+dx))^{5/2}dx-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{2030}$$

$$\frac{2a^2\int(e\cot(c+dx))^{7/2}dx}{e}-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\int\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{e}-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3954}$$

$$\frac{2a^2\left(-e^2\int(e\cot(c+dx))^{3/2}dx-\frac{2e(e\cot(c+dx))^{5/2}}{5d}\right)}{e}-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\left(-e^2\int\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2e(e\cot(c+dx))^{5/2}}{5d}\right)}{e}-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3954}$$

$$\frac{2a^2\left(-e^2\left(e^2\left(-\int\frac{1}{\sqrt{e\cot(c+dx)}}dx\right)-\frac{2e\sqrt{e\cot(c+dx)}}{d}\right)-\frac{2e(e\cot(c+dx))^{5/2}}{5d}\right)}{e}-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\left(-e^2\left(e^2\left(-\int\frac{1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{2e\sqrt{e\cot(c+dx)}}{d}\right)-\frac{2e(e\cot(c+dx))^{5/2}}{5d}\right)}{e}-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{3957}$$

$$\frac{2a^2\left(-e^2\left(\frac{e^3\int\frac{1}{\sqrt{e\cot(c+dx)}\left(\cot^2(c+dx)e^2+e^2\right)}d(e\cot(c+dx))}{d}-\frac{2e\sqrt{e\cot(c+dx)}}{d}\right)-\frac{2e(e\cot(c+dx))^{5/2}}{5d}\right)}{e}-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{266}$$

$$\frac{2a^2\left(-e^2\left(\frac{2e^3\int\frac{1}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2e\sqrt{e\cot(c+dx)}}{d}\right)-\frac{2e(e\cot(c+dx))^{5/2}}{5d}\right)}{e}-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$$\Big\downarrow \text{755}$$

3.8. $\int(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx$

$$\frac{2a^2\left(-e^2\left(\frac{2e^3\left(\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\int \frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}\right)}{d}-\frac{2e\sqrt{e\cot(c+dx)}}{d}\right)-\frac{2e(e\cot(c+dx))^{5/2}}{5d}\right)}{e}-\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$\downarrow$ 1476

$$\frac{2a^2\left(-e^2\left(\frac{2e^3\left(\frac{\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d}{2e}\right.}{d}\right.\right.}{e}$$

$$\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$\downarrow$ 1082

$$\frac{2a^2\left(-e^2\left(\frac{2e^3\left(\frac{\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}\right)}{d}-2\right.\right.}{e}$$

$$\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$\downarrow$ 217

$$\frac{2a^2\left(-e^2\left(\frac{2e^3\left(\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}\right)}{d}-\frac{2e\sqrt{e\cot(c+dx)}}{d}\right)-\frac{2e(e\cot}{}\right.}{e}$$

$$\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$\downarrow$ 1479

3.8. $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx$

$$\frac{2a^2\left(-e^2\left(\frac{2e^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c\cdots\right)}{\sqrt{2}\sqrt{e}}\cdots\right)}{d}\cdots\right)\cdots\right)}{e\cdots}$$

$$\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$\big\downarrow$ 25

$$\frac{2a^2\left(-e^2\left(\frac{2e^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)\cdots\right)}{\sqrt{2}\sqrt{e}}\cdots\right)}{d}\cdots\right)\cdots\right)}{e\cdots}$$

$$\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$\big\downarrow$ 27

$$\frac{2a^2\left(-e^2\left(\frac{2e^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}}{2e}+\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)\cdots\right)}{\sqrt{2}\sqrt{e}}\cdots\right)}{d}\cdots\right)\cdots\right)}{e\cdots}$$

$$\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

$\big\downarrow$ 1103

3.8. $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx$

$$\frac{2a^2\left(-e^2\left(\frac{2e^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2\cdots\right.}{2\sqrt{2}\sqrt{e}}}{2e}\cdots}{d}\cdots\right.\right.}{e}$$

$$\frac{2a^2(e\cot(c+dx))^{7/2}}{7de}$$

input
```
Int[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2,x]
```

output
```
(-2*a^2*(e*Cot[c + d*x])^(7/2))/(7*d*e) + (2*a^2*((-2*e*(e*Cot[c + d*x])^(
5/2))/(5*d) - e^2*((-2*e*Sqrt[e*Cot[c + d*x]])/d + (2*e^3*((-(ArcTan[1 - S
qrt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt[2]*Sqrt[
e]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) + (-1/2*Log[e - Sqrt[2]*e^(3/2)*
Cot[c + d*x] + e^2*Cot[c + d*x]^2]/(Sqrt[2]*Sqrt[e]) + Log[e + Sqrt[2]*e^(
3/2)*Cot[c + d*x] + e^2*Cot[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/(2*e)))/d)))/
e
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.8. $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx$

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

3.8.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{2e(e\cot(dx+c))^{\frac{5}{2}}}{5}-2\sqrt{e\cot(dx+c)}\,e^3+\frac{e^3(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right.\right.}{}\right.}{de}$
default	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{2e(e\cot(dx+c))^{\frac{5}{2}}}{5}-2\sqrt{e\cot(dx+c)}\,e^3+\frac{e^3(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right.\right.}{}\right.}{de}$
parts	$-\frac{2a^2e\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^2\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8(e^2)^{\frac{1}{4}}}\right.}{d}$

input
```
int((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^2/e*(1/7*(e*cot(d*x+c))^(7/2)+2/5*e*(e*cot(d*x+c))^(5/2)-2*(e*cot(d
*x+c))^(1/2)*e^3+1/4*e^3*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

3.8.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.88

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx =$$

$$-\frac{35\left(-\frac{a^8e^{10}}{d^4}\right)^{\frac{1}{4}}(d\cos(2\,dx+2\,c)-d)\log\left(a^2e^2\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(-\frac{a^8e^{10}}{d^4}\right)^{\frac{1}{4}}d\right)\sin(2\,dx+2\,c)+35\left(-\frac{a^8}{\cdots}\right.}{\cdots}$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/35*(35*(-a^8*e^10/d^4)^(1/4)*(d*cos(2*d*x + 2*c) - d)*log(a^2*e^2*sqrt(
(e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (-a^8*e^10/d^4)^(1/4)*d)*sin(
2*d*x + 2*c) + 35*(-a^8*e^10/d^4)^(1/4)*(I*d*cos(2*d*x + 2*c) - I*d)*log(a
^2*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + I*(-a^8*e^10/d^4)
^(1/4)*d)*sin(2*d*x + 2*c) + 35*(-a^8*e^10/d^4)^(1/4)*(-I*d*cos(2*d*x + 2*
c) + I*d)*log(a^2*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - I*
(-a^8*e^10/d^4)^(1/4)*d)*sin(2*d*x + 2*c) - 35*(-a^8*e^10/d^4)^(1/4)*(d*co
s(2*d*x + 2*c) - d)*log(a^2*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
2*c)) - (-a^8*e^10/d^4)^(1/4)*d)*sin(2*d*x + 2*c) - 2*(5*a^2*e^2*cos(2*d*x
 + 2*c)^2 + 10*a^2*e^2*cos(2*d*x + 2*c) + 5*a^2*e^2 + 28*(3*a^2*e^2*cos(2*
d*x + 2*c) - 2*a^2*e^2)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c)))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x + 2*c))
```

3.8.6 Sympy [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx = a^2\left(\int (e\cot(c+dx))^{\frac{5}{2}}\,dx + \int 2(e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)\,dx + \int (e\cot(c+dx))^{\frac{5}{2}}\cot^2(c+dx)\,dx\right)$$

input
```
integrate((e*cot(d*x+c))**(5/2)*(a+a*cot(d*x+c))**2,x)
```

output
```
a**2*(Integral((e*cot(c + d*x))**(5/2), x) + Integral(2*(e*cot(c + d*x))**
(5/2)*cot(c + d*x), x) + Integral((e*cot(c + d*x))**(5/2)*cot(c + d*x)**2,
 x))
```

3.8.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.8.8 Giac [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx = \int (a\cot(dx+c)+a)^2(e\cot(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(5/2), x)
```

3.8.9 Mupad [B] (verification not implemented)

Time = 13.91 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.46

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx = \frac{4\,a^2\,e^2\,\sqrt{e\cot(c+d\,x)}}{d} - \frac{4\,a^2\,(e\cot(c+d\,x))^{5/2}}{5\,d} - \frac{2\,a^2\,(e\cot(c+d\,x))^{7/2}}{7\,d\,e} + \frac{(-1)^{1/4}\,a^2\,e^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,2\mathrm{i}}{d} + \frac{2\,(-1)^{1/4}\,a^2\,e^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}\,1\mathrm{i}}{\sqrt{e}}\right)}{d}$$

input
```
int((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^2,x)
```

output
```
(4*a^2*e^2*(e*cot(c + d*x))^(1/2))/d - (4*a^2*(e*cot(c + d*x))^(5/2))/(5*d
) - (2*a^2*(e*cot(c + d*x))^(7/2))/(7*d*e) + ((-1)^(1/4)*a^2*e^(5/2)*atan(
((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*2i)/d + (2*(-1)^(1/4)*a^2*e^(
5/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2)*1i)/e^(1/2)))/d
```

3.8. $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2\,dx$

3.9 $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx$

3.9.1 Optimal result

Integrand size = 25, antiderivative size = 246

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx=-\frac{\sqrt{2}a^2e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{\sqrt{2}a^2e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{4a^2(e\cot(c+dx))^{3/2}}{3d}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}+\frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}-\frac{a^2e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}$$

output
```
-4/3*a^2*(e*cot(d*x+c))^(3/2)/d-2/5*a^2*(e*cot(d*x+c))^(5/2)/d/e+1/2*a^2*e
^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/
2)-1/2*a^2*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1
/2))/d*2^(1/2)-a^2*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*
2^(1/2)/d+a^2*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/
2)/d
```

3.9.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.46

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx =$$

$$-\frac{2a^2(e\cot(c+dx))^{3/2}\left(-15\arctan\left(\sqrt[4]{-\cot^2(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}+15\text{arctanh}\left(\sqrt[4]{-\cot^2(c+dx)}\right)\right.}{15d\cot^{\frac{7}{4}}(c+dx)}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2,x]
```

output
```
(-2*a^2*(e*Cot[c + d*x])^(3/2)*(-15*ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[
c + d*x])^(1/4) + 15*ArcTanh[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4
) + Cot[c + d*x]^(7/4)*(10 + 3*Cot[c + d*x])))/(15*d*Cot[c + d*x]^(7/4))
```

3.9.3 Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.94, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4026, 27, 2030, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4026}$$

$$\int 2a^2\cot(c+dx)(e\cot(c+dx))^{3/2}dx-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{27}$$

$$2a^2\int \cot(c+dx)(e\cot(c+dx))^{3/2}dx-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{2030}$$

$$\frac{2a^2\int (e\cot(c+dx))^{5/2}dx}{e}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$\big\downarrow$ 3042

$$\frac{2a^2\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{e}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$\big\downarrow$ 3954

$$\frac{2a^2\left(e^2\left(-\int \sqrt{e\cot(c+dx)}dx\right)-\frac{2e(e\cot(c+dx))^{3/2}}{3d}\right)}{e}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$\big\downarrow$ 3042

$$\frac{2a^2\left(e^2\left(-\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}dx\right)-\frac{2e(e\cot(c+dx))^{3/2}}{3d}\right)}{e}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$\big\downarrow$ 3957

$$\frac{2a^2\left(\frac{e^3\int \frac{\sqrt{e\cot(c+dx)}}{\cot^2(c+dx)e^2+e^2}d(e\cot(c+dx))}{d}-\frac{2e(e\cot(c+dx))^{3/2}}{3d}\right)}{e}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$\big\downarrow$ 266

$$\frac{2a^2\left(\frac{2e^3\int \frac{e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2e(e\cot(c+dx))^{3/2}}{3d}\right)}{e}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$\big\downarrow$ 826

$$\frac{2a^2\left(\frac{2e^3\left(\frac{1}{2}\int \frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-\frac{2e(e\cot(c+dx))^{3/2}}{3d}\right)}{e}-\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$\big\downarrow$ 1476

$$\frac{2a^2\left(\frac{2e^3\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}\right)-\frac{1}{2}\int \frac{e-e^2\cot^2(c+d}{e^4\cot^4(c+dx)+}\right.}{d}}{e}$$

$$\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$\big\downarrow$ 1082

3.9. $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx$

$$2a^2\left(\frac{\frac{2e^3\left(\frac{1}{2}\left(\frac{\int\frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e^2\cot^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int\frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-}{e}\right.$$

$$\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$\big\downarrow$ 217

$$2a^2\left(\frac{\frac{2e^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int\frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-\frac{2e(e\cot(c+dx))^{3/2}}{3d}}{e}\right)-$$

$$\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$\big\downarrow$ 1479

$$2a^2\left(\frac{\frac{2e^3\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}}{e}\right.$$

$$\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$\big\downarrow$ 25

$$2a^2\left(\frac{\frac{2e^3\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+d\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}}{e}\right.$$

$$\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$\big\downarrow$ 27

3.9. $\int(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx$

$$2a^2\left(\frac{2e^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+d\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}\right.}{e}$$

$$\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \; 1103$$

$$2a^2\left(\frac{2e^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot\right.}{2\sqrt{2}\sqrt{e}}\right.\right.}{d}\right.}{e}$$

$$\frac{2a^2(e\cot(c+dx))^{5/2}}{5de}$$

input
```
Int[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2,x]
```

output
```
(-2*a^2*(e*Cot[c + d*x])^(5/2))/(5*d*e) + (2*a^2*((-2*e*(e*Cot[c + d*x])^(
3/2))/(3*d) + (2*e^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2]
*Sqrt[e])) + ArcTan[1 + Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e]))/2
 + (Log[e - Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c + d*x]^2]/(2*Sqrt[2]*
Sqrt[e]) - Log[e + Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c + d*x]^2]/(2*S
qrt[2]*Sqrt[e]))/2))/d))/e
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.9. $\int (e \cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx$

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

3.9.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^3\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{(e^2)^{\frac{1}{4}}}\cdots\right)\cdots\right)}{4(e^2)^{\frac{1}{4}}}\cdots\right)}{de}$
default	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^3\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{(e^2)^{\frac{1}{4}}}\cdots\right)\cdots\right)}{4(e^2)^{\frac{1}{4}}}\cdots\right)}{de}$
parts	$-\frac{2a^2e\left(\sqrt{e\cot(dx+c)}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arcta}\cdots\right)}{8}\cdots\right)}{d}$

input
```
int((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^2/e*(1/5*(e*cot(d*x+c))^(5/2)+2/3*e*(e*cot(d*x+c))^(3/2)-1/4*e^3/(e
^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/
2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-
2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

3.9.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.72

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = \frac{15\left(-\frac{a^8e^6}{d^4}\right)^{\frac{1}{4}}(d\cos(2\,dx+2\,c)-d)\log\left(a^6e^4\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(-\frac{a^8e^6}{d^4}\right)^{\frac{3}{4}}d^3\right)-15\left(\cdots\right.}{\cdots}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/15*(15*(-a^8*e^6/d^4)^(1/4)*(d*cos(2*d*x + 2*c) - d)*log(a^6*e^4*sqrt((e
*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (-a^8*e^6/d^4)^(3/4)*d^3) - 15*
(-a^8*e^6/d^4)^(1/4)*(I*d*cos(2*d*x + 2*c) - I*d)*log(a^6*e^4*sqrt((e*cos(
2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + I*(-a^8*e^6/d^4)^(3/4)*d^3) - 15*(-a
^8*e^6/d^4)^(1/4)*(-I*d*cos(2*d*x + 2*c) + I*d)*log(a^6*e^4*sqrt((e*cos(2*
d*x + 2*c) + e)/sin(2*d*x + 2*c)) - I*(-a^8*e^6/d^4)^(3/4)*d^3) - 15*(-a^8
*e^6/d^4)^(1/4)*(d*cos(2*d*x + 2*c) - d)*log(a^6*e^4*sqrt((e*cos(2*d*x + 2
*c) + e)/sin(2*d*x + 2*c)) - (-a^8*e^6/d^4)^(3/4)*d^3) + 2*(3*a^2*e*cos(2*
d*x + 2*c) + 10*a^2*e*sin(2*d*x + 2*c) + 3*a^2*e)*sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c) - d)
```

3.9.6 Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = a^2\left(\int (e\cot(c+dx))^{\frac{3}{2}}\,dx + \int 2(e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)\,dx + \int (e\cot(c+dx))^{\frac{3}{2}}\cot^2(c+dx)\,dx\right)$$

input
```
integrate((e*cot(d*x+c))**(3/2)*(a+a*cot(d*x+c))**2,x)
```

output
```
a**2*(Integral((e*cot(c + d*x))**(3/2), x) + Integral(2*(e*cot(c + d*x))**
(3/2)*cot(c + d*x), x) + Integral((e*cot(c + d*x))**(3/2)*cot(c + d*x)**2,
 x))
```

3.9.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.9.8 Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = \int (a\cot(dx+c)+a)^2(e\cot(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(3/2), x)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.42

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2\,dx = \frac{2\,(-1)^{1/4}\,a^2\,e^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d} - \frac{2\,a^2\,(e\cot(c+d\,x))^{5/2}}{5\,d\,e} - \frac{4\,a^2\,(e\cot(c+d\,x))^{3/2}}{3\,d} + \frac{(-1)^{1/4}\,a^2\,e^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}\,1\mathrm{i}}{\sqrt{e}}\right)\,2\mathrm{i}}{d}$$

input

```
int((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^2,x)
```

output

```
(2*(-1)^(1/4)*a^2*e^(3/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)
))/d - (2*a^2*(e*cot(c + d*x))^(5/2))/(5*d*e) - (4*a^2*(e*cot(c + d*x))^(3
/2))/(3*d) + ((-1)^(1/4)*a^2*e^(3/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/
2)*1i)/e^(1/2))*2i)/d
```

3.10 $\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx$

3.10.1 Optimal result

Integrand size = 25, antiderivative size = 244

$$\begin{aligned}
&\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx \\
&= -\frac{\sqrt{2}a^2\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{\sqrt{2}a^2\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d} \\
&\quad -\frac{4a^2\sqrt{e\cot(c+dx)}}{d}-\frac{2a^2(e\cot(c+dx))^{3/2}}{3de} \\
&\quad -\frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d} \\
&\quad +\frac{a^2\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d}
\end{aligned}$$

output
```
-2/3*a^2*(e*cot(d*x+c))^(3/2)/d/e-1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^
(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)+1/2*a^2*ln(e^(1/2)+cot(d*x+c
)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-a^2*arctan(1-2^(
1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)*e^(1/2)/d+a^2*arctan(1+2^(1/2)*
(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)*e^(1/2)/d-4*a^2*(e*cot(d*x+c))^(1/2)
/d
```

3.10.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.72

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx =$$
$$-\frac{a^2\sqrt{e\cot(c+dx)}\left(6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-6\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+24\sqrt{c}\right.}{}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2,x]
```

output
```
-1/6*(a^2*Sqrt[e*Cot[c + d*x]]*(6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c +
d*x]]] - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*Sqrt[Cot[c
+ d*x]] + 4*Cot[c + d*x]^(3/2) + 3*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*
x]] + Cot[c + d*x]] - 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
 + d*x]]))/(d*Sqrt[Cot[c + d*x]])
```

3.10.3 Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.96, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3042, 4026, 27, 2030, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4026$$

$$\int 2a^2\cot(c+dx)\sqrt{e\cot(c+dx)}dx-\frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \; 27$$

$$2a^2\int \cot(c+dx)\sqrt{e\cot(c+dx)}dx-\frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{2a^2 \int (e\cot(c+dx))^{3/2}dx}{e} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2a^2 \int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{e} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{2a^2\left(e^2\left(-\int \frac{1}{\sqrt{e\cot(c+dx)}}dx\right) - \frac{2e\sqrt{e\cot(c+dx)}}{d}\right)}{e} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2a^2\left(e^2\left(-\int \frac{1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx\right) - \frac{2e\sqrt{e\cot(c+dx)}}{d}\right)}{e} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{ 3957}$$

$$\frac{2a^2\left(\frac{e^3\int \frac{1}{\sqrt{e\cot(c+dx)}\left(\cot^2(c+dx)e^2+e^2\right)}d(e\cot(c+dx))}{d} - \frac{2e\sqrt{e\cot(c+dx)}}{d}\right)}{e} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2a^2\left(\frac{2e^3\int \frac{1}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{d} - \frac{2e\sqrt{e\cot(c+dx)}}{d}\right)}{e} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{2a^2\left(\frac{2e^3\left(\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e} + \frac{\int \frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}\right)}{d} - \frac{2e\sqrt{e\cot(c+dx)}}{d}\right)}{e} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{ 1476}$$

3.10. $\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx$

$$\frac{2a^2\left(\frac{2e^3\left(\frac{\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c}}{2e}\right.}{d}\right.}{e}$$

$$\frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$\downarrow$ 1082

$$\frac{2a^2\left(\frac{2e^3\left(\frac{\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}\right)}{d}-\frac{2e\sqrt{e\cot}}{d}\right.}{e}$$

$$\frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$\downarrow$ 217

$$\frac{2a^2\left(\frac{2e^3\left(\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}\right)}{d}-\frac{2e\sqrt{e\cot(c+dx)}}{d}\right)}{e}-$$

$$\frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$\downarrow$ 1479

$$\frac{2a^2\left(\frac{2e^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{}\right.}{d}\right.}{e}$$

$$\frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

3.10. $\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx$

$$\Big\downarrow \; 25$$

$$2a^2\left(\frac{2e^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\text{arc}}{}}{2e}\right.}{d}\right.$$

$$\overline{\qquad\qquad e \qquad\qquad}$$

$$\frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \; 27$$

$$2a^2\left(\frac{2e^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\text{arc}}{}}{2e}\right.}{d}\right.$$

$$\overline{\qquad\qquad e \qquad\qquad}$$

$$\frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \; 1103$$

$$2a^2\left(\frac{2e^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right.}{d}\right.$$

$$\overline{\qquad\qquad e \qquad\qquad}$$

$$\frac{2a^2(e\cot(c+dx))^{3/2}}{3de}$$

input
```
Int[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2,x]
```

3.10. $\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx$

output
```
(-2*a^2*(e*Cot[c + d*x])^(3/2))/(3*d*e) + (2*a^2*((-2*e*Sqrt[e*Cot[c + d*x
]])/d + (2*e^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[
e])) + ArcTan[1 + Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) +
 (-1/2*Log[e - Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c + d*x]^2]/(Sqrt[2]
*Sqrt[e]) + Log[e + Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c + d*x]^2]/(2*
Sqrt[2]*Sqrt[e]))/(2*e)))/d))/e
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

3.10. $\int \sqrt{e \cot(c+dx)}(a+a\cot(c+dx))^2\,dx$

3.10.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}+2e\sqrt{e\cot(dx+c)}-\frac{e\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot}}{(e^2)}\right.\right.}{4}\right.}{de}$
default	$-\frac{2a^2\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}+2e\sqrt{e\cot(dx+c)}-\frac{e\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot}}{(e^2)}\right.\right.}{4}\right.}{de}$
parts	$-\frac{a^2e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4d\left(e^2\right)^{\frac{1}{4}}}$

input
```
int((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^2/e*(1/3*(e*cot(d*x+c))^(3/2)+2*e*(e*cot(d*x+c))^(1/2)-1/4*e*(e^2)^
(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(
1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1
/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

3.10.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.52

$$\int\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx$$
$$=\frac{3\left(-\frac{a^8e^2}{d^4}\right)^{\frac{1}{4}}d\log\left(a^2\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(-\frac{a^8e^2}{d^4}\right)^{\frac{1}{4}}d\right)\sin(2\,dx+2\,c)+3i\left(-\frac{a^8e^2}{d^4}\right)^{\frac{1}{4}}d\log\left(a^2\sqrt{\frac{e\cos(2\,dx+2}{\sin(2\,dx+2}}\right.}{}$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(3*(-a^8*e^2/d^4)^(1/4)*d*log(a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*
d*x + 2*c)) + (-a^8*e^2/d^4)^(1/4)*d)*sin(2*d*x + 2*c) + 3*I*(-a^8*e^2/d^4
)^(1/4)*d*log(a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + I*(-a^
8*e^2/d^4)^(1/4)*d)*sin(2*d*x + 2*c) - 3*I*(-a^8*e^2/d^4)^(1/4)*d*log(a^2*
sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - I*(-a^8*e^2/d^4)^(1/4)*d
)*sin(2*d*x + 2*c) - 3*(-a^8*e^2/d^4)^(1/4)*d*log(a^2*sqrt((e*cos(2*d*x +
2*c) + e)/sin(2*d*x + 2*c)) - (-a^8*e^2/d^4)^(1/4)*d)*sin(2*d*x + 2*c) - 2
*(a^2*cos(2*d*x + 2*c) + 6*a^2*sin(2*d*x + 2*c) + a^2)*sqrt((e*cos(2*d*x +
 2*c) + e)/sin(2*d*x + 2*c)))/(d*sin(2*d*x + 2*c))
```

3.10.6 Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx = a^2\left(\int \sqrt{e\cot{(c+dx)}}\,dx + \int 2\sqrt{e\cot{(c+dx)}}\cot{(c+dx)}\,dx + \int \sqrt{e\cot{(c+dx)}}\cot^2{(c+dx)}\,dx\right)$$

input
```
integrate((e*cot(d*x+c))**(1/2)*(a+a*cot(d*x+c))**2,x)
```

output
```
a**2*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(2*sqrt(e*cot(c + d*x))*
cot(c + d*x), x) + Integral(sqrt(e*cot(c + d*x))*cot(c + d*x)**2, x))
```

3.10.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.10.8 Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx = \int (a\cot(dx+c)+a)^2\sqrt{e\cot(dx+c)}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^2*sqrt(e*cot(d*x + c)), x)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.43

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2\,dx = -\frac{4a^2\sqrt{e\cot(c+dx)}}{d} - \frac{2a^2(e\cot(c+dx))^{3/2}}{3de} - \frac{(-1)^{1/4}a^2\sqrt{e}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)2i}{d} - \frac{2(-1)^{1/4}a^2\sqrt{e}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{e\cot(c+dx)}\,1i}{\sqrt{e}}\right)}{d}$$

input
```
int((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^2,x)
```

output
```
- (4*a^2*(e*cot(c + d*x))^(1/2))/d - (2*a^2*(e*cot(c + d*x))^(3/2))/(3*d*e
) - ((-1)^(1/4)*a^2*e^(1/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/
2))*2i)/d - (2*(-1)^(1/4)*a^2*e^(1/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1
/2)*1i)/e^(1/2)))/d
```

3.11 $\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

3.11.1 Optimal result

Integrand size = 25, antiderivative size = 222

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} - \frac{\sqrt{2}a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} - \frac{2a^2\sqrt{e\cot(c+dx)}}{de} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d\sqrt{e}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}d\sqrt{e}}$$

output

```
-1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(
1/2)/e^(1/2)+1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^
(1/2))/d*2^(1/2)/e^(1/2)+a^2*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2)
)*2^(1/2)/d/e^(1/2)-a^2*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(
1/2)/d/e^(1/2)-2*a^2*(e*cot(d*x+c))^(1/2)/d/e
```

3.11.2 Mathematica [A] (verified)

Time = 6.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.14

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = -\frac{2\cos(c+dx)(a+a\cot(c+dx))^2\sin(c+dx)}{d\sqrt{e\cot(c+dx)}(\cos(c+dx)+\sin(c+dx))^2}$$
$$-\frac{2\arctan\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}(a+a\cot(c+dx))^2\sin^2(c+d}{d\sqrt{e\cot(c+dx)}(\cos(c+dx)+\sin(c+dx))^2}$$
$$+\frac{2\text{arctanh}\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}(a+a\cot(c+dx))^2\sin^2(c+d}{d\sqrt{e\cot(c+dx)}(\cos(c+dx)+\sin(c+dx))^2}$$

input
```
Integrate[(a + a*Cot[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

output
```
(-2*Cos[c + d*x]*(a + a*Cot[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[e*Cot[c + d*
x]]*(Cos[c + d*x] + Sin[c + d*x])^2) - (2*ArcTan[(-Cot[c + d*x])^(1/4)*Cot
[c + d*x]^(1/4)]*(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)*(a + a*Cot[c + d
*x])^2*Sin[c + d*x]^2)/(d*Sqrt[e*Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x
])^2) + (2*ArcTanh[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*x
])^(1/4)*Cot[c + d*x]^(1/4)*(a + a*Cot[c + d*x])^2*Sin[c + d*x]^2)/(d*Sqrt
[e*Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x])^2)
```

3.11.3 Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.90, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4026, 27, 2030, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cot(c+dx)+a)^2}{\sqrt{e\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4026}$$

$$\int \frac{2a^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \text{27}$$

$$2a^2\int \frac{\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \text{2030}$$

$$\frac{2a^2\int \sqrt{e\cot(c+dx)}dx}{e}-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{e}-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{2a^2\int \frac{\sqrt{e\cot(c+dx)}}{\cot^2(c+dx)e^2+e^2}d(e\cot(c+dx))}{d}-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \text{266}$$

$$-\frac{4a^2\int \frac{e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \text{826}$$

$$-\frac{4a^2\left(\frac{1}{2}\int \frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{4a^2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}\right)\right.}{d}$$
$$\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{4a^2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot}\right.}{d}$$
$$\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

3.11. $\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

$\Big\downarrow$ 217

$$-\frac{4a^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int\frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

$\Big\downarrow$ 1479

$$-\frac{4a^2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\cdots\right.}{\cdots}\right.\cdots\right)}{d}\cdots-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

$\Big\downarrow$ 25

$$-\frac{4a^2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\cdots\right.}{\cdots}\right.\cdots\right)}{d}\cdots-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

$\Big\downarrow$ 27

$$-\frac{4a^2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\cdots\right.}{\cdots}\right.\cdots\right)}{d}\cdots-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

$\Big\downarrow$ 1103

$$-\frac{4a^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3}\cdots\right.}{\cdots}\right.\cdots\right)}{d}\cdots-\frac{2a^2\sqrt{e\cot(c+dx)}}{de}$$

input
```
Int[(a + a*Cot[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

3.11. $\int\frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

output
```
(-2*a^2*Sqrt[e*Cot[c + d*x]])/(d*e) - (4*a^2*((-(ArcTan[1 - Sqrt[2]*Sqrt[e
]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt[2]*Sqrt[e]*Cot[c + d*
x]]/(Sqrt[2]*Sqrt[e]))/2 + (Log[e - Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot
[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]) - Log[e + Sqrt[2]*e^(3/2)*Cot[c + d*x] +
e^2*Cot[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/2))/d
```

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

3.11. $\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

3.11.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(\sqrt{e\cot(dx+c)}+\frac{e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.\right.}{4(e^2)^{\frac{1}{4}}}\right)}{de}$
default	$-\frac{2a^2\left(\sqrt{e\cot(dx+c)}+\frac{e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.\right.}{4(e^2)^{\frac{1}{4}}}\right)}{de}$
parts	$-\frac{a^2(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot}}{(e^2)}\right.\right.}{4de}$

input
```
int((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^2/e*((e*cot(d*x+c))^(1/2)+1/4*e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+
c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)+1)))
```

3.11.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.45

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx =$$

$$-\frac{\left(-\frac{a^8}{d^4e^2}\right)^{\frac{1}{4}}de\log\left(a^6\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(-\frac{a^8}{d^4e^2}\right)^{\frac{3}{4}}d^3e^2\right)-i\left(-\frac{a^8}{d^4e^2}\right)^{\frac{1}{4}}de\log\left(a^6\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+i\left(-\right.\right.}{}$$

input
```
integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-((-a^8/(d^4*e^2))^(1/4)*d*e*log(a^6*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d
*x + 2*c)) + (-a^8/(d^4*e^2))^(3/4)*d^3*e^2) - I*(-a^8/(d^4*e^2))^(1/4)*d*
e*log(a^6*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + I*(-a^8/(d^4*e
^2))^(3/4)*d^3*e^2) + I*(-a^8/(d^4*e^2))^(1/4)*d*e*log(a^6*sqrt((e*cos(2*d
*x + 2*c) + e)/sin(2*d*x + 2*c)) - I*(-a^8/(d^4*e^2))^(3/4)*d^3*e^2) - (-a
^8/(d^4*e^2))^(1/4)*d*e*log(a^6*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
2*c)) - (-a^8/(d^4*e^2))^(3/4)*d^3*e^2) + 2*a^2*sqrt((e*cos(2*d*x + 2*c) +
 e)/sin(2*d*x + 2*c)))/(d*e)
```

3.11.6 Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{e\cot{(c+dx)}}}\,dx + \int \frac{2\cot{(c+dx)}}{\sqrt{e\cot{(c+dx)}}}\,dx + \int \frac{\cot^2{(c+dx)}}{\sqrt{e\cot{(c+dx)}}}\,dx\right)$$

input
```
integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

output
```
a**2*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(2*cot(c + d*x)/sqrt(e
*cot(c + d*x)), x) + Integral(cot(c + d*x)**2/sqrt(e*cot(c + d*x)), x))
```

3.11.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.11.8 Giac [F]

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx=\int \frac{(a\cot(dx+c)+a)^2}{\sqrt{e\cot(dx+c)}}\,dx$$

input
```
integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^2/sqrt(e*cot(d*x + c)), x)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 12.74 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.39

$$\int \frac{(a+a\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx=\frac{2\,(-1)^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,\sqrt{e}}-\frac{2\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,\sqrt{e}}-\frac{2\,a^2\,\sqrt{e\cot\left(c+d\,x\right)}}{d\,e}$$

input
```
int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(1/2),x)
```

output
```
(2*(-1)^(1/4)*a^2*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/(d*e
^(1/2)) - (2*(-1)^(1/4)*a^2*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/
2)))/(d*e^(1/2)) - (2*a^2*(e*cot(c + d*x))^(1/2))/(d*e)
```

3.12 $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

3.12.1 Optimal result

Integrand size = 25, antiderivative size = 222

$$\begin{aligned}\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx &= \frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} \\ &- \frac{\sqrt{2}a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{2a^2}{de\sqrt{e\cot(c+dx)}} \\ &+ \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{3/2}} \\ &- \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{3/2}}\end{aligned}$$

output
```
1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(3
/2)*2^(1/2)-1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(
1/2))/d/e^(3/2)*2^(1/2)+a^2*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))
*2^(1/2)/d/e^(3/2)-a^2*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1
/2)/d/e^(3/2)+2*a^2/d/e/(e*cot(d*x+c))^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.83

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{a^2\left(4+2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\sqrt{\cot(c+dx)}-2\sqrt{2}\arctan\left(1+\right.\right.}{}$$

input
```
Integrate[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

output
```
(a^2*(4 + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sqrt[Cot[c + d*
x]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sqrt[Cot[c + d*x]]
+ Sqrt[2]*Sqrt[Cot[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
d*x]] - Sqrt[2]*Sqrt[Cot[c + d*x]]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Co
t[c + d*x]]))/(2*d*e*Sqrt[e*Cot[c + d*x]])
```

3.12.3 Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.93, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4025, 27, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cot(c+dx)+a)^2}{(e\cot(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4025}$$

$$\frac{\int \frac{2a^2e}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{2a^2}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a^2\int \frac{1}{\sqrt{e\cot(c+dx)}}dx}{e}+\frac{2a^2}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\int \frac{1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e}+\frac{2a^2}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{2a^2\int \frac{1}{\sqrt{e\cot(c+dx)}\left(\cot^2(c+dx)e^2+e^2\right)}d(e\cot(c+dx))}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{4a^2\int \frac{1}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{755}$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{4a^2\left(\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\int \frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}\right)}{d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{4a^2\left(\frac{\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}\right)}{d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{4a^2\left(\frac{\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}\right)}{d}$$

$$\Big\downarrow \text{217}$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{4a^2\left(\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}\right)}{d}$$

$$\Big\downarrow \text{1479}$$

3.12. $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{4a^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\text{a}}{2e}\right.}{d}$$

$$\downarrow 25$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{4a^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\text{arctan}}{2e}\right.}{d}$$

$$\downarrow 27$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{4a^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\text{arctan}}{2e}\right.}{d}$$

$$\downarrow 1103$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{4a^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right.}{d}$$

input
```
Int[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

output
```
(2*a^2)/(d*e*Sqrt[e*Cot[c + d*x]]) - (4*a^2*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]
*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt[2]*Sqrt[e]*Cot[c + d*x
]]/(Sqrt[2]*Sqrt[e]))/(2*e) + (-1/2*Log[e - Sqrt[2]*e^(3/2)*Cot[c + d*x] +
 e^2*Cot[c + d*x]^2]/(Sqrt[2]*Sqrt[e]) + Log[e + Sqrt[2]*e^(3/2)*Cot[c + d
*x] + e^2*Cot[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/(2*e)))/d
```

3.12. $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4025
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e
+ f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
 && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

3.12.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{2a^2\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{de}$
default	$-\frac{2a^2\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{de}$
parts	$-\frac{2a^2e\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e^2\left(e^2\right)^{\frac{1}{4}}}\right.}{d}$

input
```
int((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^2/e*(1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/(e*cot(
d*x+c))^(1/2))
```

3.12.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.86

$$\int\frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx=\frac{2\,a^2\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sin\left(2\,dx+2\,c\right)-\left(de^2\cos\left(2\,dx+2\,c\right)+de^2\right)\left(-\frac{a^8}{d^4e^6}\right)^{\frac{1}{4}}\log}{}$$

input
```
integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(2*a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) -
(d*e^2*cos(2*d*x + 2*c) + d*e^2)*(-a^8/(d^4*e^6))^(1/4)*log(d*e^2*(-a^8/(d
^4*e^6))^(1/4) + a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (-
I*d*e^2*cos(2*d*x + 2*c) - I*d*e^2)*(-a^8/(d^4*e^6))^(1/4)*log(I*d*e^2*(-a
^8/(d^4*e^6))^(1/4) + a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))
 + (I*d*e^2*cos(2*d*x + 2*c) + I*d*e^2)*(-a^8/(d^4*e^6))^(1/4)*log(-I*d*e^
2*(-a^8/(d^4*e^6))^(1/4) + a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2
*c))) + (d*e^2*cos(2*d*x + 2*c) + d*e^2)*(-a^8/(d^4*e^6))^(1/4)*log(-d*e^2
*(-a^8/(d^4*e^6))^(1/4) + a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c))))/(d*e^2*cos(2*d*x + 2*c) + d*e^2)
```

3.12.6 Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = a^2\left(\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{2\cot(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\cot^2(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(3/2),x)
```

output
```
a**2*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(2*cot(c + d*x)/(e*c
ot(c + d*x))**(3/2), x) + Integral(cot(c + d*x)**2/(e*cot(c + d*x))**(3/2)
, x))
```

3.12.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.12.8 Giac [F]

$$\int \frac{(a + a\cot(c + dx))^2}{(e\cot(c + dx))^{3/2}}\,dx = \int \frac{(a\cot(dx + c) + a)^2}{(e\cot(dx + c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(3/2), x)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 12.88 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.39

$$\int \frac{(a + a\cot(c + dx))^2}{(e\cot(c + dx))^{3/2}}\,dx = \frac{2a^2}{d\,e\,\sqrt{e\cot(c + d\,x)}} + \frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,2\mathrm{i}}{d\,e^{3/2}} + \frac{(-1)^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,2\mathrm{i}}{d\,e^{3/2}}$$

input
```
int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(3/2),x)
```

output
```
(2*a^2)/(d*e*(e*cot(c + d*x))^(1/2)) + ((-1)^(1/4)*a^2*atan(((-1)^(1/4)*(e
*cot(c + d*x))^(1/2))/e^(1/2))*2i)/(d*e^(3/2)) + ((-1)^(1/4)*a^2*atanh(((-
1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*2i)/(d*e^(3/2))
```

3.13 $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx$

3.13.1 Optimal result

Integrand size = 25, antiderivative size = 247

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = -\frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{\sqrt{2}a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{2a^2}{3de(e\cot(c+dx))^{3/2}} + \frac{4a^2}{de^2\sqrt{e\cot(c+dx)}} + \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{5/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{5/2}}$$

output
```
2/3*a^2/d/e/(e*cot(d*x+c))^(3/2)+1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(
1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)-1/2*a^2*ln(e^(1/2)+cot(d*x+c)
*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)-a^2*arctan(1-2^(1
/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d/e^(5/2)+a^2*arctan(1+2^(1/2)*(
e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d/e^(5/2)+4*a^2/d/e^2/(e*cot(d*x+c))^
(1/2)
```

3.13.2 Mathematica [A] (warning: unable to verify)

Time = 3.06 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.62

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \frac{a^2\left(48\cos^2(c+dx)+6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\cot^{\frac{5}{2}}(c+dx)\sin^2(c \cdots\right.}{\cdots}$$

input
```
Integrate[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(5/2),x]
```

output
```
(a^2*(48*Cos[c + d*x]^2 + 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]
*Cot[c + d*x]^(5/2)*Sin[c + d*x]^2 - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot
[c + d*x]]]*Cot[c + d*x]^(5/2)*Sin[c + d*x]^2 + 3*Sqrt[2]*Cot[c + d*x]^(5/
2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 - 3*S
qrt[2]*Cot[c + d*x]^(5/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x
]]*Sin[c + d*x]^2 + 4*Sin[2*(c + d*x)] + 6*ArcTan[(-Cot[c + d*x]^2)^(1/4)]
*(4*(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(9/4)*Sin[c + d*x]^2 - (-Cot[c + d*
x]^2)^(3/4)*Sin[2*(c + d*x)]) - 6*ArcTanh[(-Cot[c + d*x]^2)^(1/4)]*(4*Cos[
c + d*x]^2*(-Cot[c + d*x]^2)^(1/4) + (-Cot[c + d*x]^2)^(3/4)*Sin[2*(c + d*
x)]))*(1 + Tan[c + d*x])^2)/(12*d*e^2*Sqrt[e*Cot[c + d*x]]*(Cos[c + d*x] +
 Sin[c + d*x])^2)
```

3.13.3 Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.94, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4025, 27, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cot(c+dx)+a)^2}{(e\cot(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4025}$$

$$\frac{\int \frac{2a^2 e}{(e\cot(c+dx))^{3/2}}dx}{e^2}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a^2\int \frac{1}{(e\cot(c+dx))^{3/2}}dx}{e}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\int \frac{1}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{e}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{2a^2\left(\frac{2}{de\sqrt{e\cot(c+dx)}}-\frac{\int \sqrt{e\cot(c+dx)}dx}{e^2}\right)}{e}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\left(\frac{2}{de\sqrt{e\cot(c+dx)}}-\frac{\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{e^2}\right)}{e}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{2a^2\left(\frac{\int \frac{\sqrt{e\cot(c+dx)}}{\cot^2(c+dx)e^2+e^2}d(e\cot(c+dx))}{de}+\frac{2}{de\sqrt{e\cot(c+dx)}}\right)}{e}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2a^2\left(\frac{2\int \frac{e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{de}+\frac{2}{de\sqrt{e\cot(c+dx)}}\right)}{e}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{826}$$

$$\frac{2a^2\left(\frac{2\left(\frac{1}{2}\int \frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}\right)}{de}+\frac{2}{de\sqrt{e\cot(c+dx)}}\right)}{e}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{1476}$$

3.13. $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}dx$

$$2a^2\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}\right)-\frac{1}{2}\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}\right.}{de}\right. \over e$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 1082

$$2a^2\left(\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}\right)}{de}+\frac{\ }{de}\right. \over e$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 217

$$\frac{2a^2\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}\right)}{de}+\frac{2}{de\sqrt{e\cot(c+dx)}}\right)}{e}+$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 1479

$$2a^2\left(\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+da\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{de}\right. \over e$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 25

3.13. $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx$

$$\frac{2a^2\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)\ldots}{\sqrt{2}\sqrt{e}}\ldots}{de}\ldots\right.}{e}$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2a^2\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)\ldots}{\sqrt{2}\sqrt{e}}\ldots}{de}\ldots\right.}{e}$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 1103$$

$$\frac{2a^2\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(\ldots\right.}{2\sqrt{2}\sqrt{e}}\ldots}{de}\ldots\right.}{e}$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

input
```
Int[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(5/2),x]
```

output
```
(2*a^2)/(3*d*e*(e*Cot[c + d*x])^(3/2)) + (2*a^2*(2/(d*e*Sqrt[e*Cot[c + d*x
]]) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e])) +
 ArcTan[1 + Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e]))/2 + (Log[e -
Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]) - L
og[e + Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c + d*x]^2]/(2*Sqrt[2]*Sqrt[
e]))/2))/(d*e)))/e
```

3.13. $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx$

3.13.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4025
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e
+ f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
 && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

3.13.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+\right.\right.}{4e(e^2)^{\frac{1}{4}}}}{de}$
default	$-\frac{2a^2\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+\right.\right.}{4e(e^2)^{\frac{1}{4}}}}{de}$
parts	$-\frac{2a^2e\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot}}{(e^2)}\right.\right.}{8e^4}}{d}$

input
```
int((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^2/e*(-1/4/e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/3/(e*c
ot(d*x+c))^(3/2)-2/e/(e*cot(d*x+c))^(1/2))
```

3.13.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.81

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \frac{3\left(de^3\cos(2\,dx+2\,c)+de^3\right)\left(-\frac{a^8}{d^4e^{10}}\right)^{\frac{1}{4}}\log\left(d^3e^8\left(-\frac{a^8}{d^4e^{10}}\right)^{\frac{3}{4}}+a^6\sqrt{\frac{e\cos(2\,dx+2\,c)}{\sin(2\,dx+2\,c)}}\right.}{}$$

input
```
integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*(-a^8/(d^4*e^10))^(1/4)*log(d^3*e^
8*(-a^8/(d^4*e^10))^(3/4) + a^6*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
2*c))) - 3*(I*d*e^3*cos(2*d*x + 2*c) + I*d*e^3)*(-a^8/(d^4*e^10))^(1/4)*lo
g(I*d^3*e^8*(-a^8/(d^4*e^10))^(3/4) + a^6*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c))) - 3*(-I*d*e^3*cos(2*d*x + 2*c) - I*d*e^3)*(-a^8/(d^4*e^10
))^(1/4)*log(-I*d^3*e^8*(-a^8/(d^4*e^10))^(3/4) + a^6*sqrt((e*cos(2*d*x +
2*c) + e)/sin(2*d*x + 2*c))) - 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*(-a^8/(d
^4*e^10))^(1/4)*log(-d^3*e^8*(-a^8/(d^4*e^10))^(3/4) + a^6*sqrt((e*cos(2*d
*x + 2*c) + e)/sin(2*d*x + 2*c))) - 2*(a^2*cos(2*d*x + 2*c) - 6*a^2*sin(2*
d*x + 2*c) - a^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^3*
cos(2*d*x + 2*c) + d*e^3)
```

3.13.6 Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = a^2\left(\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{2\cot(c+dx)}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{\cot^2(c+dx)}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx\right)$$

input
```
integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(5/2),x)
```

output
```
a**2*(Integral((e*cot(c + d*x))**(-5/2), x) + Integral(2*cot(c + d*x)/(e*c
ot(c + d*x))**(5/2), x) + Integral(cot(c + d*x)**2/(e*cot(c + d*x))**(5/2)
, x))
```

3.13.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.13.8 Giac [F]

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \int \frac{(a\cot(dx+c)+a)^2}{(e\cot(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(5/2), x)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 13.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.40

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \frac{4\,a^2\cot(c+d\,x)+\frac{2\,a^2}{3}}{d\,e\,(e\cot(c+d\,x))^{3/2}} + \frac{2\,(-1)^{1/4}\,a^2\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,e^{5/2}} - \frac{2\,(-1)^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,e^{5/2}}$$

input
```
int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(5/2),x)
```

output
```
(4*a^2*cot(c + d*x) + (2*a^2)/3)/(d*e*(e*cot(c + d*x))^(3/2)) + (2*(-1)^(1
/4)*a^2*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/(d*e^(5/2)) - (
2*(-1)^(1/4)*a^2*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/(d*e^
(5/2))
```

3.14 $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

3.14.1 Optimal result

Integrand size = 25, antiderivative size = 249

$$\begin{aligned}\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx &= -\frac{\sqrt{2}a^2\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}} \\ &+ \frac{\sqrt{2}a^2\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{de^{7/2}} + \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} \\ &+ \frac{4a^2}{3de^2(e\cot(c+dx))^{3/2}} - \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{7/2}} \\ &+ \frac{a^2\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\sqrt{2}de^{7/2}}\end{aligned}$$

output
```
2/5*a^2/d/e/(e*cot(d*x+c))^(5/2)+4/3*a^2/d/e^2/(e*cot(d*x+c))^(3/2)-1/2*a^
2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(7/2)*2^
(1/2)+1/2*a^2*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/
d/e^(7/2)*2^(1/2)-a^2*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/
2)/d/e^(7/2)+a^2*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*2^(1/2)/d/
e^(7/2)
```

3.14.2 Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.69

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx =$$

$$-\frac{a^2\sqrt{e\cot(c+dx)}(1+\cot(c+dx))^2\left(-20\cos^2(c+dx)+30\arctan\left(\sqrt[4]{-\cot^2(c+dx)}\right)(-\cot(c+dx))^3\right.}{}$$

input
```
Integrate[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(7/2),x]
```

output
```
-1/15*(a^2*Sqrt[e*Cot[c + d*x]]*(1 + Cot[c + d*x])^2*(-20*Cos[c + d*x]^2 +
 30*ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(3/4)*Cot[c + d*x]^(11
/4)*Sin[c + d*x]^2 + 30*ArcTanh[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(
3/4)*Cot[c + d*x]^(11/4)*Sin[c + d*x]^2 - 3*Sin[2*(c + d*x)])*Tan[c + d*x]
^4)/(d*e^4*(Cos[c + d*x] + Sin[c + d*x])^2)
```

3.14.3 Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.96, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4025, 27, 3042, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cot(c+dx)+a)^2}{(e\cot(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4025}$$

$$\frac{\int \frac{2a^2e}{(e\cot(c+dx))^{5/2}}dx}{e^2}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a^2\int \frac{1}{(e\cot(c+dx))^{5/2}}dx}{e}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{2a^2\int \frac{1}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{e}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 3955

$$\frac{2a^2\left(\frac{2}{3de(e\cot(c+dx))^{3/2}}-\frac{\int \frac{1}{\sqrt{e\cot(c+dx)}}dx}{e^2}\right)}{e}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{2a^2\left(\frac{2}{3de(e\cot(c+dx))^{3/2}}-\frac{\int \frac{1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}\right)}{e}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 3957

$$\frac{2a^2\left(\frac{\int \frac{1}{\sqrt{e\cot(c+dx)}\left(\cot^2(c+dx)e^2+e^2\right)}d(e\cot(c+dx))}{de}+\frac{2}{3de(e\cot(c+dx))^{3/2}}\right)}{e}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 266

$$\frac{2a^2\left(\frac{2\int \frac{1}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{de}+\frac{2}{3de(e\cot(c+dx))^{3/2}}\right)}{e}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 755

$$\frac{2a^2\left(\frac{2\left(\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\int \frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}\right)}{de}+\frac{2}{3de(e\cot(c+dx))^{3/2}}\right)}{e}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 1476

3.14. $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

$$2a^2\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+a}}{2e}\right.}{de}\right.$$
$$\overline{\qquad\qquad\qquad\qquad e\qquad\qquad\qquad\qquad}$$
$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\Big\downarrow$ 1082

$$2a^2\left(\frac{2\left(\frac{\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}\right)}{de}+\frac{2}{3de(e\cot(c\right.$$
$$\overline{\qquad\qquad\qquad\qquad e\qquad\qquad\qquad\qquad}$$
$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\Big\downarrow$ 217

$$\frac{2a^2\left(\frac{2\left(\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}\right)}{de}+\frac{2}{3de(e\cot(c+dx))^{3/2}}\right)}{e}+$$
$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\Big\downarrow$ 1479

$$2a^2\left(\frac{2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-}{2e}\right.}{de}\right.$$
$$\overline{\qquad\qquad\qquad\qquad e\qquad\qquad\qquad\qquad}$$
$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

3.14. $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

$\Big\downarrow$ 25

$$2a^2\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\text{arcta}}{2e}\right.}{de}\right.$$

$$e$$

$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\Big\downarrow$ 27

$$2a^2\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\text{arcta}}{2e}\right.}{de}\right.$$

$$e$$

$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\Big\downarrow$ 1103

$$2a^2\left(\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right.}{2\sqrt{2}\sqrt{e}}}{2e}\right.}{de}\right.$$

$$e$$

$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

input
```
Int[(a + a*Cot[c + d*x])^2/(e*Cot[c + d*x])^(7/2),x]
```

3.14. $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

output
```
(2*a^2)/(5*d*e*(e*Cot[c + d*x])^(5/2)) + (2*a^2*(2/(3*d*e*(e*Cot[c + d*x])
^(3/2)) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e]
)) + ArcTan[1 + Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) + (
-1/2*Log[e - Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c + d*x]^2]/(Sqrt[2]*S
qrt[e]) + Log[e + Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c + d*x]^2]/(2*Sq
rt[2]*Sqrt[e]))/(2*e)))/(d*e)))/e
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4025
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e
+ f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
 && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

3.14. $\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

3.14.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2a^2\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4e^3}\right.}{de}$
default	$-\frac{2a^2\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4e^3}\right.}{de}$
parts	$-\frac{2a^2e\left(-\frac{1}{5e^2(e\cot(dx+c))^{\frac{5}{2}}}+\frac{1}{e^4\sqrt{e\cot(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(d}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e^4\left(e^2\right)^{\frac{1}{4}}}\right.}{d}$

input
```
int((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^2/e*(-1/4/e^3*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/5/(e
*cot(d*x+c))^(5/2)-2/3/e/(e*cot(d*x+c))^(3/2))
```

3.14.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.18

$$\int\frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx=\frac{15\left(de^4\cos\left(2\,dx+2\,c\right)^2+2\,de^4\cos\left(2\,dx+2\,c\right)+de^4\right)\left(-\frac{a^8}{d^4e^{14}}\right)^{\frac{1}{4}}\log\left(de^4\left(-\frac{}{d}\right.\right.}{}$$

input
```
integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/15*(15*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)*(-a
^8/(d^4*e^14))^(1/4)*log(d*e^4*(-a^8/(d^4*e^14))^(1/4) + a^2*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - 15*(-I*d*e^4*cos(2*d*x + 2*c)^2 - 2*
I*d*e^4*cos(2*d*x + 2*c) - I*d*e^4)*(-a^8/(d^4*e^14))^(1/4)*log(I*d*e^4*(-
a^8/(d^4*e^14))^(1/4) + a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
)) - 15*(I*d*e^4*cos(2*d*x + 2*c)^2 + 2*I*d*e^4*cos(2*d*x + 2*c) + I*d*e^4
)*(-a^8/(d^4*e^14))^(1/4)*log(-I*d*e^4*(-a^8/(d^4*e^14))^(1/4) + a^2*sqrt(
(e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - 15*(d*e^4*cos(2*d*x + 2*c)^2
 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)*(-a^8/(d^4*e^14))^(1/4)*log(-d*e^4*(-
a^8/(d^4*e^14))^(1/4) + a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
)) - 2*(10*a^2*cos(2*d*x + 2*c)^2 - 10*a^2 + 3*(a^2*cos(2*d*x + 2*c) - a^2
)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^
4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)
```

3.14.6 Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = a^2\left(\int \frac{1}{(e\cot(c+dx))^{\frac{7}{2}}}\,dx + \int \frac{2\cot(c+dx)}{(e\cot(c+dx))^{\frac{7}{2}}}\,dx + \int \frac{\cot^2(c+dx)}{(e\cot(c+dx))^{\frac{7}{2}}}\,dx\right)$$

input
```
integrate((a+a*cot(d*x+c))**2/(e*cot(d*x+c))**(7/2),x)
```

output
```
a**2*(Integral((e*cot(c + d*x))**(-7/2), x) + Integral(2*cot(c + d*x)/(e*c
ot(c + d*x))**(7/2), x) + Integral(cot(c + d*x)**2/(e*cot(c + d*x))**(7/2)
, x))
```

3.14.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.14.8 Giac [F]

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx=\int \frac{(a\cot(dx+c)+a)^2}{(e\cot(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(7/2), x)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.40

$$\int \frac{(a+a\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx=\frac{\frac{4\,a^2\cot(c+d\,x)}{3}+\frac{2\,a^2}{5}}{d\,e\,(e\cot(c+d\,x))^{5/2}}-\frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,2\mathrm{i}}{d\,e^{7/2}}-\frac{(-1)^{1/4}\,a^2\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,2\mathrm{i}}{d\,e^{7/2}}$$

input
```
int((a + a*cot(c + d*x))^2/(e*cot(c + d*x))^(7/2),x)
```

output
```
((4*a^2*cot(c + d*x))/3 + (2*a^2)/5)/(d*e*(e*cot(c + d*x))^(5/2)) - ((-1)^
(1/4)*a^2*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*2i)/(d*e^(7/2)
) - ((-1)^(1/4)*a^2*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*2i)
/(d*e^(7/2))
```

3.15 $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx$

3.15.1 Optimal result

Integrand size = 25, antiderivative size = 186

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = \frac{2\sqrt{2}a^3e^{5/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} + \frac{4a^3e^2\sqrt{e\cot(c+dx)}}{d} + \frac{4a^3e(e\cot(c+dx))^{3/2}}{3d} - \frac{4a^3(e\cot(c+dx))^{5/2}}{5d} - \frac{40a^3(e\cot(c+dx))^{7/2}}{63de} - \frac{2(e\cot(c+dx))^{7/2}\left(a^3+a^3\cot(c+dx)\right)}{9de}$$

output
```
4/3*a^3*e*(e*cot(d*x+c))^(3/2)/d-4/5*a^3*(e*cot(d*x+c))^(5/2)/d-40/63*a^3*
(e*cot(d*x+c))^(7/2)/d/e-2/9*(e*cot(d*x+c))^(7/2)*(a^3+a^3*cot(d*x+c))/d/e
+2*a^3*e^(5/2)*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+
c))^(1/2))*2^(1/2)/d+4*a^3*e^2*(e*cot(d*x+c))^(1/2)/d
```

3.15.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 847 vs. 2(186) = 372.

Time = 6.16 (sec) , antiderivative size = 847, normalized size of antiderivative = 4.55

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx =$$

$$-\frac{2\cos^2(c+dx)(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin(c+dx)}{9d(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{6\cos(c+dx)(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^2(c+dx)}{7d(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{4(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)}{5d(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{2\arctan\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+}{d\cot^{\frac{11}{4}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{2\text{arctanh}\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+}{d\cot^{\frac{11}{4}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)}{d\cot^{\frac{5}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)}{d\cot^{\frac{5}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^3(c+dx)}{\sqrt{2}d\cot^{\frac{5}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\log\left(1+\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)\sin^3(c+dx)}{\sqrt{2}d\cot^{\frac{5}{2}}(c+dx)(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{4(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)\tan(c+dx)}{3d(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{4(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\sin^3(c+dx)\tan^2(c+dx)}{d(\cos(c+dx)+\sin(c+dx))^3}$$

input
```
Integrate[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3,x]
```

3.15. $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx$

output
```
(-2*Cos[c + d*x]^2*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d
*x])/(9*d*(Cos[c + d*x] + Sin[c + d*x])^3) - (6*Cos[c + d*x]*(e*Cot[c + d*
x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^2)/(7*d*(Cos[c + d*x] + Sin[
c + d*x])^3) - (4*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*
x]^3)/(5*d*(Cos[c + d*x] + Sin[c + d*x])^3) - (2*ArcTan[(-Cot[c + d*x])^(1
/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*x])^(1/4)*(e*Cot[c + d*x])^(5/2)*(a +
a*Cot[c + d*x])^3*Sin[c + d*x]^3)/(d*Cot[c + d*x]^(11/4)*(Cos[c + d*x] + S
in[c + d*x])^3) + (2*ArcTanh[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-C
ot[c + d*x])^(1/4)*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d
*x]^3)/(d*Cot[c + d*x]^(11/4)*(Cos[c + d*x] + Sin[c + d*x])^3) + (Sqrt[2]*
ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c
 + d*x])^3*Sin[c + d*x]^3)/(d*Cot[c + d*x]^(5/2)*(Cos[c + d*x] + Sin[c + d
*x])^3) - (Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*(e*Cot[c + d*x])
^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3)/(d*Cot[c + d*x]^(5/2)*(Cos[c
 + d*x] + Sin[c + d*x])^3) + ((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^
3*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^3)/(Sqrt
[2]*d*Cot[c + d*x]^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) - ((e*Cot[c + d*
x])^(5/2)*(a + a*Cot[c + d*x])^3*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[
c + d*x]]*Sin[c + d*x]^3)/(Sqrt[2]*d*Cot[c + d*x]^(5/2)*(Cos[c + d*x] + Si
n[c + d*x])^3) + (4*(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3*Sin[c...
```

3.15.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4049, 25, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4011, 3042, 4015, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(c+dx)+a)^3(e\cot(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4049}$$

$$-\frac{2\int -(e\cot(c+dx))^{5/2}\left(10e\cot^2(c+dx)a^3+ea^3+9e\cot(c+dx)a^3\right)dx}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

$$\Big\downarrow \; 25$$

$$\frac{2\int (e\cot(c+dx))^{5/2}\left(10e\cot^2(c+dx)a^3+ea^3+9e\cot(c+dx)a^3\right)dx}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(10e\tan\left(c+dx+\frac{\pi}{2}\right)^2a^3+ea^3-9e\tan\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

$$\Big\downarrow \; 4113$$

$$\frac{2\left(\int (e\cot(c+dx))^{5/2}\left(9a^3e\cot(c+dx)-9a^3e\right)dx-\frac{20a^3(e\cot(c+dx))^{7/2}}{7d}\right)}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(-9ea^3-9e\tan\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx-\frac{20a^3(e\cot(c+dx))^{7/2}}{7d}\right)}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

$$\Big\downarrow \; 4011$$

$$\frac{2\left(\int (e\cot(c+dx))^{3/2}\left(-9e^2a^3-9e^2\cot(c+dx)a^3\right)dx-\frac{20a^3(e\cot(c+dx))^{7/2}}{7d}-\frac{18a^3e(e\cot(c+dx))^{5/2}}{5d}\right)}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(9a^3e^2\tan\left(c+dx+\frac{\pi}{2}\right)-9a^3e^2\right)dx-\frac{20a^3(e\cot(c+dx))^{7/2}}{7d}-\frac{18a^3e(e\cot(c+dx))^{5/2}}{5d}\right)}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

$$\Big\downarrow \; 4011$$

3.15. $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx$

$$\frac{2\left(\int \sqrt{e\cot(c+dx)}\left(9a^3e^3-9a^3e^3\cot(c+dx)\right)dx+\frac{6a^3e^2(e\cot(c+dx))^{3/2}}{d}-\frac{20a^3(e\cot(c+dx))^{7/2}}{7d}-\frac{18a^3e(e\cot(c+dx))^{5/2}}{5d}\right)}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

$\big\downarrow$ 3042

$$\frac{2\left(\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(9a^3e^3+9a^3\tan\left(c+dx+\frac{\pi}{2}\right)e^3\right)dx+\frac{6a^3e^2(e\cot(c+dx))^{3/2}}{d}-\frac{20a^3(e\cot(c+dx))^{7/2}}{7d}-\frac{18a^3e(e\cot(c+dx))^{5/2}}{5d}\right)}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

$\big\downarrow$ 4011

$$\frac{2\left(\int \frac{9a^3e^4+9a^3\cot(c+dx)e^4}{\sqrt{e\cot(c+dx)}}dx+\frac{18a^3e^3\sqrt{e\cot(c+dx)}}{d}+\frac{6a^3e^2(e\cot(c+dx))^{3/2}}{d}-\frac{20a^3(e\cot(c+dx))^{7/2}}{7d}-\frac{18a^3e(e\cot(c+dx))^{5/2}}{5d}\right)}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

$\big\downarrow$ 3042

$$\frac{2\left(\int \frac{9a^3e^4-9a^3e^4\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{18a^3e^3\sqrt{e\cot(c+dx)}}{d}+\frac{6a^3e^2(e\cot(c+dx))^{3/2}}{d}-\frac{20a^3(e\cot(c+dx))^{7/2}}{7d}-\frac{18a^3e(e\cot(c+dx))^{5/2}}{5d}\right)}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

$\big\downarrow$ 4015

$$\frac{2\left(-\frac{162a^6e^8\int \frac{1}{-162a^6e^8-81\left(a^3e^4-a^3e^4\cot(c+dx)\right)^2\tan(c+dx)}d\frac{9\left(a^3e^4-a^3e^4\cot(c+dx)\right)}{\sqrt{e\cot(c+dx)}}}{d}+\frac{18a^3e^3\sqrt{e\cot(c+dx)}}{d}+\frac{6a^3e^2(e\cot(c+dx))^{3/2}}{d}-\frac{20a^3(e\cot(c+dx))^{7/2}}{7d}-\frac{18a^3e(e\cot(c+dx))^{5/2}}{5d}\right)}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

$\big\downarrow$ 218

$$\frac{2\left(\frac{9\sqrt{2}a^3e^{7/2}\arctan\left(\frac{a^3e^4-a^3e^4\cot(c+dx)}{\sqrt{2}a^3e^{7/2}\sqrt{e\cot(c+dx)}}\right)}{d}+\frac{18a^3e^3\sqrt{e\cot(c+dx)}}{d}+\frac{6a^3e^2(e\cot(c+dx))^{3/2}}{d}-\frac{18a^3e(e\cot(c+dx))^{5/2}}{5d}-\frac{20a^3(e\cot(c+dx))^{7/2}}{7d}\right)}{9e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{7/2}}{9de}$$

3.15. $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx$

input
```
Int[(e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3,x]
```

output
```
(-2*(e*Cot[c + d*x])^(7/2)*(a^3 + a^3*Cot[c + d*x]))/(9*d*e) + (2*((9*Sqrt
[2]*a^3*e^(7/2)*ArcTan[(a^3*e^4 - a^3*e^4*Cot[c + d*x])/(Sqrt[2]*a^3*e^(7/
2)*Sqrt[e*Cot[c + d*x]])])/d + (18*a^3*e^3*Sqrt[e*Cot[c + d*x]])/d + (6*a^
3*e^2*(e*Cot[c + d*x])^(3/2))/d - (18*a^3*e*(e*Cot[c + d*x])^(5/2))/(5*d)
- (20*a^3*(e*Cot[c + d*x])^(7/2))/(7*d)))/(9*e)
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.15.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(157) = 314$.

Time = 0.36 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.90

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{9}{2}}}{9}+\frac{3e(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{2e^2(e\cot(dx+c))^{\frac{5}{2}}}{5}-\frac{2e^3(e\cot(dx+c))^{\frac{3}{2}}}{3}-2\sqrt{e\cot(dx+c)}\,e^4+2e^5\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\cdots\right.}{\cdots}\right.\right.}{\cdots}$
default	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{9}{2}}}{9}+\frac{3e(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{2e^2(e\cot(dx+c))^{\frac{5}{2}}}{5}-\frac{2e^3(e\cot(dx+c))^{\frac{3}{2}}}{3}-2\sqrt{e\cot(dx+c)}\,e^4+2e^5\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\cdots\right.}{\cdots}\right.\right.}{\cdots}$
parts	$-\frac{2a^3e\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}-\frac{e^2\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\cdots\right.\right.}{8(e^2)^{\frac{1}{4}}}\right.}{d}$

input
```
int((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^3/e^2*(1/9*(e*cot(d*x+c))^(9/2)+3/7*e*(e*cot(d*x+c))^(7/2)+2/5*e^2*
(e*cot(d*x+c))^(5/2)-2/3*e^3*(e*cot(d*x+c))^(3/2)-2*(e*cot(d*x+c))^(1/2)*e
^4+2*e^5*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8/(e^2)^(
1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1
/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.88

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = \left[\frac{315\sqrt{2}\left(a^3e^2\cos(2\,dx+2\,c)^2-2\,a^3e^2\cos(2\,dx+2\,c)+a^3e^2\right)\sqrt{-e}\log\left(-\sqrt{2}\sqrt{-e}\sqrt{\right.}\right.}{}$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/315*(315*sqrt(2)*(a^3*e^2*cos(2*d*x + 2*c)^2 - 2*a^3*e^2*cos(2*d*x + 2*
c) + a^3*e^2)*sqrt(-e)*log(-sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) - 1) - 2*e*sin(2*d
*x + 2*c) + e) + 2*(721*a^3*e^2*cos(2*d*x + 2*c)^2 - 1330*a^3*e^2*cos(2*d*
x + 2*c) + 469*a^3*e^2 - 15*(23*a^3*e^2*cos(2*d*x + 2*c) - 5*a^3*e^2)*sin(
2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*cos(2*d*
x + 2*c)^2 - 2*d*cos(2*d*x + 2*c) + d), 2/315*(315*sqrt(2)*(a^3*e^2*cos(2*
d*x + 2*c)^2 - 2*a^3*e^2*cos(2*d*x + 2*c) + a^3*e^2)*sqrt(e)*arctan(-1/2*s
qrt(2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x
+ 2*c) - sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x + 2*c) + e)) + (721*a^3*e^2*co
s(2*d*x + 2*c)^2 - 1330*a^3*e^2*cos(2*d*x + 2*c) + 469*a^3*e^2 - 15*(23*a^
3*e^2*cos(2*d*x + 2*c) - 5*a^3*e^2)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x +
2*c) + e)/sin(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c)^2 - 2*d*cos(2*d*x + 2*c)
+ d)]
```

3.15.6 Sympy [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = a^3\left(\int (e\cot(c+dx))^{\frac{5}{2}}\,dx + \int 3(e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)\,dx + \int 3(e\cot(c+dx))^{\frac{5}{2}}\cot^2(c+dx)\,dx + \int (e\cot(c+dx))^{\frac{5}{2}}\cot^3(c+dx)\,dx\right)$$

input
```
integrate((e*cot(d*x+c))**(5/2)*(a+a*cot(d*x+c))**3,x)
```

output
```
a**3*(Integral((e*cot(c + d*x))**(5/2), x) + Integral(3*(e*cot(c + d*x))**
(5/2)*cot(c + d*x), x) + Integral(3*(e*cot(c + d*x))**(5/2)*cot(c + d*x)**
2, x) + Integral((e*cot(c + d*x))**(5/2)*cot(c + d*x)**3, x))
```

3.15.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.15.8 Giac [F]

$$\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx = \int (a\cot(dx+c)+a)^3(e\cot(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((e*cot(d*x+c))^(5/2)*(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(5/2), x)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

$$\begin{aligned}\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx &= \frac{4\,a^3\,e^2\,\sqrt{e\cot(c+d\,x)}}{d}\\ &- \frac{4\,a^3\,(e\cot(c+d\,x))^{5/2}}{5\,d} - \frac{6\,a^3\,(e\cot(c+d\,x))^{7/2}}{7\,d\,e}\\ &- \frac{2\,a^3\,(e\cot(c+d\,x))^{9/2}}{9\,d\,e^2} + \frac{4\,a^3\,e\,(e\cot(c+d\,x))^{3/2}}{3\,d}\\ &- \frac{\sqrt{2}\,a^3\,e^{5/2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{d}\end{aligned}$$

input
```
int((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^3,x)
```

output
```
(4*a^3*e^2*(e*cot(c + d*x))^(1/2))/d - (4*a^3*(e*cot(c + d*x))^(5/2))/(5*d
) - (6*a^3*(e*cot(c + d*x))^(7/2))/(7*d*e) - (2*a^3*(e*cot(c + d*x))^(9/2)
)/(9*d*e^2) + (4*a^3*e*(e*cot(c + d*x))^(3/2))/(3*d) - (2^(1/2)*a^3*e^(5/2
)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*
(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*
e^(3/2)))))/d
```

3.15. $\int (e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3\,dx$

3.16 $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx$

3.16.1 Optimal result

Integrand size = 25, antiderivative size = 160

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = -\frac{2\sqrt{2}a^3e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} + \frac{4a^3e\sqrt{e\cot(c+dx)}}{d} - \frac{4a^3(e\cot(c+dx))^{3/2}}{3d} - \frac{32a^3(e\cot(c+dx))^{5/2}}{35de} - \frac{2(e\cot(c+dx))^{5/2}\left(a^3+a^3\cot(c+dx)\right)}{7de}$$

output
```
-4/3*a^3*(e*cot(d*x+c))^(3/2)/d-32/35*a^3*(e*cot(d*x+c))^(5/2)/d/e-2/7*(e*
cot(d*x+c))^(5/2)*(a^3+a^3*cot(d*x+c))/d/e-2*a^3*e^(3/2)*arctanh(1/2*(e^(1
/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d+4*a^3*e*(e
*cot(d*x+c))^(1/2)/d
```

3.16.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 380 vs. 2(160) = 320.

Time = 5.18 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.38

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = -\frac{a^3(e\cot(c+dx))^{3/2}(1+\cot(c+dx))^3\left(-420\arctan\left(\sqrt[4]{-\cot^2(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\sin^3(c+dx)+\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^3,x]
```

output
```
-1/210*(a^3*(e*Cot[c + d*x])^(3/2)*(1 + Cot[c + d*x])^3*(-420*ArcTan[(-Cot
[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4)*Sin[c + d*x]^3 + 420*ArcTanh[(-C
ot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4)*Sin[c + d*x]^3 + Cot[c + d*x]^
(1/4)*Sin[c + d*x]*(60*Cos[c + d*x]^2*Cot[c + d*x]^(3/2) - 210*Sqrt[2]*Arc
Tan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sin[c + d*x]^2 + 210*Sqrt[2]*ArcTan[1
+ Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sin[c + d*x]^2 - 840*Sqrt[Cot[c + d*x]]*Sin[
c + d*x]^2 + 280*Cot[c + d*x]^(3/2)*Sin[c + d*x]^2 - 105*Sqrt[2]*Log[1 - S
qrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 + 105*Sqrt[2]*Log
[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 + 126*Cot[c
 + d*x]^(3/2)*Sin[2*(c + d*x)])))/(d*Cot[c + d*x]^(7/4)*(Cos[c + d*x] + Si
n[c + d*x])^3)
```

3.16.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4049, 25, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4015, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(c+dx)+a)^3(e\cot(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4049}$$

$$-\frac{2\int -(e\cot(c+dx))^{3/2}\left(8e\cot^2(c+dx)a^3+ea^3+7e\cot(c+dx)a^3\right)dx}{7e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{5/2}}{7de}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int (e\cot(c+dx))^{3/2}\left(8e\cot^2(c+dx)a^3+ea^3+7e\cot(c+dx)a^3\right)dx}{7e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{5/2}}{7de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(8e\tan\left(c+dx+\frac{\pi}{2}\right)^2a^3+ea^3-7e\tan\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx}{7e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{5/2}}{7de}$$

$\Big\downarrow$ 4113

$$\frac{2\left(\int (e\cot(c+dx))^{3/2}\left(7a^3e\cot(c+dx)-7a^3e\right)dx-\frac{16a^3(e\cot(c+dx))^{5/2}}{5d}\right)}{7e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{5/2}}{7de}$$

$\Big\downarrow$ 3042

$$\frac{2\left(\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(-7ea^3-7e\tan\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx-\frac{16a^3(e\cot(c+dx))^{5/2}}{5d}\right)}{7e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{5/2}}{7de}$$

$\Big\downarrow$ 4011

$$\frac{2\left(\int \sqrt{e\cot(c+dx)}\left(-7e^2a^3-7e^2\cot(c+dx)a^3\right)dx-\frac{16a^3(e\cot(c+dx))^{5/2}}{5d}-\frac{14a^3e(e\cot(c+dx))^{3/2}}{3d}\right)}{7e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{5/2}}{7de}$$

$\Big\downarrow$ 3042

$$\frac{2\left(\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(7a^3e^2\tan\left(c+dx+\frac{\pi}{2}\right)-7a^3e^2\right)dx-\frac{16a^3(e\cot(c+dx))^{5/2}}{5d}-\frac{14a^3e(e\cot(c+dx))^{3/2}}{3d}\right)}{7e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{5/2}}{7de}$$

$\Big\downarrow$ 4011

$$\frac{2\left(\int \frac{7a^3e^3-7a^3e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx+\frac{14a^3e^2\sqrt{e\cot(c+dx)}}{d}-\frac{16a^3(e\cot(c+dx))^{5/2}}{5d}-\frac{14a^3e(e\cot(c+dx))^{3/2}}{3d}\right)}{7e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{5/2}}{7de}$$

$\Big\downarrow$ 3042

$$\frac{2\left(\int \frac{7a^3e^3+7a^3\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{14a^3e^2\sqrt{e\cot(c+dx)}}{d}-\frac{16a^3(e\cot(c+dx))^{5/2}}{5d}-\frac{14a^3e(e\cot(c+dx))^{3/2}}{3d}\right)}{7e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{5/2}}{7de}$$

3.16. $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx$

$$\Big\downarrow \; 4015$$

$$\frac{2\left(-\frac{98a^6e^6\int \frac{1}{98a^6e^6-49\left(a^3e^3+a^3\cot(c+dx)e^3\right)^2\tan(c+dx)}d\frac{7\left(a^3e^3+a^3\cot(c+dx)e^3\right)}{\sqrt{e\cot(c+dx)}}}{d}+\frac{14a^3e^2\sqrt{e\cot(c+dx)}}{d}-\frac{14a^3e(e\cot(c+dx))^{3/2}}{3d}-\frac{16a^3 \cdots}{}\right.}{7e}$$

$$\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{5/2}}{7de}$$

$$\Big\downarrow \; 221$$

$$\frac{2\left(-\frac{7\sqrt{2}a^3e^{5/2}\text{arctanh}\left(\frac{a^3e^3\cot(c+dx)+a^3e^3}{\sqrt{2}a^3e^{5/2}\sqrt{e\cot(c+dx)}}\right)}{d}+\frac{14a^3e^2\sqrt{e\cot(c+dx)}}{d}-\frac{16a^3(e\cot(c+dx))^{5/2}}{5d}-\frac{14a^3e(e\cot(c+dx))^{3/2}}{3d}\right)}{7e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{5/2}}{7de}$$

input
```
Int[(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^3,x]
```

output
```
(-2*(e*Cot[c + d*x])^(5/2)*(a^3 + a^3*Cot[c + d*x]))/(7*d*e) + (2*((-7*Sqr
t[2]*a^3*e^(5/2)*ArcTanh[(a^3*e^3 + a^3*e^3*Cot[c + d*x])/(Sqrt[2]*a^3*e^(
5/2)*Sqrt[e*Cot[c + d*x]])])/d + (14*a^3*e^2*Sqrt[e*Cot[c + d*x]])/d - (14
*a^3*e*(e*Cot[c + d*x])^(3/2))/(3*d) - (16*a^3*(e*Cot[c + d*x])^(5/2))/(5*
d)))/(7*e)
```

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.16. $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx$

3.16.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(135) = 270$.

Time = 0.04 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.12

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{3e(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e^2(e\cot(dx+c))^{\frac{3}{2}}}{3}-2\sqrt{e\cot(dx+c)}\,e^3+2e^4\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)}{e\cot(dx+c)-(e^2)}\right.\right.}{}\right.\right.}{}$
default	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{3e(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2e^2(e\cot(dx+c))^{\frac{3}{2}}}{3}-2\sqrt{e\cot(dx+c)}\,e^3+2e^4\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)}{e\cot(dx+c)-(e^2)}\right.\right.}{}\right.\right.}{}$
parts	$-\frac{2a^3e\left(\sqrt{e\cot(dx+c)}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\right.}{8}\right.}{d}$

input
```
int((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^3/e^2*(1/7*(e*cot(d*x+c))^(7/2)+3/5*e*(e*cot(d*x+c))^(5/2)+2/3*e^2*
(e*cot(d*x+c))^(3/2)-2*(e*cot(d*x+c))^(1/2)*e^3+2*e^4*(1/8/e*(e^2)^(1/4)*2
^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1
/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+
2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c
)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)+1))))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.04

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = \left[\frac{105\sqrt{2}(a^3e\cos(2\,dx+2\,c)-a^3e)\sqrt{e}\log\left(\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\text{s}\right.}{}\right.$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/105*(105*sqrt(2)*(a^3*e*cos(2*d*x + 2*c) - a^3*e)*sqrt(e)*log(sqrt(2)*s
qrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) -
 sin(2*d*x + 2*c) - 1) + 2*e*sin(2*d*x + 2*c) + e)*sin(2*d*x + 2*c) - 2*(5
5*a^3*e*cos(2*d*x + 2*c)^2 - 30*a^3*e*cos(2*d*x + 2*c) - 85*a^3*e - 21*(13
*a^3*e*cos(2*d*x + 2*c) - 7*a^3*e)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2
*c) + e)/sin(2*d*x + 2*c)))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x + 2*c)), 2
/105*(105*sqrt(2)*(a^3*e*cos(2*d*x + 2*c) - a^3*e)*sqrt(-e)*arctan(1/2*sqr
t(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x +
 2*c) + sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x + 2*c) + e))*sin(2*d*x + 2*c) -
 (55*a^3*e*cos(2*d*x + 2*c)^2 - 30*a^3*e*cos(2*d*x + 2*c) - 85*a^3*e - 21*
(13*a^3*e*cos(2*d*x + 2*c) - 7*a^3*e)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x
+ 2*c) + e)/sin(2*d*x + 2*c)))/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x + 2*c))
]
```

3.16.6 Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = a^3\left(\int (e\cot(c+dx))^{\frac{3}{2}}\,dx + \int 3(e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)\,dx + \int 3(e\cot(c+dx))^{\frac{3}{2}}\cot^2(c+dx)\,dx + \int (e\cot(c+dx))^{\frac{3}{2}}\cot^3(c+dx)\,dx\right)$$

input
```
integrate((e*cot(d*x+c))**(3/2)*(a+a*cot(d*x+c))**3,x)
```

output
```
a**3*(Integral((e*cot(c + d*x))**(3/2), x) + Integral(3*(e*cot(c + d*x))**
(3/2)*cot(c + d*x), x) + Integral(3*(e*cot(c + d*x))**(3/2)*cot(c + d*x)**
2, x) + Integral((e*cot(c + d*x))**(3/2)*cot(c + d*x)**3, x))
```

3.16.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.16.8 Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = \int (a\cot(dx+c)+a)^3(e\cot(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2), x)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx = \frac{4\,a^3\,e\,\sqrt{e\cot(c+d\,x)}}{d} - \frac{6\,a^3\,(e\cot(c+d\,x))^{5/2}}{5\,d\,e} - \frac{2\,a^3\,(e\cot(c+d\,x))^{7/2}}{7\,d\,e^2} - \frac{4\,a^3\,(e\cot(c+d\,x))^{3/2}}{3\,d} + \frac{\sqrt{2}\,a^3\,e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,a^6\,e^{9/2}\,\sqrt{e\cot(c+d\,x)}\,32\mathrm{i}}{32\,a^6\,e^5+32\,a^6\,e^5\cot(c+d\,x)}\right)\,2\mathrm{i}}{d}$$

input
```
int((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^3,x)
```

output
```
(4*a^3*e*(e*cot(c + d*x))^(1/2))/d - (6*a^3*(e*cot(c + d*x))^(5/2))/(5*d*e
) - (2*a^3*(e*cot(c + d*x))^(7/2))/(7*d*e^2) - (4*a^3*(e*cot(c + d*x))^(3/
2))/(3*d) + (2^(1/2)*a^3*e^(3/2)*atan((2^(1/2)*a^6*e^(9/2)*(e*cot(c + d*x)
)^(1/2)*32i)/(32*a^6*e^5 + 32*a^6*e^5*cot(c + d*x)))*2i)/d
```

3.16. $\int (e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3\,dx$

3.17 $\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx$

3.17.1 Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx = -\frac{2\sqrt{2}a^3\sqrt{e}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d} - \frac{4a^3\sqrt{e\cot(c+dx)}}{d} - \frac{8a^3(e\cot(c+dx))^{3/2}}{5de} - \frac{2(e\cot(c+dx))^{3/2}\left(a^3+a^3\cot(c+dx)\right)}{5de}$$

output
```
-8/5*a^3*(e*cot(d*x+c))^(3/2)/d/e-2/5*(e*cot(d*x+c))^(3/2)*(a^3+a^3*cot(d*
x+c))/d/e-2*a^3*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x
+c))^(1/2))*2^(1/2)*e^(1/2)/d-4*a^3*(e*cot(d*x+c))^(1/2)/d
```

3.17.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 360 vs. 2(138) = 276.

Time = 3.43 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.61

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx = -\frac{a^3\sqrt{e\cot(c+dx)}(1+\cot(c+dx))^3\left(-20\arctan\left(\sqrt[4]{-\cot^2(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\sin^3(c+dx)+\right.}{}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^3,x]
```

output
```
-1/10*(a^3*Sqrt[e*Cot[c + d*x]]*(1 + Cot[c + d*x])^3*(-20*ArcTan[(-Cot[c +
 d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4)*Sin[c + d*x]^3 + 20*ArcTanh[(-Cot[c
+ d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4)*Sin[c + d*x]^3 + Cot[c + d*x]^(1/4)
*Sin[c + d*x]*(4*Cos[c + d*x]^2*Sqrt[Cot[c + d*x]] + 10*Sqrt[2]*ArcTan[1 -
 Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sin[c + d*x]^2 - 10*Sqrt[2]*ArcTan[1 + Sqrt[2
]*Sqrt[Cot[c + d*x]]]*Sin[c + d*x]^2 + 40*Sqrt[Cot[c + d*x]]*Sin[c + d*x]^
2 + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d
*x]^2 - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c
 + d*x]^2 + 10*Sqrt[Cot[c + d*x]]*Sin[2*(c + d*x)])))/(d*Cot[c + d*x]^(3/4
)*(Cos[c + d*x] + Sin[c + d*x])^3)
```

3.17.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4049, 25, 3042, 4113, 3042, 4011, 3042, 4015, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(c+dx)+a)^3\sqrt{e\cot(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4049}$$

$$-\frac{2\int -\sqrt{e\cot(c+dx)}\left(6e\cot^2(c+dx)a^3+ea^3+5e\cot(c+dx)a^3\right)dx}{5e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{3/2}}{5de}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \sqrt{e\cot(c+dx)}\left(6e\cot^2(c+dx)a^3+ea^3+5e\cot(c+dx)a^3\right)dx}{5e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{3/2}}{5de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(6e\tan\left(c+dx+\frac{\pi}{2}\right)^2a^3+ea^3-5e\tan\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx}{5e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{3/2}}{5de}$$

$$\Big\downarrow \text{4113}$$

$$\frac{2\left(\int \sqrt{e\cot(c+dx)}\left(5a^3e\cot(c+dx)-5a^3e\right)dx-\frac{4a^3(e\cot(c+dx))^{3/2}}{d}\right)}{5e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{3/2}}{5de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-5ea^3-5e\tan\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx-\frac{4a^3(e\cot(c+dx))^{3/2}}{d}\right)}{5e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{3/2}}{5de}$$

$$\Big\downarrow \text{4011}$$

$$\frac{2\left(\int \frac{-5e^2a^3-5e^2\cot(c+dx)a^3}{\sqrt{e\cot(c+dx)}}dx-\frac{4a^3(e\cot(c+dx))^{3/2}}{d}-\frac{10a^3e\sqrt{e\cot(c+dx)}}{d}\right)}{5e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{3/2}}{5de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\int \frac{5a^3e^2\tan\left(c+dx+\frac{\pi}{2}\right)-5a^3e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{4a^3(e\cot(c+dx))^{3/2}}{d}-\frac{10a^3e\sqrt{e\cot(c+dx)}}{d}\right)}{5e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{3/2}}{5de}$$

$$\Big\downarrow \text{4015}$$

$$\frac{2\left(-\frac{50a^6e^4\int \frac{1}{-50e^4a^6-25\left(a^3e^2-a^3e^2\cot(c+dx)\right)^2\tan(c+dx)}d\left(-\frac{5\left(a^3e^2-a^3e^2\cot(c+dx)\right)}{\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{4a^3(e\cot(c+dx))^{3/2}}{d}-\frac{10a^3e\sqrt{e\cot(c+dx)}}{d}\right)}{5e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{3/2}}{5de}$$

$$\Big\downarrow \text{218}$$

3.17. $\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx$

$$\frac{2\left(-\frac{5\sqrt{2}a^3e^{3/2}\arctan\left(\frac{a^3e^2-a^3e^2\cot(c+dx)}{\sqrt{2}a^3e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{4a^3(e\cot(c+dx))^{3/2}}{d}-\frac{10a^3e\sqrt{e\cot(c+dx)}}{d}\right)}{5e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)(e\cot(c+dx))^{3/2}}{5de}$$

input
```
Int[Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^3,x]
```

output
```
(-2*(e*Cot[c + d*x])^(3/2)*(a^3 + a^3*Cot[c + d*x]))/(5*d*e) + (2*((-5*Sqr
t[2]*a^3*e^(3/2)*ArcTan[(a^3*e^2 - a^3*e^2*Cot[c + d*x])/(Sqrt[2]*a^3*e^(3
/2)*Sqrt[e*Cot[c + d*x]])])/d - (10*a^3*e*Sqrt[e*Cot[c + d*x]])/d - (4*a^3
*(e*Cot[c + d*x])^(3/2))/d))/(5*e)
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.17.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(117) = 234$.

Time = 0.05 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.34

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{5}{2}}}{5}+e(e\cot(dx+c))^{\frac{3}{2}}+2\sqrt{e\cot(dx+c)}\,e^2-2e^3\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\cdots}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\cdots}\right.\right.}{\cdots}\right.\right.}{\cdots}$
default	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{5}{2}}}{5}+e(e\cot(dx+c))^{\frac{3}{2}}+2\sqrt{e\cot(dx+c)}\,e^2-2e^3\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\cdots}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\cdots}\right.\right.}{\cdots}\right.\right.}{\cdots}$
parts	$-\frac{a^3e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c\cdots}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4d\left(e^2\right)^{\frac{1}{4}}}$

input
```
int((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^3/e^2*(1/5*(e*cot(d*x+c))^(5/2)+e*(e*cot(d*x+c))^(3/2)+2*(e*cot(d*x
+c))^(1/2)*e^2-2*e^3*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))
+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.65

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx$$

$$= \left[\frac{5\sqrt{2}(a^3\cos(2\,dx+2\,c)-a^3)\sqrt{-e}\log\left(\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)-1)\right.}{5\,(d\cos(2\,dx+}\right.$$

$$-\frac{2\left(5\sqrt{2}(a^3\cos(2\,dx+2\,c)-a^3)\sqrt{e}\arctan\left(-\frac{\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)+1)}{2\,(e\cos(2\,dx+2\,c)+e)}\right)+(9\,a^3\cos\right.}{5\,(d\cos(2\,dx+2\,c)-d)}$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/5*(5*sqrt(2)*(a^3*cos(2*d*x + 2*c) - a^3)*sqrt(-e)*log(sqrt(2)*sqrt(-e)
*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2
*d*x + 2*c) - 1) - 2*e*sin(2*d*x + 2*c) + e) - 2*(9*a^3*cos(2*d*x + 2*c) -
 5*a^3*sin(2*d*x + 2*c) - 11*a^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x
+ 2*c)))/(d*cos(2*d*x + 2*c) - d), -2/5*(5*sqrt(2)*(a^3*cos(2*d*x + 2*c) -
 a^3)*sqrt(e)*arctan(-1/2*sqrt(2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x + 2
*c) + e)) + (9*a^3*cos(2*d*x + 2*c) - 5*a^3*sin(2*d*x + 2*c) - 11*a^3)*sqr
t((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*cos(2*d*x + 2*c) - d)]
```

3.17.6 Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx = a^3\left(\int \sqrt{e\cot\left(c+dx\right)}\,dx + \int 3\sqrt{e\cot\left(c+dx\right)}\cot\left(c+dx\right)dx + \int 3\sqrt{e\cot\left(c+dx\right)}\cot^2\left(c+dx\right)dx + \int \sqrt{e\cot\left(c+dx\right)}\cot^3\left(c+dx\right)dx\right)$$

input
```
integrate((e*cot(d*x+c))**(1/2)*(a+a*cot(d*x+c))**3,x)
```

output
```
a**3*(Integral(sqrt(e*cot(c + d*x)), x) + Integral(3*sqrt(e*cot(c + d*x))*
cot(c + d*x), x) + Integral(3*sqrt(e*cot(c + d*x))*cot(c + d*x)**2, x) + I
ntegral(sqrt(e*cot(c + d*x))*cot(c + d*x)**3, x))
```

3.17.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.17.8 Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx = \int (a\cot(dx+c)+a)^3\sqrt{e\cot(dx+c)}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^3*sqrt(e*cot(d*x + c)), x)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3\,dx = \frac{\sqrt{2}\,a^3\sqrt{e}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{d} - \frac{2\,a^3\,(e\cot(c+d\,x))^{3/2}}{d\,e} - \frac{2\,a^3\,(e\cot(c+d\,x))^{5/2}}{5\,d\,e^2} - \frac{4\,a^3\,\sqrt{e\cot(c+d\,x)}}{d}$$

input
```
int((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^3,x)
```

output
```
(2^(1/2)*a^3*e^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)))
 + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c
 + d*x))^(3/2))/(2*e^(3/2)))))/d - (2*a^3*(e*cot(c + d*x))^(3/2))/(d*e) -
(2*a^3*(e*cot(c + d*x))^(5/2))/(5*d*e^2) - (4*a^3*(e*cot(c + d*x))^(1/2))/
d
```

3.18 $\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx$

3.18.1 Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = \frac{2\sqrt{2}a^3\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{d\sqrt{e}} - \frac{16a^3\sqrt{e\cot(c+dx)}}{3de} - \frac{2\sqrt{e\cot(c+dx)}(a^3+a^3\cot(c+dx))}{3de}$$

output
```
2*a^3*arctanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2
))*2^(1/2)/d/e^(1/2)-16/3*a^3*(e*cot(d*x+c))^(1/2)/d/e-2/3*(a^3+a^3*cot(d*
x+c))*(e*cot(d*x+c))^(1/2)/d/e
```

3.18.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(117) = 234.

Time = 1.63 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.92

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = -\frac{a^3(1+\cot(c+dx))^3\sin(c+dx)\left(4\cos^2(c+dx)+6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)\sqrt{\cot(c+dx)}\,\mathrm{s}\right.}{}$$

input
```
Integrate[(a + a*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]],x]
```

output
```
-1/6*(a^3*(1 + Cot[c + d*x])^3*Sin[c + d*x]*(4*Cos[c + d*x]^2 + 6*Sqrt[2]*
ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sin[c + d*x]^2 -
 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]*Sqrt[Cot[c + d*x]]*Sin[c
 + d*x]^2 + 12*ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x]^2)^(1/4)*Sin
[c + d*x]^2 - 12*ArcTanh[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x]^2)^(1/4)*
Sin[c + d*x]^2 + 3*Sqrt[2]*Sqrt[Cot[c + d*x]]*Log[1 - Sqrt[2]*Sqrt[Cot[c +
 d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 - 3*Sqrt[2]*Sqrt[Cot[c + d*x]]*Log[1
 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 + 18*Sin[2*(c
 + d*x)]))/(d*Sqrt[e*Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x])^3)
```

3.18.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4049, 25, 3042, 4113, 3042, 4015, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cot(c+dx)+a)^3}{\sqrt{e\cot(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4049}$$

$$-\frac{2\int -\frac{4e\cot^2(c+dx)a^3+ea^3+3e\cot(c+dx)a^3}{\sqrt{e\cot(c+dx)}}dx}{3e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)\sqrt{e\cot(c+dx)}}{3de}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\int \frac{4e\cot^2(c+dx)a^3+ea^3+3e\cot(c+dx)a^3}{\sqrt{e\cot(c+dx)}}dx}{3e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)\sqrt{e\cot(c+dx)}}{3de}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int \frac{4e\tan\left(c+dx+\frac{\pi}{2}\right)^2a^3+ea^3-3e\tan\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)\sqrt{e\cot(c+dx)}}{3de}$$

$$\Big\downarrow \text{4113}$$

$$\frac{2\left(\int \frac{3a^3e\cot(c+dx)-3a^3e}{\sqrt{e\cot(c+dx)}}dx-\frac{8a^3\sqrt{e\cot(c+dx)}}{d}\right)}{3e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)\sqrt{e\cot(c+dx)}}{3de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\int \frac{-3ea^3-3e\tan\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{8a^3\sqrt{e\cot(c+dx)}}{d}\right)}{3e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)\sqrt{e\cot(c+dx)}}{3de}$$

$$\Big\downarrow \text{4015}$$

$$\frac{2\left(-\frac{18a^6e^2\int \frac{1}{18a^6e^2-9\left(ea^3+e\cot(c+dx)a^3\right)^2\tan(c+dx)}d\left(-\frac{3\left(ea^3+e\cot(c+dx)a^3\right)}{\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{8a^3\sqrt{e\cot(c+dx)}}{d}\right)}{3e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)\sqrt{e\cot(c+dx)}}{3de}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(\frac{3\sqrt{2}a^3\sqrt{e}\text{arctanh}\left(\frac{a^3e\cot(c+dx)+a^3e}{\sqrt{2}a^3\sqrt{e}\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{8a^3\sqrt{e\cot(c+dx)}}{d}\right)}{3e}-\frac{2\left(a^3\cot(c+dx)+a^3\right)\sqrt{e\cot(c+dx)}}{3de}$$

input
```
Int[(a + a*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]],x]
```

output
```
(-2*Sqrt[e*Cot[c + d*x]]*(a^3 + a^3*Cot[c + d*x]))/(3*d*e) + (2*((3*Sqrt[2
]*a^3*Sqrt[e]*ArcTanh[(a^3*e + a^3*e*Cot[c + d*x])/(Sqrt[2]*a^3*Sqrt[e]*Sq
rt[e*Cot[c + d*x]])])/d - (8*a^3*Sqrt[e*Cot[c + d*x]])/d))/(3*e)
```

3.18.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.18.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(98) = 196$.

Time = 0.06 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.64

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}+3e\sqrt{e\cot(dx+c)}-2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\ldots}}{\ldots}\right.\right.}{8e}\right.\right.}{\ldots}$
default	$-\frac{2a^3\left(\frac{(e\cot(dx+c))^{\frac{3}{2}}}{3}+3e\sqrt{e\cot(dx+c)}-2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\ldots}}{\ldots}\right.\right.}{8e}\right.\right.}{\ldots}$
parts	$-\frac{a^3(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot\ldots}}{(e^2)\ldots}\right.\right.}{4de}$

input
```
int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^3/e^2*(1/3*(e*cot(d*x+c))^(3/2)+3*e*(e*cot(d*x+c))^(1/2)-2*e^2*(1/8
/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*
2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2
)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arc
tan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8/(e^2)^(1/4)*2^(1/2)*
(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e
*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arcta
n(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)+1))))
```

3.18. $\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx$

3.18.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.98

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = \left[\frac{3\sqrt{2}a^3\sqrt{e}\log\left(-\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)-1)}{\sqrt{e}}+2\,\sin(2\,dx+2\,c)+1\right)\sin(2\,dx+2\,c)-2}{3\,de\sin(2\,dx+2\,c)}\right.$$

$$-\frac{2\left(3\sqrt{2}a^3e\sqrt{-\frac{1}{e}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sqrt{-\frac{1}{e}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)+1)}{2\,(\cos(2\,dx+2\,c)+1)}\right)\sin(2\,dx+2\,c)+(a^3\cos(2\,dx\right.}{3\,de\sin(2\,dx+2\,c)}$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(2)*a^3*sqrt(e)*log(-sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1)/sqrt(e) + 2*sin(2
*d*x + 2*c) + 1)*sin(2*d*x + 2*c) - 2*(a^3*cos(2*d*x + 2*c) + 9*a^3*sin(2*
d*x + 2*c) + a^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e*si
n(2*d*x + 2*c)), -2/3*(3*sqrt(2)*a^3*e*sqrt(-1/e)*arctan(1/2*sqrt(2)*sqrt(
(e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) +
sin(2*d*x + 2*c) + 1)/(cos(2*d*x + 2*c) + 1))*sin(2*d*x + 2*c) + (a^3*cos(
2*d*x + 2*c) + 9*a^3*sin(2*d*x + 2*c) + a^3)*sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c)))/(d*e*sin(2*d*x + 2*c))]
```

3.18.6 Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = a^3\left(\int \frac{1}{\sqrt{e\cot(c+dx)}}\,dx + \int \frac{3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx + \int \frac{3\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx + \int \frac{\cot^3(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(1/2),x)
```

output
```
a**3*(Integral(1/sqrt(e*cot(c + d*x)), x) + Integral(3*cot(c + d*x)/sqrt(e
*cot(c + d*x)), x) + Integral(3*cot(c + d*x)**2/sqrt(e*cot(c + d*x)), x) +
 Integral(cot(c + d*x)**3/sqrt(e*cot(c + d*x)), x))
```

3.18.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.18.8 Giac [F]

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{(a\cot(dx+c)+a)^3}{\sqrt{e\cot(dx+c)}}\,dx$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^3/sqrt(e*cot(d*x + c)), x)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 13.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = \frac{2\sqrt{2}\,a^3\,\mathrm{atanh}\left(\frac{32\sqrt{2}\,a^6\,\sqrt{e}\,\sqrt{e\cot(c+d\,x)}}{32\,a^6\,e+32\,a^6\,e\cot(c+d\,x)}\right)}{d\,\sqrt{e}} - \frac{2\,a^3\,(e\cot(c+d\,x))^{3/2}}{3\,d\,e^2} - \frac{6\,a^3\,\sqrt{e\cot(c+d\,x)}}{d\,e}$$

input
```
int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(1/2),x)
```

output
```
(2*2^(1/2)*a^3*atanh((32*2^(1/2)*a^6*e^(1/2)*(e*cot(c + d*x))^(1/2))/(32*a
^6*e + 32*a^6*e*cot(c + d*x))))/(d*e^(1/2)) - (2*a^3*(e*cot(c + d*x))^(3/2
))/(3*d*e^2) - (6*a^3*(e*cot(c + d*x))^(1/2))/(d*e)
```

3.18. $\int \frac{(a+a\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx$

3.19 $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx$

3.19.1 Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2\sqrt{2}a^3\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{3/2}} - \frac{4a^3\sqrt{e\cot(c+dx)}}{de^2} + \frac{2(a^3+a^3\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

output
```
2*a^3*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2)
)*2^(1/2)/d/e^(3/2)+2*(a^3+a^3*cot(d*x+c))/d/e/(e*cot(d*x+c))^(1/2)-4*a^3*
(e*cot(d*x+c))^(1/2)/d/e^2
```

3.19.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 357 vs. 2(114) = 228.

Time = 3.12 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.13

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \frac{a^3(1+\cot(c+dx))^3\sin(c+dx)\left(-4\cos^2(c+dx)+4\arctan\left(\sqrt[4]{-\cot^2(c+d}\right.\right.}{}$$

input
```
Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2),x]
```

output
```
(a^3*(1 + Cot[c + d*x])^3*Sin[c + d*x]*(-4*Cos[c + d*x]^2 + 4*ArcTan[(-Cot
[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(5/4)*Cot[c + d*x]^(1/4)*Sin[c + d*x]^
2 + 4*ArcTanh[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^
(5/4)*Sin[c + d*x]^2 + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*Co
t[c + d*x]^(3/2)*Sin[c + d*x]^2 - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c
+ d*x]]]*Cot[c + d*x]^(3/2)*Sin[c + d*x]^2 + Sqrt[2]*Cot[c + d*x]^(3/2)*Lo
g[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^2 - Sqrt[2]*
Cot[c + d*x]^(3/2)*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]*Sin[
c + d*x]^2 + 2*Sin[2*(c + d*x)]))/(2*d*(e*Cot[c + d*x])^(3/2)*(Cos[c + d*x
] + Sin[c + d*x])^3)
```

3.19.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4048, 25, 3042, 4113, 3042, 4015, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cot(c+dx)+a)^3}{(e\cot(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4048}$$

$$\frac{2\left(a^3\cot(c+dx)+a^3\right)}{de\sqrt{e\cot(c+dx)}}-\frac{2\int -\frac{2e^2a^3+e^2\cot^2(c+dx)a^3+e^2\cot(c+dx)a^3}{\sqrt{e\cot(c+dx)}}dx}{e^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\int \frac{2e^2a^3+e^2\cot^2(c+dx)a^3+e^2\cot(c+dx)a^3}{\sqrt{e\cot(c+dx)}}dx}{e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int \frac{2e^2a^3+e^2\tan\left(c+dx+\frac{\pi}{2}\right)^2a^3-e^2\tan\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{4113}$$

$$\frac{2\left(\int \frac{e^2a^3+e^2\cot(c+dx)a^3}{\sqrt{e\cot(c+dx)}}dx-\frac{2a^3e\sqrt{e\cot(c+dx)}}{d}\right)}{e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\int \frac{a^3e^2-a^3e^2\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a^3e\sqrt{e\cot(c+dx)}}{d}\right)}{e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{4015}$$

$$\frac{2\left(-\frac{2a^6e^4\int \frac{1}{-2e^4a^6-\left(a^3e^2-a^3e^2\cot(c+dx)\right)^2\tan(c+dx)}d\frac{a^3e^2-a^3e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}}{d}-\frac{2a^3e\sqrt{e\cot(c+dx)}}{d}\right)}{e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\left(\frac{\sqrt{2}a^3e^{3/2}\arctan\left(\frac{a^3e^2-a^3e^2\cot(c+dx)}{\sqrt{2}a^3e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{2a^3e\sqrt{e\cot(c+dx)}}{d}\right)}{e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{de\sqrt{e\cot(c+dx)}}$$

input
```
Int[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2),x]
```

output
```
(2*(a^3 + a^3*Cot[c + d*x]))/(d*e*Sqrt[e*Cot[c + d*x]]) + (2*((Sqrt[2]*a^3
*e^(3/2)*ArcTan[(a^3*e^2 - a^3*e^2*Cot[c + d*x])/(Sqrt[2]*a^3*e^(3/2)*Sqrt
[e*Cot[c + d*x]])])/d - (2*a^3*e*Sqrt[e*Cot[c + d*x]])/d))/e^3
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.19.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(99) = 198$.

Time = 0.04 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.68

method	result
derivativedivides	$-\frac{2a^3\left(\sqrt{e\cot(dx+c)}+2e\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{a}}{8e}\right.\right.}{}$
default	$-\frac{2a^3\left(\sqrt{e\cot(dx+c)}+2e\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{a}}{8e}\right.\right.}{}$
parts	$-\frac{2a^3e\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e^2(e^2)^{\frac{1}{4}}}\right.}{d}$

input
```
int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^3/e^2*((e*cot(d*x+c))^(1/2)+2*e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*c
ot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x
+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2
)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1))+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-e/(
e*cot(d*x+c))^(1/2))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.26

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{2}(a^3e\cos(2\,dx+2\,c)+a^3e)\sqrt{-\frac{1}{e}}\log\left(-\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sqrt{-\frac{1}{e}}(\cos(2\,dx\right.}{}\right.$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[(sqrt(2)*(a^3*e*cos(2*d*x + 2*c) + a^3*e)*sqrt(-1/e)*log(-sqrt(2)*sqrt((e
*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + si
n(2*d*x + 2*c) - 1) - 2*sin(2*d*x + 2*c) + 1) - 2*(a^3*cos(2*d*x + 2*c) -
a^3*sin(2*d*x + 2*c) + a^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
))/(d*e^2*cos(2*d*x + 2*c) + d*e^2), 2*(sqrt(2)*(a^3*e*cos(2*d*x + 2*c) +
a^3*e)*arctan(-1/2*sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))
*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(sqrt(e)*(cos(2*d*x + 2*c) + 1)
))/sqrt(e) - (a^3*cos(2*d*x + 2*c) - a^3*sin(2*d*x + 2*c) + a^3)*sqrt((e*c
os(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^2*cos(2*d*x + 2*c) + d*e^2)]
```

3.19.6 Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = a^3\left(\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{3\cot(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{3\cot^2(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{\cot^3(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(3/2),x)
```

output
```
a**3*(Integral((e*cot(c + d*x))**(-3/2), x) + Integral(3*cot(c + d*x)/(e*c
ot(c + d*x))**(3/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(3/
2), x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(3/2), x))
```

3.19.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.19.8 Giac [F]

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx=\int \frac{(a\cot(dx+c)+a)^3}{(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^3/(e*cot(d*x + c))^(3/2), x)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 12.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx=\frac{2a^3}{de\sqrt{e\cot(c+dx)}}-\frac{2a^3\sqrt{e\cot(c+dx)}}{de^2}-\frac{\sqrt{2}\,a^3\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{2\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+dx))^{3/2}}{2e^{3/2}}\right)\right)}{de^{3/2}}$$

input
```
int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(3/2),x)
```

output
```
(2*a^3)/(d*e*(e*cot(c + d*x))^(1/2)) - (2*a^3*(e*cot(c + d*x))^(1/2))/(d*e
^2) - (2^(1/2)*a^3*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) +
 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c +
 d*x))^(3/2))/(2*e^(3/2)))))/(d*e^(3/2))
```

3.20 $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx$

3.20.1 Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{2}a^3\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{5/2}} + \frac{16a^3}{3de^2\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

output
```
2/3*(a^3+a^3*cot(d*x+c))/d/e/(e*cot(d*x+c))^(3/2)-2*a^3*arctanh(1/2*(e^(1/
2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d/e^(5/2)+16/
3*a^3/d/e^2/(e*cot(d*x+c))^(1/2)
```

3.20.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 599 vs. $2(117) = 234$.

Time = 6.21 (sec) , antiderivative size = 599, normalized size of antiderivative = 5.12

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \frac{6\cos^2(c+dx)(a+a\cot(c+dx))^3\sin(c+dx)}{d(e\cot(c+dx))^{5/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{2\cos(c+dx)\left(1-\frac{3}{2}\arctan\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)(-\cot(c+dx))^{3/4}\cot^{\frac{3}{4}}(c+dx)-\frac{3}{2}\text{arctanh}\left(\right.\right.}{3d(e\cot(c+dx))^{5/2}(\cos(c+}$$
$$+\frac{2\arctan\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\cot^{\frac{9}{4}}(c+dx)(a+a\cot(c+dx))^3\sin^3(c+dx)}{d(e\cot(c+dx))^{5/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{2\text{arctanh}\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\cot^{\frac{9}{4}}(c+dx)(a+a\cot(c+dx))^3\sin^3(c+dx)}{d(e\cot(c+dx))^{5/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{3\cot^{\frac{5}{2}}(c+dx)(a+a\cot(c+dx))^3\left(2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(c+\right.\right.}{4d(e\cot(c+dx))^{5/}}$$

input
```
Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(5/2),x]
```

output
```
(6*Cos[c + d*x]^2*(a + a*Cot[c + d*x])^3*Sin[c + d*x])/(d*(e*Cot[c + d*x])
^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (2*Cos[c + d*x]*(1 - (3*ArcTan[(
-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*x])^(3/4)*Cot[c + d*x
]^(3/4))/2 - (3*ArcTanh[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c
+ d*x])^(3/4)*Cot[c + d*x]^(3/4))/2)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^2
)/(3*d*(e*Cot[c + d*x])^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (2*ArcTan
[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*x])^(1/4)*Cot[c + d
*x]^(9/4)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3)/(d*(e*Cot[c + d*x])^(5/2)
*(Cos[c + d*x] + Sin[c + d*x])^3) - (2*ArcTanh[(-Cot[c + d*x])^(1/4)*Cot[c
 + d*x]^(1/4)]*(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(9/4)*(a + a*Cot[c + d*x
])^3*Sin[c + d*x]^3)/(d*(e*Cot[c + d*x])^(5/2)*(Cos[c + d*x] + Sin[c + d*x
])^3) + (3*Cot[c + d*x]^(5/2)*(a + a*Cot[c + d*x])^3*(2*Sqrt[2]*ArcTan[1 -
 Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d
*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[
2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])*Sin[c + d*x]^3)/(4*
d*(e*Cot[c + d*x])^(5/2)*(Cos[c + d*x] + Sin[c + d*x])^3)
```

3.20.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4048, 25, 3042, 4111, 27, 3042, 4015, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cot(c+dx)+a)^3}{(e\cot(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4048}$$

$$\frac{2\left(a^3\cot(c+dx)+a^3\right)}{3de(e\cot(c+dx))^{3/2}}-\frac{2\int -\frac{4e^2a^3+e^2\cot^2(c+dx)a^3+3e^2\cot(c+dx)a^3}{(e\cot(c+dx))^{3/2}}dx}{3e^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{4e^2a^3+e^2\cot^2(c+dx)a^3+3e^2\cot(c+dx)a^3}{(e\cot(c+dx))^{3/2}}dx}{3e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{4e^2a^3+e^2\tan\left(c+dx+\frac{\pi}{2}\right)^2a^3-3e^2\tan\left(c+dx+\frac{\pi}{2}\right)a^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4111}$$

$$\frac{2\left(\frac{\int \frac{3\left(a^3e^3-a^3e^3\cot(c+dx)\right)}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{8a^3e}{d\sqrt{e\cot(c+dx)}}\right)}{3e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{3\int \frac{a^3e^3-a^3e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{8a^3e}{d\sqrt{e\cot(c+dx)}}\right)}{3e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{3\int \frac{a^3e^3+a^3\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}+\frac{8a^3e}{d\sqrt{e\cot(c+dx)}}\right)}{3e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4015}$$

$$\frac{2\left(\frac{8a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{6a^6e^4\int \frac{1}{2a^6e^6-\left(a^3e^3+a^3\cot(c+dx)e^3\right)^2\tan(c+dx)}d\frac{a^3e^3+a^3\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}}}{d}\right)}{3e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(\frac{8a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{3\sqrt{2}a^3\sqrt{e}\,\text{arctanh}\left(\frac{a^3e^3\cot(c+dx)+a^3e^3}{\sqrt{2}a^3e^{5/2}\sqrt{e\cot(c+dx)}}\right)}{d}\right)}{3e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{3de(e\cot(c+dx))^{3/2}}$$

input
```
Int[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(5/2),x]
```

output
```
(2*(a^3 + a^3*Cot[c + d*x]))/(3*d*e*(e*Cot[c + d*x])^(3/2)) + (2*((-3*Sqrt
[2]*a^3*Sqrt[e]*ArcTanh[(a^3*e^3 + a^3*e^3*Cot[c + d*x])/(Sqrt[2]*a^3*e^(5
/2)*Sqrt[e*Cot[c + d*x]])])/d + (8*a^3*e)/(d*Sqrt[e*Cot[c + d*x]])))/(3*e^
3)
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(98) = 196$.

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.59

method	result
derivativedivides	$-\frac{2a^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
default	$-\frac{2a^3\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}{(e^2)^{\frac{1}{4}}}\right.\right.}{4e}\right.}{}$
parts	$-\frac{2a^3e\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot}{(e^2)}\right.\right.}{8e^4}\right.}{d}$

input
```
int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^3/e^2*(1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4/(e^
2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2
)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2
)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2
^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/3*e/(e*cot(d*x+c))^(3/2)-3/(
e*cot(d*x+c))^(1/2))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.23

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \left[\frac{\frac{3\sqrt{2}(a^3e\cos(2\,dx+2\,c)+a^3e)\log\left(\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)-1)}{\sqrt{e}}+2\,\sin(2\,dx+2\,c)+1\right.}{\sqrt{e}}}{3\,(de^3\cos(2\,dx+2\right.$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(2)*(a^3*e*cos(2*d*x + 2*c) + a^3*e)*log(sqrt(2)*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) -
1)/sqrt(e) + 2*sin(2*d*x + 2*c) + 1)/sqrt(e) - 2*(a^3*cos(2*d*x + 2*c) - 9
*a^3*sin(2*d*x + 2*c) - a^3)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c
)))/(d*e^3*cos(2*d*x + 2*c) + d*e^3), 2/3*(3*sqrt(2)*(a^3*e*cos(2*d*x + 2*
c) + a^3*e)*sqrt(-1/e)*arctan(1/2*sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(cos(
2*d*x + 2*c) + 1)) - (a^3*cos(2*d*x + 2*c) - 9*a^3*sin(2*d*x + 2*c) - a^3)
*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^3*cos(2*d*x + 2*c)
+ d*e^3)]
```

3.20.6 Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = a^3\left(\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{3\cot(c+dx)}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{3\cot^2(c+dx)}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx + \int \frac{\cot^3(c+dx)}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx\right)$$

input
```
integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(5/2),x)
```

output
```
a**3*(Integral((e*cot(c + d*x))**(-5/2), x) + Integral(3*cot(c + d*x)/(e*c
ot(c + d*x))**(5/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(5/
2), x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(5/2), x))
```

3.20.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.20.8 Giac [F]

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx=\int \frac{(a\cot(dx+c)+a)^3}{(e\cot(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cot(d*x + c) + a)^3/(e*cot(d*x + c))^(5/2), x)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx=\frac{\frac{2a^3e}{3}+6a^3e\cot(c+dx)}{d\,e^2\,(e\cot(c+dx))^{3/2}}-\frac{2\sqrt{2}\,a^3\,\mathrm{atanh}\left(\frac{32\sqrt{2}\,a^6\,d\,e^{5/2}\,\sqrt{e\cot(c+dx)}}{32\,a^6\,d\,e^3+32\,a^6\,d\,e^3\cot(c+dx)}\right)}{d\,e^{5/2}}$$

input
```
int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(5/2),x)
```

output
```
((2*a^3*e)/3 + 6*a^3*e*cot(c + d*x))/(d*e^2*(e*cot(c + d*x))^(3/2)) - (2*2
^(1/2)*a^3*atanh((32*2^(1/2)*a^6*d*e^(5/2)*(e*cot(c + d*x))^(1/2))/(32*a^6
*d*e^3 + 32*a^6*d*e^3*cot(c + d*x))))/(d*e^(5/2))
```

3.21 $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

3.21.1 Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = -\frac{2\sqrt{2}a^3\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{7/2}} + \frac{8a^3}{5de^2(e\cot(c+dx))^{3/2}} + \frac{4a^3}{de^3\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

output
```
8/5*a^3/d/e^2/(e*cot(d*x+c))^(3/2)+2/5*(a^3+a^3*cot(d*x+c))/d/e/(e*cot(d*x
+c))^(5/2)-2*a^3*arctan(1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*
x+c))^(1/2))*2^(1/2)/d/e^(7/2)+4*a^3/d/e^3/(e*cot(d*x+c))^(1/2)
```

3.21.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 415 vs. 2(141) = 282.

Time = 5.75 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.94

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \frac{a^3\left(80\cos^3(c+dx)+40\cos^2(c+dx)\sin(c+dx)+8\cos(c+dx)\sin^2(c+dx)\right.}{}$$

input
```
Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(7/2),x]
```

output
```
(a^3*(80*Cos[c + d*x]^3 + 40*Cos[c + d*x]^2*Sin[c + d*x] + 8*Cos[c + d*x]*
Sin[c + d*x]^2 + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]*Cot[c +
 d*x]^(7/2)*Sin[c + d*x]^3 - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*
x]]]*Cot[c + d*x]^(7/2)*Sin[c + d*x]^3 - 20*ArcTanh[(-Cot[c + d*x]^2)^(1/4
)]*(2*(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(13/4) - 3*(-Cot[c + d*x]^2)^(7/4
))*Sin[c + d*x]^3 + 20*ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(2*(-Cot[c + d*x])^
(1/4)*Cot[c + d*x]^(13/4) + 3*(-Cot[c + d*x]^2)^(7/4))*Sin[c + d*x]^3 + 5*
Sqrt[2]*Cot[c + d*x]^(7/2)*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*
x]]*Sin[c + d*x]^3 - 5*Sqrt[2]*Cot[c + d*x]^(7/2)*Log[1 + Sqrt[2]*Sqrt[Cot
[c + d*x]] + Cot[c + d*x]]*Sin[c + d*x]^3)*(1 + Tan[c + d*x])^3)/(20*d*e^3
*Sqrt[e*Cot[c + d*x]]*(Cos[c + d*x] + Sin[c + d*x])^3)
```

3.21.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 4048, 25, 3042, 4111, 27, 3042, 4012, 25, 3042, 4015, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cot(c+dx)+a)^3}{(e\cot(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4048}$$

$$\frac{2\left(a^3\cot(c+dx)+a^3\right)}{5de(e\cot(c+dx))^{5/2}}-\frac{2\int -\frac{6e^2a^3+e^2\cot^2(c+dx)a^3+5e^2\cot(c+dx)a^3}{(e\cot(c+dx))^{5/2}}dx}{5e^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{6e^2a^3+e^2\cot^2(c+dx)a^3+5e^2\cot(c+dx)a^3}{(e\cot(c+dx))^{5/2}}dx}{5e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{6e^2a^3+e^2\tan\left(c+dx+\frac{\pi}{2}\right)^2a^3-5e^2\tan\left(c+dx+\frac{\pi}{2}\right)a^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4111}$$

$$\frac{2\left(\frac{\int \frac{5\left(a^3e^3-a^3e^3\cot(c+dx)\right)}{(e\cot(c+dx))^{3/2}}dx}{e^2}+\frac{4a^3e}{d(e\cot(c+dx))^{3/2}}\right)}{5e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{5\int \frac{a^3e^3-a^3e^3\cot(c+dx)}{(e\cot(c+dx))^{3/2}}dx}{e^2}+\frac{4a^3e}{d(e\cot(c+dx))^{3/2}}\right)}{5e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{5\int \frac{a^3e^3+a^3\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{e^2}+\frac{4a^3e}{d(e\cot(c+dx))^{3/2}}\right)}{5e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4012}$$

$$\frac{2\left(\frac{5\left(\frac{\int -\frac{a^3e^4+a^3\cot(c+dx)e^4}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{2a^3e^2}{d\sqrt{e\cot(c+dx)}}\right)}{e^2}+\frac{4a^3e}{d(e\cot(c+dx))^{3/2}}\right)}{5e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{5\left(\frac{2a^3e^2}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{a^3e^4+a^3\cot(c+dx)e^4}{\sqrt{e\cot(c+dx)}}dx}{e^2}\right)}{e^2}+\frac{4a^3e}{d(e\cot(c+dx))^{3/2}}\right)}{5e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{5\left(\frac{2a^3e^2}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{a^3e^4-a^3e^4\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}\right)}{e^2}+\frac{4a^3e}{d(e\cot(c+dx))^{3/2}}\right)}{5e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{5de(e\cot(c+dx))^{5/2}}$$

3.21. $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

$$\Big\downarrow \text{4015}$$

$$\frac{2\left(\frac{5\left(\frac{2a^6e^6\int \frac{1}{-2a^6e^8-\left(a^3e^4-a^3e^4\cot(c+dx)\right)^2\tan(c+dx)}d\frac{a^3e^4-a^3e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}}{d}+\frac{2a^3e^2}{d\sqrt{e\cot(c+dx)}}\right)}{e^2}+\frac{4a^3e}{d(e\cot(c+dx))^{3/2}}\right)}{5e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\left(\frac{5\left(\frac{2a^3e^2}{d\sqrt{e\cot(c+dx)}}-\frac{\sqrt{2}a^3e^{3/2}\arctan\left(\frac{a^3e^4-a^3e^4\cot(c+dx)}{\sqrt{2}a^3e^{7/2}\sqrt{e\cot(c+dx)}}\right)}{d}\right)}{e^2}+\frac{4a^3e}{d(e\cot(c+dx))^{3/2}}\right)}{5e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{5de(e\cot(c+dx))^{5/2}}$$

input
```
Int[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(7/2),x]
```

output
```
(2*(a^3 + a^3*Cot[c + d*x]))/(5*d*e*(e*Cot[c + d*x])^(5/2)) + (2*((4*a^3*e
)/(d*(e*Cot[c + d*x])^(3/2)) + (5*(-((Sqrt[2]*a^3*e^(3/2)*ArcTan[(a^3*e^4
- a^3*e^4*Cot[c + d*x])/(Sqrt[2]*a^3*e^(7/2)*Sqrt[e*Cot[c + d*x]])])/d) +
(2*a^3*e^2)/(d*Sqrt[e*Cot[c + d*x]])))/e^2))/(5*e^3)
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.21. $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

3.21.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(120) = 240$.

Time = 0.05 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.29

method	result
derivativedivides	$-\frac{2a^3\left(\frac{-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right)\right)}{4e}}{\cdots}\right)}{\cdots}$
default	$-\frac{2a^3\left(\frac{-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right)\right)}{4e}}{\cdots}\right)}{\cdots}$
parts	$-\frac{2a^3e\left(-\frac{1}{5e^2(e\cot(dx+c))^{\frac{5}{2}}}+\frac{1}{e^4\sqrt{e\cot(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(d\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right)\right)}{8e^4(e^2)^{\frac{1}{4}}}\cdots\right)}{d}$

input
```
int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^3/e^2*(1/e*(-1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1
/4/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*
2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2
)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arc
tan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/5*e/(e*cot(d*x+c))^(5
/2)-1/(e*cot(d*x+c))^(3/2)-2/e/(e*cot(d*x+c))^(1/2))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 485, normalized size of antiderivative = 3.44

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \left[\frac{5\sqrt{2}\left(a^3e\cos\left(2\,dx+2\,c\right)^2+2\,a^3e\cos\left(2\,dx+2\,c\right)+a^3e\right)\sqrt{-\frac{1}{e}}\log\left(\sqrt{2}\sqrt{\frac{e\cos(\cdots}{\sin\cdots}}\right.}{\cdots}\right.$$

$$-\frac{2\left(\frac{5\sqrt{2}\left(a^3e\cos(2\,dx+2\,c)^2+2\,a^3e\cos(2\,dx+2\,c)+a^3e\right)\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)+1)}{2\sqrt{e}(\cos(2\,dx+2\,c)+1)}\right)}{\sqrt{e}}+\left(5\,a^3\cos\left(2\,dx\cdots\right.\right.}{5\left(de^4\cos\left(2\,dx+2\,c\right)^2+2\,de^4\cos\left(2\,dx\cdots\right.\right.}$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
[1/5*(5*sqrt(2)*(a^3*e*cos(2*d*x + 2*c)^2 + 2*a^3*e*cos(2*d*x + 2*c) + a^3
*e)*sqrt(-1/e)*log(sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))
*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) - 1) - 2*sin(2*d*x + 2*c)
 + 1) - 2*(5*a^3*cos(2*d*x + 2*c)^2 - 5*a^3 - (9*a^3*cos(2*d*x + 2*c) + 11
*a^3)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(
d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4), -2/5*(5*sqrt
(2)*(a^3*e*cos(2*d*x + 2*c)^2 + 2*a^3*e*cos(2*d*x + 2*c) + a^3*e)*arctan(-
1/2*sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2
*c) - sin(2*d*x + 2*c) + 1)/(sqrt(e)*(cos(2*d*x + 2*c) + 1)))/sqrt(e) + (5
*a^3*cos(2*d*x + 2*c)^2 - 5*a^3 - (9*a^3*cos(2*d*x + 2*c) + 11*a^3)*sin(2*
d*x + 2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^4*cos(2*
d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)]
```

3.21.6 Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = a^3\left(\int \frac{1}{(e\cot(c+dx))^{\frac{7}{2}}}\,dx + \int \frac{3\cot(c+dx)}{(e\cot(c+dx))^{\frac{7}{2}}}\,dx + \int \frac{3\cot^2(c+dx)}{(e\cot(c+dx))^{\frac{7}{2}}}\,dx + \int \frac{\cot^3(c+dx)}{(e\cot(c+dx))^{\frac{7}{2}}}\,dx\right)$$

input
```
integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(7/2),x)
```

output
```
a**3*(Integral((e*cot(c + d*x))**(-7/2), x) + Integral(3*cot(c + d*x)/(e*c
ot(c + d*x))**(7/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(7/
2), x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(7/2), x))
```

3.21.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.21.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.21.9 Mupad [B] (verification not implemented)

Time = 13.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \frac{4\,e\,a^3\cot(c+d\,x)^2+2\,e\,a^3\cot(c+d\,x)+\frac{2\,e\,a^3}{5}}{d\,e^2\,(e\cot(c+d\,x))^{5/2}} + \frac{\sqrt{2}\,a^3\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{d\,e^{7/2}}$$

input
```
int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(7/2),x)
```

output
```
((2*a^3*e)/5 + 4*a^3*e*cot(c + d*x)^2 + 2*a^3*e*cot(c + d*x))/(d*e^2*(e*co
t(c + d*x))^(5/2)) + (2^(1/2)*a^3*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))
/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(
1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/(d*e^(7/2))
```

3.22 $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

3.22.1 Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \frac{2\sqrt{2}a^3\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{de^{9/2}} + \frac{32a^3}{35de^2(e\cot(c+dx))^{5/2}} + \frac{4a^3}{3de^3(e\cot(c+dx))^{3/2}} - \frac{4a^3}{de^4\sqrt{e\cot(c+dx)}} + \frac{2(a^3+a^3\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

output
```
32/35*a^3/d/e^2/(e*cot(d*x+c))^(5/2)+4/3*a^3/d/e^3/(e*cot(d*x+c))^(3/2)+2/
7*(a^3+a^3*cot(d*x+c))/d/e/(e*cot(d*x+c))^(7/2)+2*a^3*arctanh(1/2*(e^(1/2)
+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*2^(1/2)/d/e^(9/2)-4*a^3
/d/e^4/(e*cot(d*x+c))^(1/2)
```

3.22.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 625 vs. $2(165) = 330$.

Time = 6.31 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.79

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \frac{4\cos^3(c+dx)(a+a\cot(c+dx))^3}{3d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{4\cos^3(c+dx)\cot(c+dx)(a+a\cot(c+dx))^3}{d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{6\cos^2(c+dx)(a+a\cot(c+dx))^3\sin(c+dx)}{5d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{2\cos(c+dx)(a+a\cot(c+dx))^3\sin^2(c+dx)}{7d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{2\arctan\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)(-\cot(c+dx))^{3/4}\cot^{\frac{15}{4}}(c+dx)(a+a\cot(c+dx))^3\sin^3(c+dx}{d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{2\operatorname{arctanh}\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)(-\cot(c+dx))^{3/4}\cot^{\frac{15}{4}}(c+dx)(a+a\cot(c+dx))^3\sin^3(c+d}{d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$-\frac{2\arctan\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\cot^{\frac{17}{4}}(c+dx)(a+a\cot(c+dx))^3\sin^3(c+dx)}{d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$
$$+\frac{2\operatorname{arctanh}\left(\sqrt[4]{-\cot(c+dx)}\sqrt[4]{\cot(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\cot^{\frac{17}{4}}(c+dx)(a+a\cot(c+dx))^3\sin^3(c+dx)}{d(e\cot(c+dx))^{9/2}(\cos(c+dx)+\sin(c+dx))^3}$$

input
```
Integrate[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(9/2),x]
```

output
```
(4*Cos[c + d*x]^3*(a + a*Cot[c + d*x])^3)/(3*d*(e*Cot[c + d*x])^(9/2)*(Cos
[c + d*x] + Sin[c + d*x])^3) - (4*Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Cot[c
 + d*x])^3)/(d*(e*Cot[c + d*x])^(9/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (
6*Cos[c + d*x]^2*(a + a*Cot[c + d*x])^3*Sin[c + d*x])/(5*d*(e*Cot[c + d*x]
)^(9/2)*(Cos[c + d*x] + Sin[c + d*x])^3) + (2*Cos[c + d*x]*(a + a*Cot[c +
d*x])^3*Sin[c + d*x]^2)/(7*d*(e*Cot[c + d*x])^(9/2)*(Cos[c + d*x] + Sin[c
+ d*x])^3) - (2*ArcTan[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c +
 d*x])^(3/4)*Cot[c + d*x]^(15/4)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3)/(d
*(e*Cot[c + d*x])^(9/2)*(Cos[c + d*x] + Sin[c + d*x])^3) - (2*ArcTanh[(-Co
t[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*x])^(3/4)*Cot[c + d*x]^(
15/4)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3)/(d*(e*Cot[c + d*x])^(9/2)*(Co
s[c + d*x] + Sin[c + d*x])^3) - (2*ArcTan[(-Cot[c + d*x])^(1/4)*Cot[c + d*
x]^(1/4)]*(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(17/4)*(a + a*Cot[c + d*x])^3
*Sin[c + d*x]^3)/(d*(e*Cot[c + d*x])^(9/2)*(Cos[c + d*x] + Sin[c + d*x])^3
) + (2*ArcTanh[(-Cot[c + d*x])^(1/4)*Cot[c + d*x]^(1/4)]*(-Cot[c + d*x])^(
1/4)*Cot[c + d*x]^(17/4)*(a + a*Cot[c + d*x])^3*Sin[c + d*x]^3)/(d*(e*Cot[
c + d*x])^(9/2)*(Cos[c + d*x] + Sin[c + d*x])^3)
```

3.22.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4048, 25, 3042, 4111, 27, 3042, 4012, 25, 3042, 4012, 3042, 4015, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cot(c+dx)+a)^3}{(e\cot(c+dx))^{9/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \; 4048$$

$$\frac{2\left(a^3\cot(c+dx)+a^3\right)}{7de(e\cot(c+dx))^{7/2}}-\frac{2\int -\frac{8e^2a^3+e^2\cot^2(c+dx)a^3+7e^2\cot(c+dx)a^3}{(e\cot(c+dx))^{7/2}}dx}{7e^3}$$

$$\Big\downarrow \; 25$$

$$\frac{2\int \frac{8e^2a^3+e^2\cot^2(c+dx)a^3+7e^2\cot(c+dx)a^3}{(e\cot(c+dx))^{7/2}}dx}{7e^3}+\frac{2(a^3\cot(c+dx)+a^3)}{7de(e\cot(c+dx))^{7/2}}$$

$\big\downarrow$ 3042

$$\frac{2\int \frac{8e^2a^3+e^2\tan\left(c+dx+\frac{\pi}{2}\right)^2a^3-7e^2\tan\left(c+dx+\frac{\pi}{2}\right)a^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7e^3}+\frac{2(a^3\cot(c+dx)+a^3)}{7de(e\cot(c+dx))^{7/2}}$$

$\big\downarrow$ 4111

$$\frac{2\left(\frac{\int \frac{7\left(a^3e^3-a^3e^3\cot(c+dx)\right)}{(e\cot(c+dx))^{5/2}}dx}{e^2}+\frac{16a^3e}{5d(e\cot(c+dx))^{5/2}}\right)}{7e^3}+\frac{2(a^3\cot(c+dx)+a^3)}{7de(e\cot(c+dx))^{7/2}}$$

$\big\downarrow$ 27

$$\frac{2\left(\frac{7\int \frac{a^3e^3-a^3e^3\cot(c+dx)}{(e\cot(c+dx))^{5/2}}dx}{e^2}+\frac{16a^3e}{5d(e\cot(c+dx))^{5/2}}\right)}{7e^3}+\frac{2(a^3\cot(c+dx)+a^3)}{7de(e\cot(c+dx))^{7/2}}$$

$\big\downarrow$ 3042

$$\frac{2\left(\frac{7\int \frac{a^3e^3+a^3\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{e^2}+\frac{16a^3e}{5d(e\cot(c+dx))^{5/2}}\right)}{7e^3}+\frac{2(a^3\cot(c+dx)+a^3)}{7de(e\cot(c+dx))^{7/2}}$$

$\big\downarrow$ 4012

$$\frac{2\left(\frac{7\left(\frac{\int -\frac{a^3e^4+a^3\cot(c+dx)e^4}{(e\cot(c+dx))^{3/2}}dx}{e^2}+\frac{2a^3e^2}{3d(e\cot(c+dx))^{3/2}}\right)}{e^2}+\frac{16a^3e}{5d(e\cot(c+dx))^{5/2}}\right)}{7e^3}+\frac{2(a^3\cot(c+dx)+a^3)}{7de(e\cot(c+dx))^{7/2}}$$

$\big\downarrow$ 25

$$\frac{2\left(\frac{7\left(\frac{2a^3e^2}{3d(e\cot(c+dx))^{3/2}}-\frac{\int \frac{a^3e^4+a^3\cot(c+dx)e^4}{(e\cot(c+dx))^{3/2}}dx}{e^2}\right)}{e^2}+\frac{16a^3e}{5d(e\cot(c+dx))^{5/2}}\right)}{7e^3}+\frac{2(a^3\cot(c+dx)+a^3)}{7de(e\cot(c+dx))^{7/2}}$$

$\big\downarrow$ 3042

3.22. $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

$$\frac{2\left(\frac{7\left(\frac{2a^3e^2}{3d(e\cot(c+dx))^{3/2}}-\frac{\int \frac{a^3e^4-a^3e^4\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{e^2}\right)}{e^2}+\frac{16a^3e}{5d(e\cot(c+dx))^{5/2}}\right)}{7e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{7de(e\cot(c+dx))^{7/2}}$$

$\downarrow$ 4012

$$\frac{2\left(\frac{7\left(\frac{2a^3e^2}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{\int \frac{a^3e^5-a^3e^5\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{2a^3e^3}{d\sqrt{e\cot(c+dx)}}}{e^2}\right)}{e^2}+\frac{16a^3e}{5d(e\cot(c+dx))^{5/2}}\right)}{7e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{7de(e\cot(c+dx))^{7/2}}$$

$\downarrow$ 3042

$$\frac{2\left(\frac{7\left(\frac{2a^3e^2}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{\int \frac{a^3e^5+a^3\tan\left(c+dx+\frac{\pi}{2}\right)e^5}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}+\frac{2a^3e^3}{d\sqrt{e\cot(c+dx)}}}{e^2}\right)}{e^2}+\frac{16a^3e}{5d(e\cot(c+dx))^{5/2}}\right)}{7e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{7de(e\cot(c+dx))^{7/2}}$$

$\downarrow$ 4015

3.22. $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

$$\frac{2\left(\frac{7\left(\frac{2a^3e^2}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2a^3e^3}{d\sqrt{e\cot(c+dx)}}-\frac{2a^6e^8\int\frac{1}{2a^6e^{10}-\left(a^3e^5+a^3\cot(c+dx)e^5\right)^2\tan(c+dx)}d\frac{a^3e^5+a^3\cot(c+dx)e^5}{\sqrt{e\cot(c+dx)}}}{d}}{e^2}\right)}{e^2}+\frac{16a^3e}{5d(e\cot(c+dx))^{5/2}}\right)}{7e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{7de(e\cot(c+dx))^{7/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{2\left(\frac{7\left(\frac{2a^3e^2}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2a^3e^3}{d\sqrt{e\cot(c+dx)}}-\frac{\sqrt{2}a^3e^{5/2}\operatorname{arctanh}\left(\frac{a^3e^5\cot(c+dx)+a^3e^5}{\sqrt{2}a^3e^{9/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{e^2}\right)}{e^2}+\frac{16a^3e}{5d(e\cot(c+dx))^{5/2}}\right)}{7e^3}+\frac{2\left(a^3\cot(c+dx)+a^3\right)}{7de(e\cot(c+dx))^{7/2}}$$

input
```
Int[(a + a*Cot[c + d*x])^3/(e*Cot[c + d*x])^(9/2),x]
```

output
```
(2*(a^3 + a^3*Cot[c + d*x]))/(7*d*e*(e*Cot[c + d*x])^(7/2)) + (2*((16*a^3*
e)/(5*d*(e*Cot[c + d*x])^(5/2)) + (7*((2*a^3*e^2)/(3*d*(e*Cot[c + d*x])^(3
/2)) - (-((Sqrt[2]*a^3*e^(5/2)*ArcTanh[(a^3*e^5 + a^3*e^5*Cot[c + d*x])/(S
qrt[2]*a^3*e^(9/2)*Sqrt[e*Cot[c + d*x]])])/d) + (2*a^3*e^3)/(d*Sqrt[e*Cot[
c + d*x]]))/e^2))/e^2))/(7*e^3)
```

3.22. $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

3.22. $\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.22.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(140) = 280$.

Time = 0.09 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.05

method	result
derivativedivides	$-\frac{2a^3\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}\cdots}\right.\right.}{4e}\cdots\right)}{\cdots}$
default	$-\frac{2a^3\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}\cdots}\right.\right.}{4e}\cdots\right)}{\cdots}$
parts	$-\frac{2a^3e\left(-\frac{1}{7e^2(e\cot(dx+c))^{\frac{7}{2}}}+\frac{1}{3e^4(e\cot(dx+c))^{\frac{3}{2}}}+\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right.\right.}{8e^6}\cdots\right)}{d}$

input
```
int((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*a^3/e^2*(1/e^2*(-1/4/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))
+1/4/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/7*e/(e*cot(d*x+c))^
(7/2)-3/5/(e*cot(d*x+c))^(5/2)+2/e^2/(e*cot(d*x+c))^(1/2)-2/3/e/(e*cot(d*x
+c))^(3/2))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.12

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \left[\frac{\frac{105\sqrt{2}\left(a^3e\cos(2\,dx+2\,c)^2+2\,a^3e\cos(2\,dx+2\,c)+a^3e\right)\log\left(-\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+}{\sqrt{e}}\right.}{\sqrt{e}}}{} \right.$$

$$-\frac{2\left(105\sqrt{2}\left(a^3e\cos(2\,dx+2\,c)^2+2\,a^3e\cos(2\,dx+2\,c)+a^3e\right)\sqrt{-\frac{1}{e}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\sqrt{-\frac{1}{e}}(\cos(2\,dx+2}{2\,(\cos(2\,dx+2\,c)+1}\right.\right.}{105\,(de^5}$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
[1/105*(105*sqrt(2)*(a^3*e*cos(2*d*x + 2*c)^2 + 2*a^3*e*cos(2*d*x + 2*c) +
 a^3*e)*log(-sqrt(2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(
2*d*x + 2*c) - sin(2*d*x + 2*c) - 1)/sqrt(e) + 2*sin(2*d*x + 2*c) + 1)/sqr
t(e) - 2*(55*a^3*cos(2*d*x + 2*c)^2 + 30*a^3*cos(2*d*x + 2*c) - 85*a^3 + 2
1*(13*a^3*cos(2*d*x + 2*c) + 7*a^3)*sin(2*d*x + 2*c))*sqrt((e*cos(2*d*x +
2*c) + e)/sin(2*d*x + 2*c)))/(d*e^5*cos(2*d*x + 2*c)^2 + 2*d*e^5*cos(2*d*x
 + 2*c) + d*e^5), -2/105*(105*sqrt(2)*(a^3*e*cos(2*d*x + 2*c)^2 + 2*a^3*e*
cos(2*d*x + 2*c) + a^3*e)*sqrt(-1/e)*arctan(1/2*sqrt(2)*sqrt((e*cos(2*d*x
+ 2*c) + e)/sin(2*d*x + 2*c))*sqrt(-1/e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2
*c) + 1)/(cos(2*d*x + 2*c) + 1)) + (55*a^3*cos(2*d*x + 2*c)^2 + 30*a^3*cos
(2*d*x + 2*c) - 85*a^3 + 21*(13*a^3*cos(2*d*x + 2*c) + 7*a^3)*sin(2*d*x +
2*c))*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^5*cos(2*d*x +
2*c)^2 + 2*d*e^5*cos(2*d*x + 2*c) + d*e^5)]
```

3.22.6 Sympy [F]

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = a^3\left(\int \frac{1}{(e\cot(c+dx))^{\frac{9}{2}}}\,dx + \int \frac{3\cot(c+dx)}{(e\cot(c+dx))^{\frac{9}{2}}}\,dx + \int \frac{3\cot^2(c+dx)}{(e\cot(c+dx))^{\frac{9}{2}}}\,dx + \int \frac{\cot^3(c+dx)}{(e\cot(c+dx))^{\frac{9}{2}}}\,dx\right)$$

input
```
integrate((a+a*cot(d*x+c))**3/(e*cot(d*x+c))**(9/2),x)
```

output
```
a**3*(Integral((e*cot(c + d*x))**(-9/2), x) + Integral(3*cot(c + d*x)/(e*c
ot(c + d*x))**(9/2), x) + Integral(3*cot(c + d*x)**2/(e*cot(c + d*x))**(9/
2), x) + Integral(cot(c + d*x)**3/(e*cot(c + d*x))**(9/2), x))
```

3.22.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.22.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a + a\cot(c + dx))^3}{(e\cot(c + dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

3.22.9 Mupad [B] (verification not implemented)

Time = 14.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.78

$$\int \frac{(a + a\cot(c + dx))^3}{(e\cot(c + dx))^{9/2}}\,dx = \frac{-4\,e\,a^3\cot(c+d\,x)^3 + \frac{4\,e\,a^3\cot(c+d\,x)^2}{3} + \frac{6\,e\,a^3\cot(c+d\,x)}{5} + \frac{2\,e\,a^3}{7}}{d\,e^2\,(e\cot(c+d\,x))^{7/2}} + \frac{2\sqrt{2}\,a^3\,\mathrm{atanh}\left(\frac{32\sqrt{2}\,a^6\,d\,e^{9/2}\,\sqrt{e\cot(c+d\,x)}}{32\,a^6\,d\,e^5+32\,a^6\,d\,e^5\cot(c+d\,x)}\right)}{d\,e^{9/2}}$$

input
```
int((a + a*cot(c + d*x))^3/(e*cot(c + d*x))^(9/2),x)
```

output
```
((2*a^3*e)/7 + (4*a^3*e*cot(c + d*x)^2)/3 - 4*a^3*e*cot(c + d*x)^3 + (6*a^
3*e*cot(c + d*x))/5)/(d*e^2*(e*cot(c + d*x))^(7/2)) + (2*2^(1/2)*a^3*atanh
((32*2^(1/2)*a^6*d*e^(9/2)*(e*cot(c + d*x))^(1/2))/(32*a^6*d*e^5 + 32*a^6*
d*e^5*cot(c + d*x))))/(d*e^(9/2))
```

3.23 $\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx$

3.23.1 Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx=\frac{e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad}-\frac{e^{5/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

output
```
e^(5/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d-1/2*e^(5/2)*arctan(1/2*(e
^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a/d*2^(1/2)-2*e^2
*(e*cot(d*x+c))^(1/2)/a/d
```

3.23.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 296 vs. 2(111) = 222.

Time = 0.92 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.67

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx=\frac{e\left(8e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)-4(-e^2)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)-2\sqrt{2}e^{3/2}\arctan\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x]),x]
```

output
```
(e*(8*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] - 4*(-e^2)^(3/4)*ArcTan
[Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] - 2*Sqrt[2]*e^(3/2)*ArcTan[1 - (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 2*Sqrt[2]*e^(3/2)*ArcTan[1 + (Sqrt[2]*S
qrt[e*Cot[c + d*x]])/Sqrt[e]] + 4*(-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d*x]
]/(-e^2)^(1/4)] - 16*e*Sqrt[e*Cot[c + d*x]] - Sqrt[2]*e^(3/2)*Log[Sqrt[e]
+ Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]] + Sqrt[2]*e^(3/2)*L
og[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]]))/(8*a*d
)
```

3.23.3 Rubi [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 4049, 27, 3042, 4136, 3042, 4015, 218, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{5/2}}{a\cot(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{a-a\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4049}$$

$$-\frac{2\int \frac{a\cot^2(c+dx)e^3+ae^3+a\cot(c+dx)e^3}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{a}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a\cot^2(c+dx)e^3+ae^3+a\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{a}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{a\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3+ae^3-a\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{4136}$$

$$-\frac{\frac{\int \frac{a^2e^3+a^2\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}}dx}{2a^2}+\frac{1}{2}ae^3\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{a}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\int \frac{a^2e^3-a^2e^3\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{1}{2}ae^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{ 4015}$$

$$-\frac{\frac{1}{2}ae^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{a^2e^6\int \frac{1}{-2a^4e^6-\left(a^2e^3-a^2e^3\cot(c+dx)\right)^2\tan(c+dx)}d\frac{a^2e^3-a^2e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}}{d}}{a}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{\frac{1}{2}ae^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{e^{5/2}\arctan\left(\frac{a^2e^3-a^2e^3\cot(c+dx)}{\sqrt{2}a^2e^{5/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}d}}{a}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{ 4117}$$

$$-\frac{\frac{ae^3\int \frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{2d}+\frac{e^{5/2}\arctan\left(\frac{a^2e^3-a^2e^3\cot(c+dx)}{\sqrt{2}a^2e^{5/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}d}}{a}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{e^3\int \frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{2d}+\frac{e^{5/2}\arctan\left(\frac{a^2e^3-a^2e^3\cot(c+dx)}{\sqrt{2}a^2e^{5/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}d}}{a}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\frac{e^{5/2}\arctan\left(\frac{a^2e^3-a^2e^3\cot(c+dx)}{\sqrt{2}a^2e^{5/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}d}-\frac{e^2\int \frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{d}}{a}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{\frac{e^{5/2}\arctan\left(\frac{a^2e^3-a^2e^3\cot(c+dx)}{\sqrt{2}a^2e^{5/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}d}+\frac{e^{5/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{d}}{a}-\frac{2e^2\sqrt{e\cot(c+dx)}}{ad}$$

3.23. $\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}dx$

input
```
Int[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x]),x]
```

output
```
-(((e^(5/2)*ArcTan[Cot[c + d*x]/Sqrt[e]])/d + (e^(5/2)*ArcTan[(a^2*e^3 - a
^2*e^3*Cot[c + d*x])/(Sqrt[2]*a^2*e^(5/2)*Sqrt[e*Cot[c + d*x]])])/(Sqrt[2]
*d))/a) - (2*e^2*Sqrt[e*Cot[c + d*x]])/(a*d)
```

3.23.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.23.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(92) = 184$.

Time = 0.10 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.81

method	result
derivativedivides	$-\frac{2e^2\left(\sqrt{e\cot(dx+c)}-\frac{e\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arc}}{8e}}{}}{}$
default	$-\frac{2e^2\left(\sqrt{e\cot(dx+c)}-\frac{e\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arc}}{8e}}{}}{}$

input
```
int((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a*e^2*((e*cot(d*x+c))^(1/2)-1/2*e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*c
ot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x
+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2
)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1))+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/2
*e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2)))
```

3.23. $\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx$

3.23.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.60

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \left[\frac{\sqrt{2}\sqrt{-e}e^2\log\left(\left(\sqrt{2}\cos(2\,dx+2\,c)+\sqrt{2}\sin(2\,dx+2\,c)-\sqrt{2}\right)\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}\right.}{}\right.$$

$$-\frac{\sqrt{2}e^{\frac{5}{2}}\arctan\left(-\frac{\left(\sqrt{2}\cos(2\,dx+2\,c)-\sqrt{2}\sin(2\,dx+2\,c)+\sqrt{2}\right)\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{2\,(e\cos(2\,dx+2\,c)+e)}\right)-2\,e^{\frac{5}{2}}\arctan\left(\frac{\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{\sqrt{e}}\right)+4\,e^2\sqrt{}}{2\,ad}$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*sqrt(-e)*e^2*log((sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d
*x + 2*c) - sqrt(2))*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c)) - 2*e*sin(2*d*x + 2*c) + e) + 2*sqrt(-e)*e^2*log((e*cos(2*d*x + 2*c) -
 e*sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) -
8*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(a*d), -1/2*(sqrt(2
)*e^(5/2)*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c)
 + sqrt(2))*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/(e*cos
(2*d*x + 2*c) + e)) - 2*e^(5/2)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2
*d*x + 2*c))/sqrt(e)) + 4*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c)))/(a*d)]
```

3.23.6 Sympy [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \frac{\int \frac{(e\cot(c+dx))^{\frac{5}{2}}}{\cot(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c)),x)
```

output
```
Integral((e*cot(c + d*x))**(5/2)/(cot(c + d*x) + 1), x)/a
```

3.23.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.23.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \int \frac{(e\cot(dx+c))^{\frac{5}{2}}}{a\cot(dx+c)+a}\,dx$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(5/2)/(a*cot(d*x + c) + a), x)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 12.93 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx = \frac{e^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{a\,d} - \frac{2\,e^2\,\sqrt{e\cot\left(c+d\,x\right)}}{a\,d} + \frac{\sqrt{2}\,e^{5/2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{4\,a\,d}$$

input
```
int((e*cot(c + d*x))^(5/2)/(a + a*cot(c + d*x)),x)
```

output
```
(e^(5/2)*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(a*d) - (2*e^2*(e*cot(c + d
*x))^(1/2))/(a*d) + (2^(1/2)*e^(5/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/
2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (
2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/(4*a*d)
```

3.23. $\int \frac{(e\cot(c+dx))^{5/2}}{a+a\cot(c+dx)}\,dx$

3.24 $\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx$

3.24.1 Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx=-\frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad}+\frac{e^{3/2}\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}$$

output
```
-e^(3/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d+1/2*e^(3/2)*arctanh(1/2*
(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a/d*2^(1/2)
```

3.24.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(87) = 174.

Time = 0.55 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.22

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx=$$
$$-\frac{8e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)-4(-e^2)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)+2\sqrt{2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)-2\sqrt{2}\cdots}{\cdots}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x]),x]
```

output
```
-1/8*(8*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] - 4*(-e^2)^(3/4)*ArcT
an[Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] + 2*Sqrt[2]*e^(3/2)*ArcTan[1 - (Sqrt
[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - 2*Sqrt[2]*e^(3/2)*ArcTan[1 + (Sqrt[2]
*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 4*(-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d*
x]]/(-e^2)^(1/4)] + Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - S
qrt[2]*Sqrt[e*Cot[c + d*x]]] - Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c
 + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(a*d)
```

3.24.3 Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4056, 25, 3042, 4015, 221, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a\cot(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{a-a\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4056}$$

$$\frac{\int -\frac{ae^2-ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}+\frac{1}{2}e^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}e^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx-\frac{\int \frac{ae^2-ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{\int \frac{ae^2+a\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$$\Big\downarrow \text{4015}$$

$$\frac{e^4\int \frac{1}{2a^2e^4-(ae^2+a\cot(c+dx)e^2)^2\tan(c+dx)}d\frac{ae^2+a\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}}{d}+\frac{1}{2}e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{e^{3/2}\text{arctanh}\left(\frac{ae^2\cot(c+dx)+ae^2}{\sqrt{2}ae^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}$$

$$\Big\downarrow \; 4117$$

$$\frac{e^2\int \frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{2d}+\frac{e^{3/2}\text{arctanh}\left(\frac{ae^2\cot(c+dx)+ae^2}{\sqrt{2}ae^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2\int \frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{2ad}+\frac{e^{3/2}\text{arctanh}\left(\frac{ae^2\cot(c+dx)+ae^2}{\sqrt{2}ae^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}$$

$$\Big\downarrow \; 73$$

$$\frac{e^{3/2}\text{arctanh}\left(\frac{ae^2\cot(c+dx)+ae^2}{\sqrt{2}ae^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}-\frac{e\int \frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{ad}$$

$$\Big\downarrow \; 216$$

$$\frac{e^{3/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{ad}+\frac{e^{3/2}\text{arctanh}\left(\frac{ae^2\cot(c+dx)+ae^2}{\sqrt{2}ae^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}$$

input
```
Int[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x]),x]
```

output
```
(e^(3/2)*ArcTan[Cot[c + d*x]/Sqrt[e]])/(a*d) + (e^(3/2)*ArcTanh[(a*e^2 + a
*e^2*Cot[c + d*x])/(Sqrt[2]*a*e^(3/2)*Sqrt[e*Cot[c + d*x]])])/(Sqrt[2]*a*d
)
```

3.24. $\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx$

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4056
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(c^2 + d^2)   Int[Simp[a^2*c - b^2*c +
2*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x
]], x], x] + Simp[(b*c - a*d)^2/(c^2 + d^2)   Int[(1 + Tan[e + f*x]^2)/(Sqr
t[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.24. $\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx$

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(71) = 142$.

Time = 0.05 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.43

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{16e}\right.}{}$
default	$-\frac{2e^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(c}}{(e^2)^{\frac{1}{4}}}\right.\right.}{16e}\right.}{}$

input
```
int((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a*e^2*(-1/16/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/16/(e
^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/
2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-
2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/2/e^(1/2)*arctan((e*cot(d*x
+c))^(1/2)/e^(1/2)))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.83

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx = \left[-\frac{\sqrt{2}\sqrt{-e}e\arctan\left(\frac{\left(\sqrt{2}\cos(2\,dx+2\,c)+\sqrt{2}\sin(2\,dx+2\,c)+\sqrt{2}\right)\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{2\,(e\cos(2\,dx+2\,c)+e)}\right)-\sqrt{-e}}{2\,ad}\right.$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(2)*sqrt(-e)*e*arctan(1/2*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*s
in(2*d*x + 2*c) + sqrt(2))*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*
x + 2*c))/(e*cos(2*d*x + 2*c) + e)) - sqrt(-e)*e*log((e*cos(2*d*x + 2*c) -
 e*sin(2*d*x + 2*c) - 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)))/(
a*d), 1/4*(sqrt(2)*e^(3/2)*log(-(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*
d*x + 2*c) - sqrt(2))*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c)) + 2*e*sin(2*d*x + 2*c) + e) - 4*e^(3/2)*arctan(sqrt((e*cos(2*d*x + 2*c
) + e)/sin(2*d*x + 2*c))/sqrt(e)))/(a*d)]
```

3.24.6 Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx = \frac{\int \frac{(e\cot(c+dx))^{\frac{3}{2}}}{\cot(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c)),x)
```

output
```
Integral((e*cot(c + d*x))**(3/2)/(cot(c + d*x) + 1), x)/a
```

3.24.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.24.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{a\cot(dx+c)+a}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(3/2)/(a*cot(d*x + c) + a), x)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 12.78 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+a\cot(c+dx)}\,dx = \frac{\sqrt{2}\,e^{3/2}\,\mathrm{atanh}\left(\frac{12\sqrt{2}\,e^{25/2}\,\sqrt{e\cot(c+d\,x)}}{12\,e^{13}\cot(c+d\,x)+12\,e^{13}}\right)}{2\,a\,d} - \frac{e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{a\,d}$$

input
```
int((e*cot(c + d*x))^(3/2)/(a + a*cot(c + d*x)),x)
```

output
```
(2^(1/2)*e^(3/2)*atanh((12*2^(1/2)*e^(25/2)*(e*cot(c + d*x))^(1/2))/(12*e^
13*cot(c + d*x) + 12*e^13)))/(2*a*d) - (e^(3/2)*atan((e*cot(c + d*x))^(1/2
)/e^(1/2)))/(a*d)
```

3.25 $\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx$

3.25.1 Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx=\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad}+\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}$$

output
```
arctan((e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a/d+1/2*arctan(1/2*(e^(1/2)-c
ot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*e^(1/2)/a/d*2^(1/2)
```

3.25.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 283 vs. 2(87) = 174.

Time = 0.43 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.25

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx$$
$$=\frac{8e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)+4(-e^2)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)+2\sqrt{2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)-2\sqrt{\cdots}}{\cdots}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x]),x]
```

output
```
(8*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] + 4*(-e^2)^(3/4)*ArcTan[Sq
rt[e*Cot[c + d*x]]/(-e^2)^(1/4)] + 2*Sqrt[2]*e^(3/2)*ArcTan[1 - (Sqrt[2]*S
qrt[e*Cot[c + d*x]])/Sqrt[e]] - 2*Sqrt[2]*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[e*Cot[c + d*x]])/Sqrt[e]] - 4*(-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d*x]]/(
-e^2)^(1/4)] + Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2
]*Sqrt[e*Cot[c + d*x]]] - Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*
x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(8*a*d*e)
```

3.25.3 Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4055, 3042, 4015, 218, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\cot(c+dx)}}{a\cot(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}{a-a\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4055}$$

$$\frac{\int \frac{ae+a\cot(c+dx)e}{\sqrt{e\cot(c+dx)}}dx}{2a^2}-\frac{1}{2}e\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{ae-ae\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{1}{2}e\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4015}$$

$$-\frac{e^2\int \frac{1}{-2a^2e^2-(ae-ae\cot(c+dx))^2\tan(c+dx)}d\frac{ae-ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}}{d}-\frac{1}{2}e\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{e}\arctan\left(\frac{ae-ae\cot(c+dx)}{\sqrt{2}a\sqrt{e}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}-\frac{1}{2}e\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4117}$$

$$\frac{\sqrt{e}\arctan\left(\frac{ae-ae\cot(c+dx)}{\sqrt{2}a\sqrt{e}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}-\frac{e\int\frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{e}\arctan\left(\frac{ae-ae\cot(c+dx)}{\sqrt{2}a\sqrt{e}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}-\frac{e\int\frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{2ad}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int\frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{ad}+\frac{\sqrt{e}\arctan\left(\frac{ae-ae\cot(c+dx)}{\sqrt{2}a\sqrt{e}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{e}\arctan\left(\frac{ae-ae\cot(c+dx)}{\sqrt{2}a\sqrt{e}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}-\frac{\sqrt{e}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{ad}$$

input
```
Int[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x]),x]
```

output
```
-((Sqrt[e]*ArcTan[Cot[c + d*x]/Sqrt[e]])/(a*d)) + (Sqrt[e]*ArcTan[(a*e - a
*e*Cot[c + d*x])/(Sqrt[2]*a*Sqrt[e]*Sqrt[e*Cot[c + d*x]])])/(Sqrt[2]*a*d)
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4055
```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[1/(c^2 + d^2)   Int[Simp[a*c + b*d + (b*c -
 a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Simp[d*((b*c - a*
d)/(c^2 + d^2))   Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d
*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.25. $\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx$

3.25.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(71) = 142$.

Time = 0.05 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.49

method	result
derivativedivides	$-\frac{2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

input
```
int((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a*e^2*(1/2/e*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/
8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2
^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)
+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arct
an(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/2/e^(3/2)*arctan((e*co
t(d*x+c))^(1/2)/e^(1/2)))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.80

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx = \left[\frac{\sqrt{2}\sqrt{-e}\log\left(-\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)+\sqrt{2}\sin\left(2\,dx+2\,c\right)-\sqrt{2}\right)\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}-2\,e\sin\left(2\,dx+2\right.\right.}{4\,ad}\right.$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*sqrt(-e)*log(-(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x
+ 2*c) - sqrt(2))*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))
 - 2*e*sin(2*d*x + 2*c) + e) + 2*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(
2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*
sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)))/(a*d), 1
/2*(sqrt(2)*sqrt(e)*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*
d*x + 2*c) + sqrt(2))*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c))/(e*cos(2*d*x + 2*c) + e)) + 2*sqrt(e)*arctan(sqrt((e*cos(2*d*x + 2*c)
+ e)/sin(2*d*x + 2*c))/sqrt(e)))/(a*d)]
```

3.25.6 Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx = \frac{\int \frac{\sqrt{e\cot(c+dx)}}{\cot(c+dx)+1}\,dx}{a}$$

input
```
integrate((e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c)),x)
```

output
```
Integral(sqrt(e*cot(c + d*x))/(cot(c + d*x) + 1), x)/a
```

3.25.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.25.8 Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx = \int \frac{\sqrt{e\cot\left(dx+c\right)}}{a\cot\left(dx+c\right)+a}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*cot(d*x + c))/(a*cot(d*x + c) + a), x)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 12.73 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx = \frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{a\,d} - \frac{\sqrt{2}\,\sqrt{e}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,{(e\cot(c+d\,x))}^{3/2}}{2\,e^{3/2}}\right)\right)}{4\,a\,d}$$

input
```
int((e*cot(c + d*x))^(1/2)/(a + a*cot(c + d*x)),x)
```

output
```
(e^(1/2)*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(a*d) - (2^(1/2)*e^(1/2)*(2
*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*c
ot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3
/2)))))/(4*a*d)
```

3.25. $\int \frac{\sqrt{e\cot(c+dx)}}{a+a\cot(c+dx)}\,dx$

3.26 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx$

3.26.1 Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx = -\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ad\sqrt{e}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}(1+\cot(c+dx))}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}}$$

output
```
-arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d/e^(1/2)-1/2*arctanh(1/2*(1+cot(d
*x+c))*e^(1/2)*2^(1/2)/(e*cot(d*x+c))^(1/2))/a/d*2^(1/2)/e^(1/2)
```

3.26.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 283 vs. 2(83) = 166.

Time = 0.52 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.41

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx =$$

$$-\frac{8e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) + 4(-e^2)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right) - 2\sqrt{2}e^{3/2}\arctan\left(1 - \frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right) + 2}{}$$

input
```
Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])),x]
```

output
```
-1/8*(8*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] + 4*(-e^2)^(3/4)*ArcT
an[Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] - 2*Sqrt[2]*e^(3/2)*ArcTan[1 - (Sqrt
[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 2*Sqrt[2]*e^(3/2)*ArcTan[1 + (Sqrt[2]
*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - 4*(-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d*
x]]/(-e^2)^(1/4)] - Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - S
qrt[2]*Sqrt[e*Cot[c + d*x]]] + Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c
 + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])/(a*d*e^2)
```

3.26.3 Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4057, 3042, 4015, 221, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(c+dx)+a)\sqrt{e\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4057}$$

$$\frac{\int \frac{a-a\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}+\frac{1}{2}\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{1}{2}\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4015}$$

$$\frac{1}{2}\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{\int \frac{1}{2a^2-(\cot(c+dx)a+a)^2\tan(c+dx)}d\frac{\cot(c+dx)a+a}{\sqrt{e\cot(c+dx)}}}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}(a\cot(c+dx)+a)}{\sqrt{2}a\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}}$$

$$\Big\downarrow \text{4117}$$

$$\frac{\int \frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{2d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}(a\cot(c+dx)+a)}{\sqrt{2}a\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{2ad}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}(a\cot(c+dx)+a)}{\sqrt{2}a\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{ade}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}(a\cot(c+dx)+a)}{\sqrt{2}a\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{ad\sqrt{e}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}(a\cot(c+dx)+a)}{\sqrt{2}a\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad\sqrt{e}}$$

input
```
Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])),x]
```

output
```
ArcTan[Cot[c + d*x]/Sqrt[e]]/(a*d*Sqrt[e]) - ArcTanh[(Sqrt[e]*(a + a*Cot[c
 + d*x]))/(Sqrt[2]*a*Sqrt[e*Cot[c + d*x]])]/(Sqrt[2]*a*d*Sqrt[e])
```

3.26.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4057
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[1/(c^2 + d^2)   Int[(a + b*Tan[e + f*x])^m
*(c - d*Tan[e + f*x]), x], x] + Simp[d^2/(c^2 + d^2)   Int[(a + b*Tan[e + f
*x])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.26. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx$

3.26.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(68) = 136$.

Time = 0.04 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.66

method	result
derivativedivides	$-\frac{2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^2\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

input
```
int(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a*e^2*(1/2/e^2*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-
1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/
2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*ar
ctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+1/2/e^(5/2)*arctan((e*
cot(d*x+c))^(1/2)/e^(1/2)))
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.87

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx$$
$$= \left[\frac{\sqrt{2}\sqrt{-e}\arctan\left(\frac{\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)+1)}{2\,(e\cos(2\,dx+2\,c)+e)}\right)-\sqrt{-e}\log\left(\frac{e\cos(2\,dx+2\,c)-e\sin(2\,dx+2\,c)+2\,\sqrt{\ldots}}{\cos(2\,dx+2\,c)+\ldots}\right)}{2\,ade}\right.$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*sqrt(-e)*arctan(1/2*sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c
) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(e*cos(
2*d*x + 2*c) + e)) - sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c)
 + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x +
2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)))/(a*d*e), 1/4*(sqrt(2
)*sqrt(e)*log(sqrt(2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1) + 2*e*sin(2*d*x + 2*c) + e)
- 4*sqrt(e)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)
))/(a*d*e)]
```

3.26.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx = \frac{\int \frac{1}{\sqrt{e\cot{(c+dx)}}\cot{(c+dx)}+\sqrt{e\cot{(c+dx)}}}\,dx}{a}$$

input
```
integrate(1/(e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c)),x)
```

output
```
Integral(1/(sqrt(e*cot(c + d*x))*cot(c + d*x) + sqrt(e*cot(c + d*x))), x)/
a
```

3.26.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.26.8 Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx = \int \frac{1}{(a\cot(dx+c)+a)\sqrt{e\cot(dx+c)}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*cot(d*x + c) + a)*sqrt(e*cot(d*x + c))), x)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 12.78 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))}\,dx = -\frac{\operatorname{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{a\,d\,\sqrt{e}} - \frac{\sqrt{2}\operatorname{atanh}\left(\frac{12\sqrt{2}\,e^{9/2}\,\sqrt{e\cot(c+d\,x)}}{12\,e^5\cot(c+d\,x)+12\,e^5}\right)}{2\,a\,d\,\sqrt{e}}$$

input
```
int(1/((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))),x)
```

output
```
- atan((e*cot(c + d*x))^(1/2)/e^(1/2))/(a*d*e^(1/2)) - (2^(1/2)*atanh((12*
2^(1/2)*e^(9/2)*(e*cot(c + d*x))^(1/2))/(12*e^5*cot(c + d*x) + 12*e^5)))/(
2*a*d*e^(1/2))
```

3.27 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx$

3.27.1 Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx=\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{3/2}}-\frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{3/2}}+\frac{2}{ade\sqrt{e\cot(c+dx)}}$$

output
```
arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d/e^(3/2)-1/2*arctan(1/2*(e^(1/2)-c
ot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a/d/e^(3/2)*2^(1/2)+2/a/d
/e/(e*cot(d*x+c))^(1/2)
```

3.27.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.06

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx=\frac{8\sqrt{e}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\cot(c+dx)\right)+8\sqrt{e}\,\text{Hyp}}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])),x]
```

output
```
(8*Sqrt[e]*Hypergeometric2F1[-1/2, 1, 1/2, -Cot[c + d*x]] + 8*Sqrt[e]*Hype
rgeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*Sqrt[e*Cot[c + d*x]
]*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 2*ArcTan[1 + (S
qrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x]
 - Sqrt[2]*Sqrt[e*Cot[c + d*x]]] + Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sq
rt[2]*Sqrt[e*Cot[c + d*x]]]))/(8*a*d*e^(3/2)*Sqrt[e*Cot[c + d*x]])
```

3.27.3 Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4052, 27, 3042, 4136, 3042, 4015, 218, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(c+dx)+a)(e\cot(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4052}$$

$$\frac{2\int -\frac{a\cot^2(c+dx)e^2+ae^2+a\cot(c+dx)e^2}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}+\frac{2}{ade\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\int \frac{a\cot^2(c+dx)e^2+ae^2+a\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\int \frac{a\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+ae^2-a\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}$$

$$\Big\downarrow \text{4136}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{\int \frac{a^2e^2+a^2\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx}{2a^2}+\frac{1}{2}ae^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}$$

$$\downarrow \text{3042}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{\int \frac{a^2e^2-a^2e^2\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{1}{2}ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}$$

$$\downarrow \text{4015}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{1}{2}ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{a^2e^4\int \frac{1}{-2a^4e^4-\left(a^2e^2-a^2e^2\cot(c+dx)\right)^2\tan(c+dx)}d\frac{a^2e^2-a^2e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}}{d}}{ae^3}$$

$$\downarrow \text{218}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{1}{2}ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{e^{3/2}\arctan\left(\frac{a^2e^2-a^2e^2\cot(c+dx)}{\sqrt{2}a^2e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}d}}{ae^3}$$

$$\downarrow \text{4117}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{ae^2\int \frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{2d}+\frac{e^{3/2}\arctan\left(\frac{a^2e^2-a^2e^2\cot(c+dx)}{\sqrt{2}a^2e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}d}}{ae^3}$$

$$\downarrow \text{27}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{e^2\int \frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{2d}+\frac{e^{3/2}\arctan\left(\frac{a^2e^2-a^2e^2\cot(c+dx)}{\sqrt{2}a^2e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}d}}{ae^3}$$

$$\downarrow \text{73}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{e^{3/2}\arctan\left(\frac{a^2e^2-a^2e^2\cot(c+dx)}{\sqrt{2}a^2e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}d}-\frac{e\int \frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{d}}{ae^3}$$

$$\downarrow \text{216}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{e^{3/2}\arctan\left(\frac{a^2e^2-a^2e^2\cot(c+dx)}{\sqrt{2}a^2e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}d}+\frac{e^{3/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{d}}{ae^3}$$

input
```
Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])),x]
```

3.27. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx$

output
```
-(((e^(3/2)*ArcTan[Cot[c + d*x]/Sqrt[e]])/d + (e^(3/2)*ArcTan[(a^2*e^2 - a
^2*e^2*Cot[c + d*x])/(Sqrt[2]*a^2*e^(3/2)*Sqrt[e*Cot[c + d*x]])])/(Sqrt[2]
*d))/(a*e^3)) + 2/(a*d*e*Sqrt[e*Cot[c + d*x]])
```

3.27.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.27.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(92) = 184$.

Time = 0.04 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.87

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots\right.}{\cdots}$
default	$-\frac{2e^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots\right.}{\cdots}$

input
```
int(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a*e^2*(1/2/e^3*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))
-1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/2/e^(7/2)*arctan((e
*cot(d*x+c))^(1/2)/e^(1/2))-1/e^3/(e*cot(d*x+c))^(1/2))
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(93) = 186$.

Time = 0.28 (sec) , antiderivative size = 472, normalized size of antiderivative = 4.25

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx = \left[-\frac{\sqrt{2}\sqrt{-e}(\cos(2\,dx+2\,c)+1)\log\left(-\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)\cdot}{\sin(2\,dx+2\,c)}}\right.}{}\right.$$

$$-\frac{\sqrt{2}\sqrt{e}(\cos(2\,dx+2\,c)+1)\arctan\left(-\frac{\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)+1)}{2\,(e\cos(2\,dx+2\,c)+e)}\right)-2\,\sqrt{e}(\cos(2\,dx+2\,c}{2\,(ade^2\cos(2\,dx+2\,c)+ade^2)}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(2)*sqrt(-e)*(cos(2*d*x + 2*c) + 1)*log(-sqrt(2)*sqrt(-e)*sqrt(
(e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x +
 2*c) - 1) - 2*e*sin(2*d*x + 2*c) + e) + 2*sqrt(-e)*(cos(2*d*x + 2*c) + 1)
*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) - 2*sqrt(-e)*sqrt((e*cos(2*d
*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) +
 sin(2*d*x + 2*c) + 1)) - 8*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
)*sin(2*d*x + 2*c))/(a*d*e^2*cos(2*d*x + 2*c) + a*d*e^2), -1/2*(sqrt(2)*sq
rt(e)*(cos(2*d*x + 2*c) + 1)*arctan(-1/2*sqrt(2)*sqrt(e)*sqrt((e*cos(2*d*x
 + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(
e*cos(2*d*x + 2*c) + e)) - 2*sqrt(e)*(cos(2*d*x + 2*c) + 1)*arctan(sqrt((e
*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) - 4*sqrt((e*cos(2*d*x +
2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c))/(a*d*e^2*cos(2*d*x + 2*c) +
a*d*e^2)]
```

3.27.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

input
```
integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c)),x)
```

output
```
Integral(1/((e*cot(c + d*x))**(3/2)*cot(c + d*x) + (e*cot(c + d*x))**(3/2)
), x)/a
```

3.27.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.27.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx = \int \frac{1}{(a\cot(dx+c)+a)\,(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*cot(d*x + c) + a)*(e*cot(d*x + c))^(3/2)), x)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx = \frac{2}{a\,d\,e\,\sqrt{e\cot(c+d\,x)}} + \frac{\operatorname{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{a\,d\,e^{3/2}} + \frac{\sqrt{2}\,\left(2\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\sqrt{e}}\right) + 2\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\sqrt{e}} + \frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{4\,a\,d\,e^{3/2}}$$

input
```
int(1/((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))),x)
```

output
```
2/(a*d*e*(e*cot(c + d*x))^(1/2)) + atan((e*cot(c + d*x))^(1/2)/e^(1/2))/(a
*d*e^(3/2)) + (2^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)
)) + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot
(c + d*x))^(3/2))/(2*e^(3/2)))))/(4*a*d*e^(3/2))
```

3.27. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))}\,dx$

3.28 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx$

3.28.1 Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = -\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{ade^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ade^{5/2}} + \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2}{ade^2\sqrt{e\cot(c+dx)}}$$

output
```
-arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a/d/e^(5/2)+2/3/a/d/e/(e*cot(d*x+c))
^(3/2)+1/2*arctanh(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))
^(1/2))/a/d/e^(5/2)*2^(1/2)-2/a/d/e^2/(e*cot(d*x+c))^(1/2)
```

3.28.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.62 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.64

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\cot(c+dx)\right) + \text{Hypergeom}}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])),x]
```

output
```
(Hypergeometric2F1[-3/2, 1, -1/2, -Cot[c + d*x]] + Hypergeometric2F1[-3/4,
 1, 1/4, -Cot[c + d*x]^2] - 3*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4,
 -Cot[c + d*x]^2])/(3*a*d*e*(e*Cot[c + d*x])^(3/2))
```

3.28.3 Rubi [A] (warning: unable to verify)

Time = 1.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4052, 27, 3042, 4132, 27, 2030, 3042, 4056, 25, 3042, 4015, 221, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(c+dx)+a)(e\cot(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4052}$$

$$\frac{2\int -\frac{3\left(a\cot^2(c+dx)e^2+ae^2+a\cot(c+dx)e^2\right)}{2(e\cot(c+dx))^{3/2}(\cot(c+dx)a+a)}dx}{3ae^3}+\frac{2}{3ade(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\int \frac{a\cot^2(c+dx)e^2+ae^2+a\cot(c+dx)e^2}{(e\cot(c+dx))^{3/2}(\cot(c+dx)a+a)}dx}{ae^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\int \frac{a\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+ae^2-a\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}$$

$$\Big\downarrow \text{4132}$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2\int -\frac{a^2e^4\cot^2(c+dx)}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}+\frac{2e}{d\sqrt{e\cot(c+dx)}}}{ae^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2e}{d\sqrt{e\cot(c+dx)}}-ae\int\frac{\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}$$

$\Big\downarrow$ 2030

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2e}{d\sqrt{e\cot(c+dx)}}-\frac{a\int\frac{(e\cot(c+dx))^{3/2}}{\cot(c+dx)a+a}dx}{e}}{ae^3}$$

$\Big\downarrow$ 3042

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2e}{d\sqrt{e\cot(c+dx)}}-\frac{a\int\frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{a-a\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{e}}{ae^3}$$

$\Big\downarrow$ 4056

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2e}{d\sqrt{e\cot(c+dx)}}-\frac{a\left(\frac{\int-\frac{ae^2-ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}+\frac{1}{2}e^2\int\frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx\right)}{e}}{ae^3}$$

$\Big\downarrow$ 25

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2e}{d\sqrt{e\cot(c+dx)}}-\frac{a\left(\frac{1}{2}e^2\int\frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx-\frac{\int\frac{ae^2-ae^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}\right)}{e}}{ae^3}$$

$\Big\downarrow$ 3042

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2e}{d\sqrt{e\cot(c+dx)}}-\frac{a\left(\frac{1}{2}e^2\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{\int\frac{ae^2+a\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}\right)}{e}}{ae^3}$$

$\Big\downarrow$ 4015

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2e}{d\sqrt{e\cot(c+dx)}}-\frac{a\left(\frac{e^4\int\frac{1}{2a^2e^4-\left(ae^2+a\cot(c+dx)e^2\right)^2\tan(c+dx)}d\frac{ae^2+a\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}}{d}+\frac{1}{2}e^2\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx\right)}{e}}{ae^3}$$

3.28. $\int\frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}dx$

$$\Big\downarrow \; 221$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2e}{d\sqrt{e\cot(c+dx)}}-\frac{a\left(\frac{1}{2}e^2\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{e^{3/2}\text{arctanh}\left(\frac{ae^2\cot(c+dx)+ae^2}{\sqrt{2}ae^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}\right)}{e}}{ae^3}$$

$$\Big\downarrow \; 4117$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2e}{d\sqrt{e\cot(c+dx)}}-\frac{a\left(\frac{e^2\int\frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{2d}+\frac{e^{3/2}\text{arctanh}\left(\frac{ae^2\cot(c+dx)+ae^2}{\sqrt{2}ae^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}\right)}{e}}{ae^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2e}{d\sqrt{e\cot(c+dx)}}-\frac{a\left(\frac{e^2\int\frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{2ad}+\frac{e^{3/2}\text{arctanh}\left(\frac{ae^2\cot(c+dx)+ae^2}{\sqrt{2}ae^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}\right)}{e}}{ae^3}$$

$$\Big\downarrow \; 73$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2e}{d\sqrt{e\cot(c+dx)}}-\frac{a\left(\frac{e^{3/2}\text{arctanh}\left(\frac{ae^2\cot(c+dx)+ae^2}{\sqrt{2}ae^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}-\frac{e\int\frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{ad}\right)}{e}}{ae^3}$$

$$\Big\downarrow \; 216$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2e}{d\sqrt{e\cot(c+dx)}}-\frac{a\left(\frac{e^{3/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{ad}+\frac{e^{3/2}\text{arctanh}\left(\frac{ae^2\cot(c+dx)+ae^2}{\sqrt{2}ae^{3/2}\sqrt{e\cot(c+dx)}}\right)}{\sqrt{2}ad}\right)}{e}}{ae^3}$$

input
```
Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])),x]
```

3.28. $\int\frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx$

output
```
2/(3*a*d*e*(e*Cot[c + d*x])^(3/2)) - (-((a*((e^(3/2)*ArcTan[Cot[c + d*x]/S
qrt[e]])/(a*d) + (e^(3/2)*ArcTanh[(a*e^2 + a*e^2*Cot[c + d*x])/(Sqrt[2]*a*
e^(3/2)*Sqrt[e*Cot[c + d*x]])])/(Sqrt[2]*a*d)))/e) + (2*e)/(d*Sqrt[e*Cot[c
 + d*x]]))/(a*e^3)
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4056
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(c^2 + d^2)   Int[Simp[a^2*c - b^2*c +
2*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x
]], x], x] + Simp[(b*c - a*d)^2/(c^2 + d^2)   Int[(1 + Tan[e + f*x]^2)/(Sqr
t[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.28. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx$

3.28.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(113) = 226$.

Time = 0.04 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.47

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^2\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

input
```
int(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a*e^2*(1/2/e^4*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))
+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/3/e^3/(e*cot(d*x+c)
)^(3/2)+1/e^4/(e*cot(d*x+c))^(1/2)+1/2/e^(9/2)*arctan((e*cot(d*x+c))^(1/2)
/e^(1/2)))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.70

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = \left[-\frac{3\sqrt{2}\sqrt{-e}(\cos(2\,dx+2\,c)+1)\arctan\left(\frac{\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+}{\sin(2\,dx+2\,c)}}}{2\,(e\,co}\right.}{}\right.$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(2)*sqrt(-e)*(cos(2*d*x + 2*c) + 1)*arctan(1/2*sqrt(2)*sqrt(-
e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin
(2*d*x + 2*c) + 1)/(e*cos(2*d*x + 2*c) + e)) + 3*sqrt(-e)*(cos(2*d*x + 2*c
) + 1)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*c
os(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x +
2*c) + sin(2*d*x + 2*c) + 1)) + 4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x
+ 2*c))*(cos(2*d*x + 2*c) + 3*sin(2*d*x + 2*c) - 1))/(a*d*e^3*cos(2*d*x +
2*c) + a*d*e^3), 1/12*(3*sqrt(2)*sqrt(e)*(cos(2*d*x + 2*c) + 1)*log(-sqrt(
2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*
c) - sin(2*d*x + 2*c) - 1) + 2*e*sin(2*d*x + 2*c) + e) - 12*sqrt(e)*(cos(2
*d*x + 2*c) + 1)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sq
rt(e)) - 8*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*
c) + 3*sin(2*d*x + 2*c) - 1))/(a*d*e^3*cos(2*d*x + 2*c) + a*d*e^3)]
```

3.28.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{5}{2}}}\,dx}{a}$$

input
```
integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c)),x)
```

output
```
Integral(1/((e*cot(c + d*x))**(5/2)*cot(c + d*x) + (e*cot(c + d*x))**(5/2)
), x)/a
```

3.28.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.28.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = \int \frac{1}{(a\cot(dx+c)+a)\,(e\cot(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*cot(d*x + c) + a)*(e*cot(d*x + c))^(5/2)), x)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.98

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{12\sqrt{2}\,a^3\,d^3\,e^{21/2}\,\sqrt{e\cot(c+d\,x)}}{12\,a^3\,d^3\,e^{11}+12\,a^3\,d^3\,e^{11}\cot(c+d\,x)}\right)}{2\,a\,d\,e^{5/2}} - \frac{\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{a\,d\,e^{5/2}} - \frac{\frac{2\cot(c+d\,x)}{e}-\frac{2}{3\,e}}{a\,d\,(e\cot(c+d\,x))^{3/2}}$$

input
```
int(1/((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))),x)
```

output
```
(2^(1/2)*atanh((12*2^(1/2)*a^3*d^3*e^(21/2)*(e*cot(c + d*x))^(1/2))/(12*a^
3*d^3*e^11 + 12*a^3*d^3*e^11*cot(c + d*x))))/(2*a*d*e^(5/2)) - atan((e*cot
(c + d*x))^(1/2)/e^(1/2))/(a*d*e^(5/2)) - ((2*cot(c + d*x))/e - 2/(3*e))/(
a*d*(e*cot(c + d*x))^(3/2))
```

3.28. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))}\,dx$

3.29 $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx$

3.29.1 Optimal result

Integrand size = 25, antiderivative size = 281

$$\begin{aligned}\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx &= -\frac{3e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} \\ &\quad - \frac{e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} + \frac{e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} \\ &\quad + \frac{e^2\sqrt{e\cot(c+dx)}}{2d\,(a^2+a^2\cot(c+dx))} - \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} \\ &\quad + \frac{e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d}\end{aligned}$$

output
```
-3/2*e^(5/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d-1/4*e^(5/2)*arctan
(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)+1/4*e^(5/2)*arctan(
1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/8*e^(5/2)*ln(e^(1/
2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)+1/8*e^(5
/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1
/2)+1/2*e^2*(e*cot(d*x+c))^(1/2)/d/(a^2+a^2*cot(d*x+c))
```

3.29.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.38 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \frac{e^2\left(-24\cot^3(c+dx)\sqrt{e\cot(c+dx)}\,\text{Hypergeometric2F1}\left(2,\frac{7}{2},\frac{9}{2},-\cot(c+dx\right)\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x])^2,x]
```

output
```
(e^2*(-24*Cot[c + d*x]^3*Sqrt[e*Cot[c + d*x]]*Hypergeometric2F1[2, 7/2, 9/
2, -Cot[c + d*x]] + 7*(24*Sqrt[e]*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] - 6
*Sqrt[2]*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 6*Sq
rt[2]*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - 48*Sqrt
[e*Cot[c + d*x]] + (8*(e*Cot[c + d*x])^(3/2))/e - 3*Sqrt[2]*Sqrt[e]*Log[Sq
rt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]] + 3*Sqrt[2]*S
qrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])
))/(168*a^2*d)
```

3.29.3 Rubi [A] (warning: unable to verify)

Time = 1.20 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.90, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4048, 27, 3042, 4136, 27, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a\cot(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4048}$$

$$\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}-\frac{\int -\frac{a^2e^3+3a^2\cot^2(c+dx)e^3-2a^2\cot(c+dx)e^3}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2e^3+3a^2\cot^2(c+dx)e^3-2a^2\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{4a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2e^3+3a^2\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3+2a^2\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{4a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{4136}$$

$$\frac{3a^2e^3\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+\frac{\int -\frac{4a^3e^3}{\sqrt{e\cot(c+dx)}}dx}{2a^2}}{4a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3a^2e^3\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx-2ae^3\int \frac{1}{\sqrt{e\cot(c+dx)}}dx}{4a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a^2e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-2ae^3\int \frac{1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{3957}$$

$$\frac{3a^2e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2ae^4\int \frac{1}{\sqrt{e\cot(c+dx)}\left(\cot^2(c+dx)e^2+e^2\right)}d(e\cot(c+dx))}{d}}{4a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{3a^2e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae^4\int \frac{1}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{d}}{4a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{755}$$

3.29. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx$

$$\frac{3a^2e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae^4\left(\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\int \frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}\right)}{d}}{4a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{3a^2e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae^4\left(\frac{\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot \ldots}}{2e}\ldots\right.}{d}}{4a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{3a^2e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae^4\left(\frac{\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\cot(c+dx)\ldots\right)}{\sqrt{2}\sqrt{e}}}{2e}\ldots\right.}{d}}{4a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 217

$$\frac{3a^2e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae^4\left(\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot\ldots\right)}{\sqrt{2}\sqrt{e}}}{2e}\ldots\right.}{d}}{4a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 1479

3.29. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx$

$$\frac{3a^2e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae^4\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot\ldots}\right.}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\ldots}}{2\sqrt{\ldots}}}{2e}\ldots\right.}{d}}{4a^3}$$

$$\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{3a^2e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae^4\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+d\ldots}\right.}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c\ldots}}{2\sqrt{2}\sqrt{e}}}{2e}\ldots\right.}{d}}{4a^3}$$

$$\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{3a^2e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae^4\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c\ldots}}{2\sqrt{e}}}{2e}\ldots\right.}{d}}{4a^3}$$

$$\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{3a^2e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae^4\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e\ldots\right.}{2\sqrt{2}\sqrt{e}}\ldots\right.}{d}}{4a^3}$$

$$\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \; 4117$$

3.29. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx$

$$\frac{\frac{3a^2e^3\int \frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}+\frac{4ae^4\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}}{4a^3}$$

$$\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{3ae^3\int \frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}+\frac{4ae^4\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}}{4a^3}$$

$$\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\downarrow$ 73

$$\frac{\frac{4ae^4\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}}{4a^3}$$

$$\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\downarrow$ 216

$$\frac{\frac{6ae^{5/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{d}+\frac{4ae^4\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}}{4a^3}$$

$$\frac{e^2\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

input
```
Int[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x])^2,x]
```

3.29. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx$

output
```
(e^2*Sqrt[e*Cot[c + d*x]])/(2*d*(a^2 + a^2*Cot[c + d*x])) + ((6*a*e^(5/2)*
ArcTan[Cot[c + d*x]/Sqrt[e]])/d + (4*a*e^4*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*
Cot[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt[2]*Sqrt[e]*Cot[c + d*x]
]/(Sqrt[2]*Sqrt[e]))/(2*e) + (-1/2*Log[e - Sqrt[2]*e^(3/2)*Cot[c + d*x] +
e^2*Cot[c + d*x]^2]/(Sqrt[2]*Sqrt[e]) + Log[e + Sqrt[2]*e^(3/2)*Cot[c + d*
x] + e^2*Cot[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/(2*e)))/d)/(4*a^3)
```

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.29. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx$

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.29.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(c}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{16e}\right.}{d\,a^2}$
default	$-\frac{2e^3\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(c}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{16e}\right.}{d\,a^2}$

input
```
int((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^2*e^3*(-1/16/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4*(
e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)+e)+3/4/e^(1/2)*arctan((e*cot(d*x+c))^(1/
2)/e^(1/2)))
```

3.29.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 1173, normalized size of antiderivative = 4.17

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/4*(2*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*
c) + 3*(e^2*cos(2*d*x + 2*c) + e^2*sin(2*d*x + 2*c) + e^2)*sqrt(-e)*log((e
*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) - 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*
c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*
d*x + 2*c) + 1)) + (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*
d)*(-e^10/(a^8*d^4))^(1/4)*log(a^2*d*(-e^10/(a^8*d^4))^(1/4) + e^2*sqrt((e
*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (-I*a^2*d*cos(2*d*x + 2*c) - I
*a^2*d*sin(2*d*x + 2*c) - I*a^2*d)*(-e^10/(a^8*d^4))^(1/4)*log(I*a^2*d*(-e
^10/(a^8*d^4))^(1/4) + e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))
) - (I*a^2*d*cos(2*d*x + 2*c) + I*a^2*d*sin(2*d*x + 2*c) + I*a^2*d)*(-e^10
/(a^8*d^4))^(1/4)*log(-I*a^2*d*(-e^10/(a^8*d^4))^(1/4) + e^2*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(
2*d*x + 2*c) + a^2*d)*(-e^10/(a^8*d^4))^(1/4)*log(-a^2*d*(-e^10/(a^8*d^4))
^(1/4) + e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))))/(a^2*d*cos(
2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d), 1/4*(2*e^2*sqrt((e*cos(2*d
*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) - 6*(e^2*cos(2*d*x + 2*c
) + e^2*sin(2*d*x + 2*c) + e^2)*sqrt(e)*arctan(sqrt((e*cos(2*d*x + 2*c) +
e)/sin(2*d*x + 2*c))/sqrt(e)) + (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x
+ 2*c) + a^2*d)*(-e^10/(a^8*d^4))^(1/4)*log(a^2*d*(-e^10/(a^8*d^4))^(1/4)
+ e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (-I*a^2*d*cos(...
```

3.29.6 Sympy [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \frac{\int \frac{(e\cot(c+dx))^{\frac{5}{2}}}{\cot^2(c+dx)+2\cot(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**2,x)
```

output
```
Integral((e*cot(c + d*x))**(5/2)/(cot(c + d*x)**2 + 2*cot(c + d*x) + 1), x
)/a**2
```

3.29.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.29.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \int \frac{(e\cot(dx+c))^{\frac{5}{2}}}{(a\cot(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(5/2)/(a*cot(d*x + c) + a)^2, x)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 13.30 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.33

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx = \frac{e^3\sqrt{e\cot(c+d\,x)}}{2\,\left(a^2\,d\,e+a^2\,d\,e\cot(c+d\,x)\right)}$$
$$-\operatorname{atan}\left(\frac{e^{20}\sqrt{e\cot(c+d\,x)}\left(-\frac{e^{10}}{256\,a^8\,d^4}\right)^{1/4}16\mathrm{i}}{\frac{36\,e^{23}}{a^2\,d}+64\,a^2\,d\,e^{18}\sqrt{-\frac{e^{10}}{256\,a^8\,d^4}}}\right.$$
$$\left.-\frac{e^{15}\sqrt{e\cot(c+d\,x)}\left(-\frac{e^{10}}{256\,a^8\,d^4}\right)^{3/4}2304\mathrm{i}}{\frac{36\,e^{23}}{a^6\,d^3}+\frac{64\,e^{18}\sqrt{-\frac{e^{10}}{256\,a^8\,d^4}}}{a^2\,d}}\right)\left(-\frac{e^{10}}{256\,a^8\,d^4}\right)^{1/4}2\mathrm{i}-\frac{\operatorname{atan}\left(\frac{4\,e^{20}\sqrt{e\cot(c+d\,x)}\left(-\frac{e^{10}}{a^8\,d^4}\right)^{1/4}}{\frac{36\,e^{23}}{a^2\,d}-4\,a^2\,d\,e^{18}\sqrt{-\frac{e^{10}}{a^8\,d^4}}}+\frac{36\,e^{15}\cdots}{\cdots}\right.}{2}$$

input
```
int((e*cot(c + d*x))^(5/2)/(a + a*cot(c + d*x))^2,x)
```

output
```
(e^3*(e*cot(c + d*x))^(1/2))/(2*(a^2*d*e + a^2*d*e*cot(c + d*x))) - atan((
e^20*(e*cot(c + d*x))^(1/2)*(-e^10/(256*a^8*d^4))^(1/4)*16i)/((36*e^23)/(a
^2*d) + 64*a^2*d*e^18*(-e^10/(256*a^8*d^4))^(1/2)) - (e^15*(e*cot(c + d*x)
)^(1/2)*(-e^10/(256*a^8*d^4))^(3/4)*2304i)/((36*e^23)/(a^6*d^3) + (64*e^18
*(-e^10/(256*a^8*d^4))^(1/2))/(a^2*d)))*(-e^10/(256*a^8*d^4))^(1/4)*2i - (
atan((4*e^20*(e*cot(c + d*x))^(1/2)*(-e^10/(a^8*d^4))^(1/4))/((36*e^23)/(a
^2*d) - 4*a^2*d*e^18*(-e^10/(a^8*d^4))^(1/2)) + (36*e^15*(e*cot(c + d*x))^
(1/2)*(-e^10/(a^8*d^4))^(3/4))/((36*e^23)/(a^6*d^3) - (4*e^18*(-e^10/(a^8*
d^4))^(1/2))/(a^2*d)))*(-e^10/(a^8*d^4))^(1/4))/2 - (atan(((e*cot(c + d*x)
)^(1/2)*(-e^5)^(1/2)*1i)/e^3)*(-e^5)^(1/2)*3i)/(2*a^2*d)
```

3.29. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^2}\,dx$

3.30 $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx$

3.30.1 Optimal result

Integrand size = 25, antiderivative size = 279

$$\begin{aligned}\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx &= \frac{e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} \\ &+ \frac{e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} - \frac{e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} \\ &- \frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} - \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} \\ &+ \frac{e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d}\end{aligned}$$

output
```
1/2*e^(3/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d+1/4*e^(3/2)*arctan(
1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/4*e^(3/2)*arctan(1
+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)-1/8*e^(3/2)*ln(e^(1/2
)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)+1/8*e^(3/
2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/
2)-1/2*e*(e*cot(d*x+c))^(1/2)/d/(a^2+a^2*cot(d*x+c))
```

3.30.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.56

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = \frac{5\left(-2e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)+\left(-e^2\right)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)-\left(-e^2\right)^{3/4}\text{arc}\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x])^2,x]
```

output
```
(5*(-2*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] + (-e^2)^(3/4)*ArcTan[
Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] - (-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d
*x]]/(-e^2)^(1/4)] + 2*e*Sqrt[e*Cot[c + d*x]]) - (2*(e*Cot[c + d*x])^(5/2)
*Hypergeometric2F1[2, 5/2, 7/2, -Cot[c + d*x]])/e)/(10*a^2*d)
```

3.30.3 Rubi [A] (warning: unable to verify)

Time = 1.22 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.87, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.880, Rules used = {3042, 4050, 27, 3042, 4136, 27, 2030, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a\cot(c+dx)+a)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 4050$$

$$-\frac{\int \frac{-a\cot^2(c+dx)e^2+ae^2-2a\cot(c+dx)e^2}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^2}-\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{-a\cot^2(c+dx)e^2+ae^2-2a\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{4a^2}-\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{-a\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+ae^2+2a\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{4a^2}-\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{ 4136}$$

$$-\frac{\frac{\int -\frac{4a^2e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}+ae^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{4a^2}-\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{ae^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx-2e^2\int \frac{\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{4a^2}-\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{ 2030}$$

$$-\frac{ae^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx-2e\int \sqrt{e\cot(c+dx)}dx}{4a^2}-\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-2e\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{4a^2}-\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{ 3957}$$

$$-\frac{ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2e^2\int \frac{\sqrt{e\cot(c+dx)}}{\cot^2(c+dx)e^2+e^2}d(e\cot(c+dx))}{d}}{4a^2}-\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4e^2\int \frac{e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{d}}{4a^2}-\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

3.30. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx$

$\downarrow$ 826

$$-\frac{ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4e^2\left(\frac{1}{2}\int\frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\int\frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}}{4a^2}-\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\downarrow$ 1476

$$-\frac{ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4e^2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int\frac{1}{e^2\cot^2(c+dx)+\sqrt{2}e^{3}\cdots}\right.\right.}{d}}{4a^2}$$
$$\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\downarrow$ 1082

$$-\frac{ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4e^2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e^2\cot^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\cot(c+d\cdots\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}}{4a^2}$$
$$\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\downarrow$ 217

$$-\frac{ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4e^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int\frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2\cdots}\right.}{d}}{4a^2}$$
$$\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\downarrow$ 1479

$$-\frac{ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+4e^2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot\cdots}\right.}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cdots}}{2\sqrt{\cdots}}\right.\right.}{4a^2}$$
$$\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

3.30. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx$

$$\Big\downarrow 25$$

$$-\frac{\frac{ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4e^2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(\right.}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\,c}}{2\sqrt{}}\right.\right.}{4a^2}}{}{\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4e^2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c\right.}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\,c}}{2}\right.\right.}{4a^2}}{}{\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}}$$

$$\Big\downarrow 1103$$

$$-\frac{\frac{ae^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4e^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(\right.}{}\right.\right.}{d}}{4a^2}}{\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}}$$

$$\Big\downarrow 4117$$

$$-\frac{\frac{ae^2\int \frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}+\frac{4e^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx\right.}{2\sqrt{2}}\right.\right.}{d}}{4a^2}}{\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{e^2\int \frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}+\frac{4e^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+\right.}{2\sqrt{2}\sqrt{e}}\right.\right.}{d}}{4a^2}}{\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}}$$

3.30. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx$

$$\Big\downarrow \; 73$$

$$-\frac{\frac{4e^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+\right.}{2\sqrt{2}\sqrt{e}}\right.}{d}}{4a^2}$$

$$\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \; 216$$

$$-\frac{\frac{2e^{3/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{d}+\frac{4e^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\log\right.}{d}}{4a^2}$$

$$\frac{e\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

input
```
Int[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x])^2,x]
```

output
```
-1/2*(e*Sqrt[e*Cot[c + d*x]])/(d*(a^2 + a^2*Cot[c + d*x])) - ((2*e^(3/2)*A
rcTan[Cot[c + d*x]/Sqrt[e]])/d + (4*e^2*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Cot
[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(
Sqrt[2]*Sqrt[e]))/2 + (Log[e - Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c +
d*x]^2]/(2*Sqrt[2]*Sqrt[e]) - Log[e + Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*C
ot[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/2))/d)/(4*a^2)
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.30. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4050
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/((m
 + 1)*(a^2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^
(n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2
*a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[
2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.30. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx$

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.30.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2e^3\left(\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1 \cdots\right)\right)}{16e\left(e^2\right)^{\frac{1}{4}}}\cdots\right)}{d\,a^2}$
default	$-\frac{2e^3\left(\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1 \cdots\right)\right)}{16e\left(e^2\right)^{\frac{1}{4}}}\cdots\right)}{d\,a^2}$

input
```
int((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^2*e^3*(1/16/e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/2/e*
(-1/2*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)+e)+1/2/e^(1/2)*arctan((e*cot(d*x+
c))^(1/2)/e^(1/2))))
```

3.30.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 1172, normalized size of antiderivative = 4.20

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/4*((e*cos(2*d*x + 2*c) + e*sin(2*d*x + 2*c) + e)*sqrt(-e)*log((e*cos(2*
d*x + 2*c) - e*sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2
*c) + 1)) - 2*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x
+ 2*c) - (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*(-e^6/(
a^8*d^4))^(1/4)*log(a^6*d^3*(-e^6/(a^8*d^4))^(3/4) + e^4*sqrt((e*cos(2*d*x
 + 2*c) + e)/sin(2*d*x + 2*c))) - (-I*a^2*d*cos(2*d*x + 2*c) - I*a^2*d*sin
(2*d*x + 2*c) - I*a^2*d)*(-e^6/(a^8*d^4))^(1/4)*log(I*a^6*d^3*(-e^6/(a^8*d
^4))^(3/4) + e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (I*a^2
*d*cos(2*d*x + 2*c) + I*a^2*d*sin(2*d*x + 2*c) + I*a^2*d)*(-e^6/(a^8*d^4))
^(1/4)*log(-I*a^6*d^3*(-e^6/(a^8*d^4))^(3/4) + e^4*sqrt((e*cos(2*d*x + 2*c
) + e)/sin(2*d*x + 2*c))) + (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*
c) + a^2*d)*(-e^6/(a^8*d^4))^(1/4)*log(-a^6*d^3*(-e^6/(a^8*d^4))^(3/4) + e
^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))))/(a^2*d*cos(2*d*x + 2*
c) + a^2*d*sin(2*d*x + 2*c) + a^2*d), 1/4*(2*(e*cos(2*d*x + 2*c) + e*sin(2
*d*x + 2*c) + e)*sqrt(e)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
2*c))/sqrt(e)) - 2*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2
*d*x + 2*c) - (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*(-
e^6/(a^8*d^4))^(1/4)*log(a^6*d^3*(-e^6/(a^8*d^4))^(3/4) + e^4*sqrt((e*cos(
2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (-I*a^2*d*cos(2*d*x + 2*c) - I*a...
```

3.30.6 Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = \frac{\int \frac{(e\cot(c+dx))^{\frac{3}{2}}}{\cot^2(c+dx)+2\cot(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**2,x)
```

output
```
Integral((e*cot(c + d*x))**(3/2)/(cot(c + d*x)**2 + 2*cot(c + d*x) + 1), x
)/a**2
```

3.30.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.30.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{(a\cot(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(3/2)/(a*cot(d*x + c) + a)^2, x)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.35

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^2}\,dx =$$

$$-\frac{\operatorname{atan}\left(\frac{4\,e^{16}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{e^6}{a^8\,d^4}\right)^{1/4}}{\frac{4\,e^{18}}{a^2\,d}+4\,a^2\,d\,e^{15}\,\sqrt{-\frac{e^6}{a^8\,d^4}}}+\frac{4\,e^{13}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{e^6}{a^8\,d^4}\right)^{3/4}}{\frac{4\,e^{18}}{a^6\,d^3}+\frac{4\,e^{15}\,\sqrt{-\frac{e^6}{a^8\,d^4}}}{a^2\,d}}\right)\left(-\frac{e^6}{a^8\,d^4}\right)^{1/4}}{2}$$

$$-\operatorname{atan}\left(\frac{e^{16}\,\sqrt{e\cot\left(c+d\,x\right)}\,\left(-\frac{e^6}{256\,a^8\,d^4}\right)^{1/4}\,16\mathrm{i}}{\frac{4\,e^{18}}{a^2\,d}-64\,a^2\,d\,e^{15}\,\sqrt{-\frac{e^6}{256\,a^8\,d^4}}}-\frac{e^{13}\,\sqrt{e\cot\left(c+d\,x\right)}\,\left(-\frac{e^6}{256\,a^8\,d^4}\right)^{3/4}\,256\mathrm{i}}{\frac{4\,e^{18}}{a^6\,d^3}-\frac{64\,e^{15}\,\sqrt{-\frac{e^6}{256\,a^8\,d^4}}}{a^2\,d}}\right)\left(-\frac{e^6}{256\,a^8\,d^4}\right)^{1/}$$

input
```
int((e*cot(c + d*x))^(3/2)/(a + a*cot(c + d*x))^2,x)
```

output
```
(atan(((e*cot(c + d*x))^(1/2)*(-e^3)^(1/2)*1i)/e^2)*(-e^3)^(1/2)*1i)/(2*a^
2*d) - atan((e^16*(e*cot(c + d*x))^(1/2)*(-e^6/(256*a^8*d^4))^(1/4)*16i)/(
(4*e^18)/(a^2*d) - 64*a^2*d*e^15*(-e^6/(256*a^8*d^4))^(1/2)) - (e^13*(e*co
t(c + d*x))^(1/2)*(-e^6/(256*a^8*d^4))^(3/4)*256i)/((4*e^18)/(a^6*d^3) - (
64*e^15*(-e^6/(256*a^8*d^4))^(1/2))/(a^2*d)))*(-e^6/(256*a^8*d^4))^(1/4)*2
i - (e^2*(e*cot(c + d*x))^(1/2))/(2*(a^2*d*e + a^2*d*e*cot(c + d*x))) - (a
tan((4*e^16*(e*cot(c + d*x))^(1/2)*(-e^6/(a^8*d^4))^(1/4))/((4*e^18)/(a^2*
d) + 4*a^2*d*e^15*(-e^6/(a^8*d^4))^(1/2)) + (4*e^13*(e*cot(c + d*x))^(1/2)
*(-e^6/(a^8*d^4))^(3/4))/((4*e^18)/(a^6*d^3) + (4*e^15*(-e^6/(a^8*d^4))^(1
/2))/(a^2*d)))*(-e^6/(a^8*d^4))^(1/4))/2
```

3.31 $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx$

3.31.1 Optimal result

Integrand size = 25, antiderivative size = 278

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = \frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d} + \frac{\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} - \frac{\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d} + \frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2+a^2\cot(c+dx)\right)} + \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d} - \frac{\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d}$$

output
```
1/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d+1/4*arctan(1-2^(1/2
)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)-1/4*arctan(1+2^(1/2)
*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^2/d*2^(1/2)+1/8*ln(e^(1/2)+cot(d*
x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/a^2/d*2^(1/2)-1/8*ln(e^
(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/a^2/d*2^(1/
2)+1/2*(e*cot(d*x+c))^(1/2)/d/(a^2+a^2*cot(d*x+c))
```

3.31.2 Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = -\frac{\sqrt{e}\left(-4\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)-2\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)+2\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)-\frac{4}{\sqrt{}}\right.}{}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^2,x]
```

output
```
-1/8*(Sqrt[e]*(-4*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] - 2*Sqrt[2]*ArcTan[
1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 2*Sqrt[2]*ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - (4*Sqrt[e*Cot[c + d*x]])/(Sqrt[e]*(1 +
Cot[c + d*x])) - Sqrt[2]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt
[e*Cot[c + d*x]]] + Sqrt[2]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*S
qrt[e*Cot[c + d*x]]]))/(a^2*d)
```

3.31.3 Rubi [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.90, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4051, 27, 3042, 4136, 27, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a\cot(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4051}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}-\frac{\int -\frac{-ae\cot^2(c+dx)+2ae\cot(c+dx)+ae}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-ae\cot^2(c+dx)+2ae\cot(c+dx)+ae}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{4a^2}+\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-ae\tan\left(c+dx+\frac{\pi}{2}\right)^2-2ae\tan\left(c+dx+\frac{\pi}{2}\right)+ae}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{4a^2}+\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{4136}$$

$$\frac{\frac{\int \frac{4a^2e}{\sqrt{e\cot(c+dx)}}dx}{2a^2}-ae\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{4a^2}+\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2e\int \frac{1}{\sqrt{e\cot(c+dx)}}dx-ae\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{4a^2}+\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2e\int \frac{1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-ae\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{4a^2}+\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{3957}$$

$$\frac{-ae\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{2e^2\int \frac{1}{\sqrt{e\cot(c+dx)}\left(\cot^2(c+dx)e^2+e^2\right)}d(e\cot(c+dx))}{d}}{4a^2}+\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{-ae\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4e^2\int \frac{1}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{d}}{4a^2}+\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{755}$$

3.31. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}dx$

$$\frac{-ae\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4e^2\left(\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\int \frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}\right)}{d}}{4a^2}+\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\downarrow$ 1476

$$\frac{-ae\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4e^2\left(\frac{\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c}}{2e}\right.}{d}}{4a^2}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\downarrow$ 1082

$$\frac{-ae\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4e^2\left(\frac{\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}}{2e}\right.}{d}}{4a^2}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\downarrow$ 217

$$\frac{-ae\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4e^2\left(\frac{\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c\right.}{\sqrt{2}\sqrt{e}}}{2e}\right.}{d}}{4a^2}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\downarrow$ 1479

3.31. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx$

$$\frac{-ae\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4e^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+\cdots}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot\cdots}}{2\sqrt{2}\sqrt{\cdots}}}{2e}\cdots\right)}{d}}{4a^2}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 25

$$\frac{-ae\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4e^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+d\cdots}}{2\sqrt{2}\sqrt{e}}}{2e}\cdots\right)}{d}}{4a^2}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 27

$$\frac{-ae\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4e^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}\cot(c+d\cdots}}{2\sqrt{e}}}{2e}\cdots\right)}{d}}{4a^2}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{-ae\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4e^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot\cdots\right.}{2\sqrt{2}\sqrt{e}}\cdots\right)}{d}}{4a^2}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 4117

3.31. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx$

$$-\frac{-\frac{ae\int \frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4e^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c\right.}{2\sqrt{2}\sqrt{e}}\right.}{d}}{4a^2}-\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{e\int \frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4e^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+\right.}{2\sqrt{2}\sqrt{e}}\right.}{d}}{4a^2}-\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 73

$$-\frac{\frac{2\int \frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{d}-\frac{4e^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\right.}{}}{2e}\right.}{d}}{4a^2}-\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 216

$$-\frac{-\frac{4e^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}}{2e}\right)}{d}}{4a^2}-\frac{\sqrt{e\cot(c+dx)}}{2d\left(a^2\cot(c+dx)+a^2\right)}$$

input
```
Int[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^2,x]
```

3.31. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx$

output
```
Sqrt[e*Cot[c + d*x]]/(2*d*(a^2 + a^2*Cot[c + d*x])) + ((-2*Sqrt[e]*ArcTan[
Cot[c + d*x]/Sqrt[e]])/d - (4*e^2*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Cot[c + d
*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2
]*Sqrt[e]))/(2*e) + (-1/2*Log[e - Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c
 + d*x]^2]/(Sqrt[2]*Sqrt[e]) + Log[e + Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*
Cot[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/(2*e)))/d)/(4*a^2)
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.31. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx$

rule 4051
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(a^2 + b^2
))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c
*(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Int
egerQ[2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.31.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{2e^3\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right)\right)}{16e^3}\cdots\right)}{d\,a^2}$
default	$-\frac{2e^3\left(\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right)\right)}{16e^3}\cdots\right)}{d\,a^2}$

input
```
int((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^2*e^3*(1/16/e^3*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/2/
e*(1/2*(e*cot(d*x+c))^(1/2)/e/(e*cot(d*x+c)+e)+1/2/e^(3/2)*arctan((e*cot(d
*x+c))^(1/2)/e^(1/2))))
```

3.31.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 1114, normalized size of antiderivative = 4.01

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)*log((e*cos(2*d*x
+ 2*c) - e*sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c)
+ 1)) - (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*(-e^2/(a
^8*d^4))^(1/4)*log(a^2*d*(-e^2/(a^8*d^4))^(1/4) + sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c))) - (I*a^2*d*cos(2*d*x + 2*c) + I*a^2*d*sin(2*d*x +
 2*c) + I*a^2*d)*(-e^2/(a^8*d^4))^(1/4)*log(I*a^2*d*(-e^2/(a^8*d^4))^(1/4)
 + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - (-I*a^2*d*cos(2*d*x
+ 2*c) - I*a^2*d*sin(2*d*x + 2*c) - I*a^2*d)*(-e^2/(a^8*d^4))^(1/4)*log(-I
*a^2*d*(-e^2/(a^8*d^4))^(1/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
2*c))) + (a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*(-e^2/(
a^8*d^4))^(1/4)*log(-a^2*d*(-e^2/(a^8*d^4))^(1/4) + sqrt((e*cos(2*d*x + 2*
c) + e)/sin(2*d*x + 2*c))) + 2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2
*c))*sin(2*d*x + 2*c))/(a^2*d*cos(2*d*x + 2*c) + a^2*d*sin(2*d*x + 2*c) +
a^2*d), 1/4*(2*sqrt(e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)*arctan(sq
rt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) - (a^2*d*cos(2*d*x
+ 2*c) + a^2*d*sin(2*d*x + 2*c) + a^2*d)*(-e^2/(a^8*d^4))^(1/4)*log(a^2*d*
(-e^2/(a^8*d^4))^(1/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))
- (I*a^2*d*cos(2*d*x + 2*c) + I*a^2*d*sin(2*d*x + 2*c) + I*a^2*d)*(-e^2/(a
^8*d^4))^(1/4)*log(I*a^2*d*(-e^2/(a^8*d^4))^(1/4) + sqrt((e*cos(2*d*x +...
```

3.31.6 Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{e\cot(c+dx)}}{\cot^2(c+dx)+2\cot(c+dx)+1}\,dx}{a^2}$$

input
```
integrate((e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**2,x)
```

output
```
Integral(sqrt(e*cot(c + d*x))/(cot(c + d*x)**2 + 2*cot(c + d*x) + 1), x)/a
**2
```

3.31.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.31.8 Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{(a\cot(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*cot(d*x + c))/(a*cot(d*x + c) + a)^2, x)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^2}\,dx$$
$$= \frac{\mathrm{atan}\left(\frac{4e^{12}\sqrt{e\cot(c+dx)}\left(-\frac{e^2}{a^8d^4}\right)^{1/4}}{\frac{4e^{13}}{a^2d}-4a^2de^{12}\sqrt{-\frac{e^2}{a^8d^4}}}+\frac{4e^{11}\sqrt{e\cot(c+dx)}\left(-\frac{e^2}{a^8d^4}\right)^{3/4}}{\frac{4e^{13}}{a^6d^3}-\frac{4e^{12}\sqrt{-\frac{e^2}{a^8d^4}}}{a^2d}}\right)\left(-\frac{e^2}{a^8d^4}\right)^{1/4}}{2}$$
$$+\mathrm{atan}\left(\frac{e^{12}\sqrt{e\cot(c+dx)}\left(-\frac{e^2}{256a^8d^4}\right)^{1/4}16\mathrm{i}}{\frac{4e^{13}}{a^2d}+64a^2de^{12}\sqrt{-\frac{e^2}{256a^8d^4}}}-\frac{e^{11}\sqrt{e\cot(c+dx)}\left(-\frac{e^2}{256a^8d^4}\right)^{3/4}256\mathrm{i}}{\frac{4e^{13}}{a^6d^3}+\frac{64e^{12}\sqrt{-\frac{e^2}{256a^8d^4}}}{a^2d}}\right)\left(-\frac{e^2}{256a^8d^4}\right.$$

input
```
int((e*cot(c + d*x))^(1/2)/(a + a*cot(c + d*x))^2,x)
```

output
```
(atan((4*e^12*(e*cot(c + d*x))^(1/2)*(-e^2/(a^8*d^4))^(1/4))/((4*e^13)/(a^
2*d) - 4*a^2*d*e^12*(-e^2/(a^8*d^4))^(1/2)) + (4*e^11*(e*cot(c + d*x))^(1/
2)*(-e^2/(a^8*d^4))^(3/4))/((4*e^13)/(a^6*d^3) - (4*e^12*(-e^2/(a^8*d^4))^
(1/2))/(a^2*d)))*(-e^2/(a^8*d^4))^(1/4))/2 + atan((e^12*(e*cot(c + d*x))^(
1/2)*(-e^2/(256*a^8*d^4))^(1/4)*16i)/((4*e^13)/(a^2*d) + 64*a^2*d*e^12*(-e
^2/(256*a^8*d^4))^(1/2)) - (e^11*(e*cot(c + d*x))^(1/2)*(-e^2/(256*a^8*d^4
))^(3/4)*256i)/((4*e^13)/(a^6*d^3) + (64*e^12*(-e^2/(256*a^8*d^4))^(1/2))/
(a^2*d)))*(-e^2/(256*a^8*d^4))^(1/4)*2i + (e*(e*cot(c + d*x))^(1/2))/(2*(a
^2*d*e + a^2*d*e*cot(c + d*x))) - ((-e)^(1/2)*atan(((e*cot(c + d*x))^(1/2)
*1i)/(-e)^(1/2))*1i)/(2*a^2*d)
```

3.32 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx$

3.32.1 Optimal result

Integrand size = 25, antiderivative size = 281

$$\begin{aligned}
\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx
&= -\frac{3\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2d\sqrt{e}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d\sqrt{e}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2d\sqrt{e}} \\
&\quad - \frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2+a^2\cot(c+dx)\right)} + \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d\sqrt{e}} \\
&\quad - \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2d\sqrt{e}}
\end{aligned}$$

output
```
-3/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(1/2)-1/4*arctan(1-2^(1/
2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)+1/4*arctan(1+2^(1/2
)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d*2^(1/2)/e^(1/2)+1/8*ln(e^(1/2)+cot(d
*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)/e^(1/2)-1/8*ln(e
^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d*2^(1/2)/e^(1
/2)-1/2*(e*cot(d*x+c))^(1/2)/d/e/(a^2+a^2*cot(d*x+c))
```

3.32.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx = -\frac{3e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)+(-e^2)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)-(-e^2)^{3/4}\text{arctanh}\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)+\frac{e\sqrt{e\cot(c+dx)}}{1+\cot(c+dx)}}{2a^2de^2}$$

input
```
Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2),x]
```

output
```
-1/2*(3*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] + (-e^2)^(3/4)*ArcTan
[Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] - (-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c +
d*x]]/(-e^2)^(1/4)] + (e*Sqrt[e*Cot[c + d*x]])/(1 + Cot[c + d*x]))/(a^2*d*
e^2)
```

3.32.3 Rubi [A] (warning: unable to verify)

Time = 1.21 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.89, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 4052, 27, 3042, 4136, 27, 2030, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4052}$$

$$-\frac{\int -\frac{e\cot^2(c+dx)a^2+3ea^2-2e\cot(c+dx)a^2}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{e\cot^2(c+dx)a^2+3ea^2-2e\cot(c+dx)a^2}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{4a^3e}-\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{e\tan\left(c+dx+\frac{\pi}{2}\right)^2a^2+3ea^2+2e\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{4a^3e}-\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{4136}$$

$$\frac{3a^2e\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+\frac{\int -\frac{4a^3e\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}}{4a^3e}-\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3a^2e\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx-2ae\int \frac{\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{4a^3e}-\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{2030}$$

$$\frac{3a^2e\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx-2a\int \sqrt{e\cot(c+dx)}dx}{4a^3e}-\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a^2e\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-2a\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{4a^3e}-\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{3957}$$

$$\frac{3a^2e\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2ae\int \frac{\sqrt{e\cot(c+dx)}}{\cot^2(c+dx)e^2+e^2}d(e\cot(c+dx))}{d}}{4a^3e}-\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{3a^2e\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae\int \frac{e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{d}}{4a^3e}-\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

3.32. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}dx$

$$\Big\downarrow \; 826$$

$$\frac{3a^2e\int \frac{\tan(c+dx+\frac{\pi}{2})^2+1}{\sqrt{-e\tan(c+dx+\frac{\pi}{2})}\,(a-a\tan(c+dx+\frac{\pi}{2}))}dx+\frac{4ae\left(\frac{1}{2}\int \frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}}{4a^3e}-\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \; 1476$$

$$\frac{3a^2e\int \frac{\tan(c+dx+\frac{\pi}{2})^2+1}{\sqrt{-e\tan(c+dx+\frac{\pi}{2})}\,(a-a\tan(c+dx+\frac{\pi}{2}))}dx+\frac{4ae\Big(\frac{1}{2}\Big(\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/}}}{d}}{4a^3e}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{3a^2e\int \frac{\tan(c+dx+\frac{\pi}{2})^2+1}{\sqrt{-e\tan(c+dx+\frac{\pi}{2})}\,(a-a\tan(c+dx+\frac{\pi}{2}))}dx+\frac{4ae\Big(\frac{1}{2}\Big(\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e^2\cot^2(c+dx)-1}d\big(\sqrt{2}\sqrt{e}\cot(c+dx}{\sqrt{2}\sqrt{e}}}{d}}{4a^3e}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{3a^2e\int \frac{\tan(c+dx+\frac{\pi}{2})^2+1}{\sqrt{-e\tan(c+dx+\frac{\pi}{2})}\,(a-a\tan(c+dx+\frac{\pi}{2}))}dx+\frac{4ae\Big(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\int \frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}}{d}}{4a^3e}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{3a^2e\int \frac{\tan(c+dx+\frac{\pi}{2})^2+1}{\sqrt{-e\tan(c+dx+\frac{\pi}{2})}\,(a-a\tan(c+dx+\frac{\pi}{2}))}dx+\frac{4ae\Big(\frac{1}{2}\Big(\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int -\frac{\sqrt{2}\big(\sqrt{e}+\sqrt{2}\sqrt{e\cot}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}}}{2\sqrt{}}}{}}{4a^3e}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

3.32. $\displaystyle\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx$

$\Big\downarrow$ 25

$$\frac{\frac{3a^2e\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c}\right.}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}c}}{2\sqrt{2}}\right.\right.}{4a^3e}}{}}{}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{3a^2e\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+}}{e^2\cot^2(c+dx)+\sqrt{2}e^{3/2}c}}{2\sqrt{}}\right.\right.}{4a^3e}}{}}{}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{\frac{3a^2e\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c\right.}{2}\right.\right.}{d}}{4a^3e}}{}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 4117

$$\frac{\frac{3a^2e\int \frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}+\frac{4ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)\right.}{2\sqrt{2}\sqrt{}}\right.\right.}{d}}{4a^3e}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{3ae\int \frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}+\frac{4ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e\right.}{2\sqrt{2}\sqrt{e}}\right.\right.}{d}}{4a^3e}$$

$$\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

3.32. $\displaystyle\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx$

$\downarrow$ 73

$$\frac{4ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx\right.}{2\sqrt{2}\sqrt{e}}\right.\right.}{d}$$

$$4a^3e$$

$$\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

$\downarrow$ 216

$$\frac{4ae\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx)+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+dx\right.}{2\sqrt{2}\sqrt{e}}\right.\right.}{d}$$

$$4a^3e$$

$$\frac{\sqrt{e\cot(c+dx)}}{2de\left(a^2\cot(c+dx)+a^2\right)}$$

input
```
Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2),x]
```

output
```
-1/2*Sqrt[e*Cot[c + d*x]]/(d*e*(a^2 + a^2*Cot[c + d*x])) + ((6*a*Sqrt[e]*A
rcTan[Cot[c + d*x]/Sqrt[e]])/d + (4*a*e*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Cot
[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(
Sqrt[2]*Sqrt[e]))/2 + (Log[e - Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c +
d*x]^2]/(2*Sqrt[2]*Sqrt[e]) - Log[e + Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*C
ot[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/2))/d)/(4*a^3*e)
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.32. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.32. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx$

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.32.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+\cdots\right)\right)}{16e^3(e^2)^{\frac{1}{4}}}\cdots\right)}{d\,a^2}$
default	$-\frac{2e^3\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+\cdots\right)\right)}{16e^3(e^2)^{\frac{1}{4}}}\cdots\right)}{d\,a^2}$

input
```
int(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^2*e^3*(-1/16/e^3/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/2
/e^3*(1/2*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)+e)+3/2/e^(1/2)*arctan((e*cot(
d*x+c))^(1/2)/e^(1/2))))
```

3.32.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 1181, normalized size of antiderivative = 4.20

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/4*(3*sqrt(-e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)*log((e*cos(2*d
*x + 2*c) - e*sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/
sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*
c) + 1)) - (a^2*d*e*cos(2*d*x + 2*c) + a^2*d*e*sin(2*d*x + 2*c) + a^2*d*e)
*(-1/(a^8*d^4*e^2))^(1/4)*log(a^6*d^3*e^2*(-1/(a^8*d^4*e^2))^(3/4) + sqrt(
(e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (I*a^2*d*e*cos(2*d*x + 2*c)
+ I*a^2*d*e*sin(2*d*x + 2*c) + I*a^2*d*e)*(-1/(a^8*d^4*e^2))^(1/4)*log(I*a
^6*d^3*e^2*(-1/(a^8*d^4*e^2))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*
d*x + 2*c))) + (-I*a^2*d*e*cos(2*d*x + 2*c) - I*a^2*d*e*sin(2*d*x + 2*c) -
 I*a^2*d*e)*(-1/(a^8*d^4*e^2))^(1/4)*log(-I*a^6*d^3*e^2*(-1/(a^8*d^4*e^2))
^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (a^2*d*e*cos(2
*d*x + 2*c) + a^2*d*e*sin(2*d*x + 2*c) + a^2*d*e)*(-1/(a^8*d^4*e^2))^(1/4)
*log(-a^6*d^3*e^2*(-1/(a^8*d^4*e^2))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c))) + 2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*si
n(2*d*x + 2*c))/(a^2*d*e*cos(2*d*x + 2*c) + a^2*d*e*sin(2*d*x + 2*c) + a^2
*d*e), -1/4*(6*sqrt(e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)*arctan(sq
rt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) - (a^2*d*e*cos(2*d*
x + 2*c) + a^2*d*e*sin(2*d*x + 2*c) + a^2*d*e)*(-1/(a^8*d^4*e^2))^(1/4)*lo
g(a^6*d^3*e^2*(-1/(a^8*d^4*e^2))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c))) + (I*a^2*d*e*cos(2*d*x + 2*c) + I*a^2*d*e*sin(2*d*x + 2...
```

3.32.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx = \frac{\int \frac{1}{\sqrt{e\cot(c+dx)}\cot^2(c+dx)+2\sqrt{e\cot(c+dx)}\cot(c+dx)+\sqrt{e\cot(c+dx)}}\,dx}{a^2}$$

input
```
integrate(1/(e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**2,x)
```

output
```
Integral(1/(sqrt(e*cot(c + d*x))*cot(c + d*x)**2 + 2*sqrt(e*cot(c + d*x))*
cot(c + d*x) + sqrt(e*cot(c + d*x))), x)/a**2
```

3.32.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.32.8 Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx = \int \frac{1}{(a\cot(dx+c)+a)^2\sqrt{e\cot(dx+c)}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*cot(d*x + c) + a)^2*sqrt(e*cot(d*x + c))), x)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{4e^8\sqrt{e\cot(c+dx)}\left(-\frac{1}{a^8d^4e^2}\right)^{1/4}}{\frac{4e^8}{a^2d}+36a^2de^9\sqrt{-\frac{1}{a^8d^4e^2}}}+\frac{36e^9\sqrt{e\cot(c+dx)}\left(-\frac{1}{a^8d^4e^2}\right)^{3/4}}{\frac{4e^8}{a^6d^3}+\frac{36e^9\sqrt{-\frac{1}{a^8d^4e^2}}}{a^2d}}\right)\left(-\frac{1}{a^8d^4e^2}\right)^{1/4}}{2}$$
$$+\operatorname{atan}\left(\frac{e^8\sqrt{e\cot(c+dx)}\left(-\frac{1}{256a^8d^4e^2}\right)^{1/4}16i}{\frac{4e^8}{a^2d}-576a^2de^9\sqrt{-\frac{1}{256a^8d^4e^2}}}-\frac{e^9\sqrt{e\cot(c+dx)}\left(-\frac{1}{256a^8d^4e^2}\right)^{3/4}2304i}{\frac{4e^8}{a^6d^3}-\frac{576e^9\sqrt{-\frac{1}{256a^8d^4e^2}}}{a^2d}}\right)\left(-\frac{1}{256a^8d\right.$$

input
```
int(1/((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^2),x)
```

output
```
(atan((4*e^8*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^2))^(1/4))/((4*e^8)/(a^
2*d) + 36*a^2*d*e^9*(-1/(a^8*d^4*e^2))^(1/2)) + (36*e^9*(e*cot(c + d*x))^(
1/2)*(-1/(a^8*d^4*e^2))^(3/4))/((4*e^8)/(a^6*d^3) + (36*e^9*(-1/(a^8*d^4*e
^2))^(1/2))/(a^2*d)))*(-1/(a^8*d^4*e^2))^(1/4))/2 + atan((e^8*(e*cot(c + d
*x))^(1/2)*(-1/(256*a^8*d^4*e^2))^(1/4)*16i)/((4*e^8)/(a^2*d) - 576*a^2*d*
e^9*(-1/(256*a^8*d^4*e^2))^(1/2)) - (e^9*(e*cot(c + d*x))^(1/2)*(-1/(256*a
^8*d^4*e^2))^(3/4)*2304i)/((4*e^8)/(a^6*d^3) - (576*e^9*(-1/(256*a^8*d^4*e
^2))^(1/2))/(a^2*d)))*(-1/(256*a^8*d^4*e^2))^(1/4)*2i - (e*cot(c + d*x))^(
1/2)/(2*(a^2*d*e + a^2*d*e*cot(c + d*x))) - (atan(((e*cot(c + d*x))^(1/2)*
1i)/(-e)^(1/2))*3i)/(2*a^2*d*(-e)^(1/2))
```

3.33 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx$

3.33.1 Optimal result

Integrand size = 25, antiderivative size = 306

$$\begin{aligned}\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx &= \frac{5\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{3/2}} \\ &- \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{3/2}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{3/2}} \\ &+ \frac{5}{2a^2de\sqrt{e\cot(c+dx)}} - \frac{1}{2de\sqrt{e\cot(c+dx)}\,(a^2+a^2\cot(c+dx))} \\ &- \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{3/2}} \\ &+ \frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{3/2}}\end{aligned}$$

output
```
5/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(3/2)-1/4*arctan(1-2^(1/2
)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(3/2)*2^(1/2)+1/4*arctan(1+2^(1/2)
*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(3/2)*2^(1/2)-1/8*ln(e^(1/2)+cot(d*
x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d/e^(3/2)*2^(1/2)+1/8*ln(e^
(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d/e^(3/2)*2^(1/
2)+5/2/a^2/d/e/(e*cot(d*x+c))^(1/2)-1/2/d/e/(a^2+a^2*cot(d*x+c))/(e*cot(d*
x+c))^(1/2)
```

3.33.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.17 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.74

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = \frac{8\sqrt{e}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\cot(c+dx)\right)+8\sqrt{e}\,\text{Hy}}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2),x]
```

output
```
(8*Sqrt[e]*Hypergeometric2F1[-1/2, 1, 1/2, -Cot[c + d*x]] + 8*Sqrt[e]*Hype
rgeometric2F1[-1/2, 2, 1/2, -Cot[c + d*x]] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]*
(-2*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 2*ArcTan[1 + (Sqr
t[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] -
 Sqrt[2]*Sqrt[e*Cot[c + d*x]]] + Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt
[2]*Sqrt[e*Cot[c + d*x]]]))/(8*a^2*d*e^(3/2)*Sqrt[e*Cot[c + d*x]])
```

3.33.3 Rubi [A] (warning: unable to verify)

Time = 1.57 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.94, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4052, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \ 4052$$

$$-\frac{\int -\frac{3e\cot^2(c+dx)a^2+5ea^2-2e\cot(c+dx)a^2}{2(e\cot(c+dx))^{3/2}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3e\cot^2(c+dx)a^2+5ea^2-2e\cot(c+dx)a^2}{(e\cot(c+dx))^{3/2}(\cot(c+dx)a+a)}dx}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\int \frac{3e\tan\left(c+dx+\frac{\pi}{2}\right)^2a^2+5ea^2+2e\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 4132

$$\frac{\frac{2\int -\frac{7a^3e^3+5a^3\cot^2(c+dx)e^3+2a^3\cot(c+dx)e^3}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}+\frac{10a}{d\sqrt{e\cot(c+dx)}}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 27

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{7a^3e^3+5a^3\cot^2(c+dx)e^3+2a^3\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{7a^3e^3+5a^3\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3-2a^3\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 4136

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+\frac{\int \frac{4a^4e^3}{\sqrt{e\cot(c+dx)}}dx}{2a^2}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 27

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+2a^2e^3\int \frac{1}{\sqrt{e\cot(c+dx)}}dx}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 3042

3.33. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx$

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+2a^2e^3\int\frac{1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 3957

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{2a^2e^4\int\frac{1}{\sqrt{e\cot(c+dx)}\left(\cot^2(c+dx)e^2+e^2\right)}d(e\cot(c+dx))}{d}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 266

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^2e^4\int\frac{1}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{d}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 755

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^2e^4\left(\frac{\int\frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\int\frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}\right)}{d}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 1476

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^2e^4\left(\frac{\frac{1}{2}\int\frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int\frac{\cdots}{e^2\cot^2(c+d\cdots}}{2e}\cdots\right)}{d}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 1082

3.33. $\int\frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx$

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^2e^4\left(\frac{\frac{\int\frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e^2\cot^2(c+dx)-1}d\left(\sqrt{2}\cdots\right.}{\sqrt{2}\sqrt{e}}}{2e}\cdots\right.}{d}}{ae^3}}{4a^3e}\cdots$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 217

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^2e^4\left(\frac{\int\frac{e-e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{2e}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\arctan\cdots}{2e}\cdots\right.}{d}}{ae^3}}{4a^3e}\cdots$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 1479

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^2e^4\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\cdots\right.}{e^2\cot^2(c+\cdots}}{\cdots}}{2e}\cdots\right.}{\cdots}}{ae^3}}{4a^3e}\cdots$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 25

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^2e^4\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\cdots\right.}{e^2\cot^2(c+dx)+\cdots}}{\cdots}}{2e}\cdots\right.}{\cdots}}{ae^3}}{4a^3e}\cdots$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 27

3.33. $\int\frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx$

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^2e^4\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{e}+\sqrt{2}\ldots}{e^2\cot^2(c+dx)+\ldots}}{2e}\ldots\right)}{\ldots}}{ae^3}}{4a^3e}$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{5a^3e^3\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^2e^4\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\log\left(\sqrt{2}e^{3/2}\ldots\right)}{\ldots}\right)}{d}}{ae^3}}{4a^3e}$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 4117$$

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{\frac{5a^3e^3\int\frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4a^2e^4\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\log\left(\sqrt{2}e^{3/2}\ldots\right)}{\ldots}\right)}{d}}{ae^3}}{4a^3e}$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{\frac{5a^2e^3\int\frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4a^2e^4\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\log\left(\sqrt{2}e^{3/2}\cot\ldots\right)}{\ldots}\right)}{d}}{ae^3}}{4a^3e}$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 73$$

3.33. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx$

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{-\frac{10a^2e^2\int\frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{d}-\frac{4a^2e^4\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e\cdots\right.}{2\sqrt{2}\sqrt{e}\cdots}\cdots}{d}}{ae^3}}{4a^3e}\cdots$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

$\big\downarrow$ 216

$$\frac{\frac{10a}{d\sqrt{e\cot(c+dx)}}-\frac{-\frac{10a^2e^{5/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{d}-\frac{4a^2e^4\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}}{2e}+\frac{\log\left(\sqrt{2}e^{3/2}\cot(c+dx)+e^2\cot^2(c+d\cdots\right.}{2\sqrt{2}\sqrt{e}}\cdots}{d}}{ae^3}}{4a^3e}\cdots$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)\sqrt{e\cot(c+dx)}}$$

input
```
Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2),x]
```

output
```
-1/2*1/(d*e*Sqrt[e*Cot[c + d*x]]*(a^2 + a^2*Cot[c + d*x])) + ((10*a)/(d*Sq
rt[e*Cot[c + d*x]]) - ((10*a^2*e^(5/2)*ArcTan[Cot[c + d*x]/Sqrt[e]])/d - (
4*a^2*e^4*((-(ArcTan[1 - Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e]))
+ ArcTan[1 + Sqrt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e]))/(2*e) + (-1/
2*Log[e - Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c + d*x]^2]/(Sqrt[2]*Sqrt
[e]) + Log[e + Sqrt[2]*e^(3/2)*Cot[c + d*x] + e^2*Cot[c + d*x]^2]/(2*Sqrt[
2]*Sqrt[e]))/(2*e)))/d)/(a*e^3))/(4*a^3*e)
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.33. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.33. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx$

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.33.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.69

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right)\right)}{16e^5}\cdots\right)}{d\,a^2}$
default	$-\frac{2e^3\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right)\right)}{16e^5}\cdots\right)}{d\,a^2}$

input
```
int(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^2*e^3*(-1/16/e^5*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/e
^4/(e*cot(d*x+c))^(1/2)-1/2/e^4*(1/2*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)+e)
+5/2/e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))))
```

3.33.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1257, normalized size of antiderivative = 4.11

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/4*(5*sqrt(-e)*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)*log((e*cos(2*d
*x + 2*c) - e*sin(2*d*x + 2*c) - 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/
sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*
c) + 1)) - (a^2*d*e^2*cos(2*d*x + 2*c) + a^2*d*e^2*sin(2*d*x + 2*c) + a^2*
d*e^2)*(-1/(a^8*d^4*e^6))^(1/4)*log(a^2*d*e^2*(-1/(a^8*d^4*e^6))^(1/4) + s
qrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (-I*a^2*d*e^2*cos(2*d*x
+ 2*c) - I*a^2*d*e^2*sin(2*d*x + 2*c) - I*a^2*d*e^2)*(-1/(a^8*d^4*e^6))^(1
/4)*log(I*a^2*d*e^2*(-1/(a^8*d^4*e^6))^(1/4) + sqrt((e*cos(2*d*x + 2*c) +
e)/sin(2*d*x + 2*c))) + (I*a^2*d*e^2*cos(2*d*x + 2*c) + I*a^2*d*e^2*sin(2*
d*x + 2*c) + I*a^2*d*e^2)*(-1/(a^8*d^4*e^6))^(1/4)*log(-I*a^2*d*e^2*(-1/(a
^8*d^4*e^6))^(1/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + (a
^2*d*e^2*cos(2*d*x + 2*c) + a^2*d*e^2*sin(2*d*x + 2*c) + a^2*d*e^2)*(-1/(a
^8*d^4*e^6))^(1/4)*log(-a^2*d*e^2*(-1/(a^8*d^4*e^6))^(1/4) + sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + 2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(
2*d*x + 2*c))*(4*cos(2*d*x + 2*c) - 5*sin(2*d*x + 2*c) - 4))/(a^2*d*e^2*co
s(2*d*x + 2*c) + a^2*d*e^2*sin(2*d*x + 2*c) + a^2*d*e^2), 1/4*(10*sqrt(e)*
(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)*arctan(sqrt((e*cos(2*d*x + 2*c)
+ e)/sin(2*d*x + 2*c))/sqrt(e)) + (a^2*d*e^2*cos(2*d*x + 2*c) + a^2*d*e^2*
sin(2*d*x + 2*c) + a^2*d*e^2)*(-1/(a^8*d^4*e^6))^(1/4)*log(a^2*d*e^2*(-1/(
a^8*d^4*e^6))^(1/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) ...
```

3.33.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\cot^2(c+dx)+2(e\cot(c+dx))^{\frac{3}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{3}{2}}}\,dx}{a^2}$$

input
```
integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**2,x)
```

output
```
Integral(1/((e*cot(c + d*x))**(3/2)*cot(c + d*x)**2 + 2*(e*cot(c + d*x))**
(3/2)*cot(c + d*x) + (e*cot(c + d*x))**(3/2)), x)/a**2
```

3.33.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.33.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = \int \frac{1}{(a\cot(dx+c)+a)^2(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(3/2)), x)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 13.24 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.35

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\,dx = \frac{\frac{5\cot(c+dx)}{2}+2}{a^2\,d\,(e\cot(c+d\,x))^{3/2}+a^2\,d\,e\,\sqrt{e\cot(c+d\,x)}}$$
$$-\frac{\operatorname{atan}\left(\frac{2048\,a^{10}\,d^5\,e^{13}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{1}{a^8\,d^4\,e^6}\right)^{1/4}}{51200\,a^8\,d^4\,e^{12}-2048\,a^{12}\,d^6\,e^{15}\,\sqrt{-\frac{1}{a^8\,d^4\,e^6}}}+\frac{51200\,a^{14}\,d^7\,e^{16}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{1}{a^8\,d^4\,e^6}\right)^{3/4}}{51200\,a^8\,d^4\,e^{12}-2048\,a^{12}\,d^6\,e^{15}\,\sqrt{-\frac{1}{a^8\,d^4\,e^6}}}\right)\,\left(-\frac{1}{a^8\,d^4\,e^6}\right)^{1/4}}{2}$$
$$-\operatorname{atan}\left(\frac{a^{10}\,d^5\,e^{13}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{1}{256\,a^8\,d^4\,e^6}\right)^{1/4}8192\mathrm{i}}{51200\,a^8\,d^4\,e^{12}+32768\,a^{12}\,d^6\,e^{15}\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^6}}}-\frac{a^{14}\,d^7\,e^{16}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{1}{256\,a^8\,d^4\,e^6}\right)^{3/4}32768\ldots}{51200\,a^8\,d^4\,e^{12}+32768\,a^{12}\,d^6\,e^{15}\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^6}}\ldots}\right.$$

input
```
int(1/((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^2),x)
```

output
```
((5*cot(c + d*x))/2 + 2)/(a^2*d*(e*cot(c + d*x))^(3/2) + a^2*d*e*(e*cot(c
+ d*x))^(1/2)) - (atan((2048*a^10*d^5*e^13*(e*cot(c + d*x))^(1/2)*(-1/(a^8
*d^4*e^6))^(1/4))/(51200*a^8*d^4*e^12 - 2048*a^12*d^6*e^15*(-1/(a^8*d^4*e^
6))^(1/2)) + (51200*a^14*d^7*e^16*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^6)
)^(3/4))/(51200*a^8*d^4*e^12 - 2048*a^12*d^6*e^15*(-1/(a^8*d^4*e^6))^(1/2)
))*(-1/(a^8*d^4*e^6))^(1/4))/2 - atan((a^10*d^5*e^13*(e*cot(c + d*x))^(1/2
)*(-1/(256*a^8*d^4*e^6))^(1/4)*8192i)/(51200*a^8*d^4*e^12 + 32768*a^12*d^6
*e^15*(-1/(256*a^8*d^4*e^6))^(1/2)) - (a^14*d^7*e^16*(e*cot(c + d*x))^(1/2
)*(-1/(256*a^8*d^4*e^6))^(3/4)*3276800i)/(51200*a^8*d^4*e^12 + 32768*a^12*
d^6*e^15*(-1/(256*a^8*d^4*e^6))^(1/2)))*(-1/(256*a^8*d^4*e^6))^(1/4)*2i +
(atan(((e*cot(c + d*x))^(1/2)*(-e^3)^(1/2)*1i)/e^2)*(-e^3)^(1/2)*5i)/(2*a^
2*d*e^3)
```

3.34 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx$

3.34.1 Optimal result

Integrand size = 25, antiderivative size = 331

$$\begin{aligned}\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx &= -\frac{7\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^2de^{5/2}}\\ &+\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{5/2}}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2\sqrt{2}a^2de^{5/2}}+\frac{7}{6a^2de(e\cot(c+dx))^{3/2}}\\ &-\frac{9}{2a^2de^2\sqrt{e\cot(c+dx)}}-\frac{1}{2de(e\cot(c+dx))^{3/2}\left(a^2+a^2\cot(c+dx)\right)}\\ &-\frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{5/2}}\\ &+\frac{\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{4\sqrt{2}a^2de^{5/2}}\end{aligned}$$

output
```
-7/2*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(5/2)+7/6/a^2/d/e/(e*cot
(d*x+c))^(3/2)-1/2/d/e/(e*cot(d*x+c))^(3/2)/(a^2+a^2*cot(d*x+c))+1/4*arcta
n(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(5/2)*2^(1/2)-1/4*arctan
(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/a^2/d/e^(5/2)*2^(1/2)-1/8*ln(e^(1
/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d/e^(5/2)*2^(1/2)
+1/8*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/a^2/d/e^(
5/2)*2^(1/2)-9/2/a^2/d/e^2/(e*cot(d*x+c))^(1/2)
```

3.34.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.25

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\cot(c+dx)\right)+\text{Hypergeor}}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2),x]
```

output
```
(Hypergeometric2F1[-3/2, 1, -1/2, -Cot[c + d*x]] + Hypergeometric2F1[-3/2,
 2, -1/2, -Cot[c + d*x]] - 3*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4,
-Cot[c + d*x]^2])/(3*a^2*d*e*(e*Cot[c + d*x])^(3/2))
```

3.34.3 Rubi [A] (warning: unable to verify)

Time = 1.95 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.95, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.120$, Rules used = {3042, 4052, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 2030, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(c+dx)+a)^2(e\cot(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4052}$$

$$-\frac{\int -\frac{5e\cot^2(c+dx)a^2+7ea^2-2e\cot(c+dx)a^2}{2(e\cot(c+dx))^{5/2}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5e\cot^2(c+dx)a^2+7ea^2-2e\cot(c+dx)a^2}{(e\cot(c+dx))^{5/2}(\cot(c+dx)a+a)}dx}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5e\tan\left(c+dx+\frac{\pi}{2}\right)^2a^2+7ea^2+2e\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4132}$$

$$\frac{\frac{2\int -\frac{3\left(9a^3e^3+7a^3\cot^2(c+dx)e^3+2a^3\cot(c+dx)e^3\right)}{2(e\cot(c+dx))^{3/2}(\cot(c+dx)a+a)}dx}{3ae^3}+\frac{14a}{3d(e\cot(c+dx))^{3/2}}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\int \frac{9a^3e^3+7a^3\cot^2(c+dx)e^3+2a^3\cot(c+dx)e^3}{(e\cot(c+dx))^{3/2}(\cot(c+dx)a+a)}dx}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\int \frac{9a^3e^3+7a^3\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3-2a^3\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4132}$$

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2\int -\frac{7a^4e^5+9a^4\cot^2(c+dx)e^5+2a^4\cot(c+dx)e^5}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}+\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{7a^4e^5+9a^4\cot^2(c+dx)e^5+2a^4\cot(c+dx)e^5}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.34. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx$

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{7a^4e^5+9a^4\tan\left(c+dx+\frac{\pi}{2}\right)^2e^5-2a^4\tan\left(c+dx+\frac{\pi}{2}\right)e^5}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4136}$$

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+\frac{\int \frac{4a^5e^5\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}}{ae^3}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+2a^3e^5\int \frac{\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{ae^3}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{2030}$$

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+2a^3e^4\int \sqrt{e\cot(c+dx)}dx}{ae^3}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+2a^3e^4\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{ae^3}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3957}$$

3.34. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx$

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{2a^3e^5\int\frac{\sqrt{e\cot(c+dx)}}{\cot^2(c+dx)e^2+e^2}d(e\cot(c+dx))}{d}}{ae^3}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 266

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^3e^5\int\frac{e^2\cot^2(c+dx)}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}}{d}}{ae^3}}{ae^3}}{4a^3e}-\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 826

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^3e^5\left(\frac{1}{2}\int\frac{e^2\cot^2(c+dx)+e}{e^4\cot^4(c+dx)+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\int\frac{e-e^2\cot^2}{e^4\cot^4(c+}\right)}{d}}{ae^3}}{ae^3}}{4a^3e}$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 1476

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-4a^3e^5\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(}\right)\right)}{ae^3}}{ae^3}}{4a^3e}$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 1082

$$\frac{\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-4a^3e^5\left(\frac{1}{2}\left(\frac{\int\frac{1}{-e^2\cot^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}-\int\right)\right)}{ae^3}}{ae^3}}{4a^3e}$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

3.34. $\displaystyle\int\frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx$

$\Big\downarrow$ 217

$$\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^3e^5\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}}{ae^3}}{ae^3}$$
$$4a^3e$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 1479

$$\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-4a^3e^5\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(\right.}{2\sqrt{2}\sqrt{e}}\right.\right.}{}}{ae}$$
$$4a^3e$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 25

$$\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-4a^3e^5\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(\right.}{2\sqrt{2}\sqrt{e}}\right.\right.}{}}{ae}$$
$$4a^3e$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-4a^3e^5\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{e^2\cot^2(c+dx)-\sqrt{2}e^{3/2}\cot(c+dx)+e}d\sqrt{e\cot(\right.}{2\sqrt{2}\sqrt{e}}\right.\right.}{}}{ae}$$
$$4a^3e$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 1103

3.34. $\displaystyle\int\frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx$

$$\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{7a^4e^5\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4a^3e^5\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{}}{ae^3}}{ae^3}}{4a^3e}$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4117$$

$$\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{\frac{7a^4e^5\int\frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4a^3e^5\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{}}{ae^3}}{ae^3}}{4a^3e}$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{\frac{7a^3e^5\int\frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4a^3e^5\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{}}{ae^3}}{ae^3}}{4a^3e}$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{-\frac{14a^3e^4\int\frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{d}-\frac{4a^3e^5\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)\right.}{}}{ae^3}}{ae^3}}{4a^3e}$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 216$$

$$\frac{14a}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{18a^2e^2}{d\sqrt{e\cot(c+dx)}}-\frac{\frac{14a^3e^{9/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{d}-\frac{4a^3e^5\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{e}\cot(c+dx)+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{e}\cot(c+dx)\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(\log\left(-\right.\right.\right.}{}}{ae^3}}{ae^3}}{4a^3e}$$

$$\frac{1}{2de\left(a^2\cot(c+dx)+a^2\right)(e\cot(c+dx))^{3/2}}$$

3.34. $\int\frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx$

input
```
Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^2),x]
```

output
```
-1/2*1/(d*e*(e*Cot[c + d*x])^(3/2)*(a^2 + a^2*Cot[c + d*x])) + ((14*a)/(3*
d*(e*Cot[c + d*x])^(3/2)) - ((18*a^2*e^2)/(d*Sqrt[e*Cot[c + d*x]]) - ((14*
a^3*e^(9/2)*ArcTan[Cot[c + d*x]/Sqrt[e]])/d - (4*a^3*e^5*((-(ArcTan[1 - Sq
rt[2]*Sqrt[e]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + Sqrt[2]*Sqrt[e
]*Cot[c + d*x]]/(Sqrt[2]*Sqrt[e]))/2 + (Log[e - Sqrt[2]*e^(3/2)*Cot[c + d*
x] + e^2*Cot[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]) - Log[e + Sqrt[2]*e^(3/2)*Cot
[c + d*x] + e^2*Cot[c + d*x]^2]/(2*Sqrt[2]*Sqrt[e]))/2))/d)/(a*e^3))/(a*e^
3))/(4*a^3*e)
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.34. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx$

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.34. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx$

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.34.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.69

method	result
derivativedivides	$-\frac{2e^{3}\left(\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^{2})^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)+(e^{2})^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^{2})^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^{2})^{\frac{1}{4}}}+1\right.\cdots}{16e^{5}(e^{2})^{\frac{1}{4}}}\cdots\right.}{d\,a^{2}}$
default	$-\frac{2e^{3}\left(\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^{2})^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)+(e^{2})^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^{2})^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^{2})^{\frac{1}{4}}}+1\right.\cdots}{16e^{5}(e^{2})^{\frac{1}{4}}}\cdots\right.}{d\,a^{2}}$

input
```
int(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^2*e^3*(1/16/e^5/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/3/
e^4/(e*cot(d*x+c))^(3/2)+2/e^5/(e*cot(d*x+c))^(1/2)+1/2/e^5*(1/2*(e*cot(d*
x+c))^(1/2)/(e*cot(d*x+c)+e)+7/2/e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/
2))))
```

3.34.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 1751, normalized size of antiderivative = 5.29

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/12*(21*(cos(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) +
 2*cos(2*d*x + 2*c) + 1)*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x +
2*c) + 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*
x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) + 3*(a^2*d*e^3*co
s(2*d*x + 2*c)^2 + 2*a^2*d*e^3*cos(2*d*x + 2*c) + a^2*d*e^3 + (a^2*d*e^3*c
os(2*d*x + 2*c) + a^2*d*e^3)*sin(2*d*x + 2*c))*(-1/(a^8*d^4*e^10))^(1/4)*l
og(a^6*d^3*e^8*(-1/(a^8*d^4*e^10))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/s
in(2*d*x + 2*c))) + 3*(-I*a^2*d*e^3*cos(2*d*x + 2*c)^2 - 2*I*a^2*d*e^3*cos
(2*d*x + 2*c) - I*a^2*d*e^3 + (-I*a^2*d*e^3*cos(2*d*x + 2*c) - I*a^2*d*e^3
)*sin(2*d*x + 2*c))*(-1/(a^8*d^4*e^10))^(1/4)*log(I*a^6*d^3*e^8*(-1/(a^8*d
^4*e^10))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) + 3*(I*
a^2*d*e^3*cos(2*d*x + 2*c)^2 + 2*I*a^2*d*e^3*cos(2*d*x + 2*c) + I*a^2*d*e^
3 + (I*a^2*d*e^3*cos(2*d*x + 2*c) + I*a^2*d*e^3)*sin(2*d*x + 2*c))*(-1/(a^
8*d^4*e^10))^(1/4)*log(-I*a^6*d^3*e^8*(-1/(a^8*d^4*e^10))^(3/4) + sqrt((e*
cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))) - 3*(a^2*d*e^3*cos(2*d*x + 2*c)^2
 + 2*a^2*d*e^3*cos(2*d*x + 2*c) + a^2*d*e^3 + (a^2*d*e^3*cos(2*d*x + 2*c)
+ a^2*d*e^3)*sin(2*d*x + 2*c))*(-1/(a^8*d^4*e^10))^(1/4)*log(-a^6*d^3*e^8*
(-1/(a^8*d^4*e^10))^(3/4) + sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
)) - 2*(20*cos(2*d*x + 2*c)^2 - (31*cos(2*d*x + 2*c) + 23)*sin(2*d*x + 2*c
) - 20)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(a^2*d*e^3*cos...
```

3.34.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\cot^2(c+dx)+2(e\cot(c+dx))^{\frac{5}{2}}\cot(c+dx)+(e\cot(c+dx))^{\frac{5}{2}}}\,dx}{a^2}$$

input
```
integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**2,x)
```

output
```
Integral(1/((e*cot(c + d*x))**(5/2)*cot(c + d*x)**2 + 2*(e*cot(c + d*x))**
(5/2)*cot(c + d*x) + (e*cot(c + d*x))**(5/2)), x)/a**2
```

3.34.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.34.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx = \int \frac{1}{(a\cot(dx+c)+a)^2\,(e\cot(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(5/2)), x)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.28

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx =$$

$$-\frac{\operatorname{atan}\left(\frac{2048\,a^{10}\,d^5\,e^{18}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{1}{a^8\,d^4\,e^{10}}\right)^{1/4}}{2048\,a^8\,d^4\,e^{16}+100352\,a^{12}\,d^6\,e^{21}\,\sqrt{-\frac{1}{a^8\,d^4\,e^{10}}}}+\frac{100352\,a^{14}\,d^7\,e^{23}\,\sqrt{e\cot(c+d\,x)}\,\left(-\frac{1}{a^8\,d^4\,e^{10}}\right)^{3/4}}{2048\,a^8\,d^4\,e^{16}+100352\,a^{12}\,d^6\,e^{21}\,\sqrt{-\frac{1}{a^8\,d^4\,e^{10}}}}\right)\,\left(-\frac{1}{a^8\,d^4\,e^{10}}\right)^{1/4}}{2}$$

$$-\operatorname{atan}\left(\frac{a^{10}\,d^5\,e^{18}\,\sqrt{e\cot\left(c+d\,x\right)}\,\left(-\frac{1}{256\,a^8\,d^4\,e^{10}}\right)^{1/4}\,8192\mathrm{i}}{2048\,a^8\,d^4\,e^{16}-1605632\,a^{12}\,d^6\,e^{21}\,\sqrt{-\frac{1}{256\,a^8\,d^4\,e^{10}}}}-\frac{a^{14}\,d^7\,e^{23}\,\sqrt{e\cot\left(c+d\,x\right)}\,\left(-\frac{1}{256\,a^8\,d^4\,e^{10}}\right)^{3/4}\,6422}{2048\,a^8\,d^4\,e^{16}-1605632\,a^{12}\,d^6\,e^{21}\,\sqrt{-\frac{1}{256\,a^8\,d^4}}}\right.$$

input
```
int(1/((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^2),x)
```

output
```
- (atan((2048*a^10*d^5*e^18*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^10))^(1/
4))/(2048*a^8*d^4*e^16 + 100352*a^12*d^6*e^21*(-1/(a^8*d^4*e^10))^(1/2)) +
 (100352*a^14*d^7*e^23*(e*cot(c + d*x))^(1/2)*(-1/(a^8*d^4*e^10))^(3/4))/(
2048*a^8*d^4*e^16 + 100352*a^12*d^6*e^21*(-1/(a^8*d^4*e^10))^(1/2)))*(-1/(
a^8*d^4*e^10))^(1/4))/2 - atan((a^10*d^5*e^18*(e*cot(c + d*x))^(1/2)*(-1/(
256*a^8*d^4*e^10))^(1/4)*8192i)/(2048*a^8*d^4*e^16 - 1605632*a^12*d^6*e^21
*(-1/(256*a^8*d^4*e^10))^(1/2)) - (a^14*d^7*e^23*(e*cot(c + d*x))^(1/2)*(-
1/(256*a^8*d^4*e^10))^(3/4)*6422528i)/(2048*a^8*d^4*e^16 - 1605632*a^12*d^
6*e^21*(-1/(256*a^8*d^4*e^10))^(1/2)))*(-1/(256*a^8*d^4*e^10))^(1/4)*2i -
((10*cot(c + d*x))/3 + (9*cot(c + d*x)^2)/2 - 2/3)/(a^2*d*(e*cot(c + d*x))
^(5/2) + a^2*d*e*(e*cot(c + d*x))^(3/2)) - (atan(((e*cot(c + d*x))^(1/2)*(
-e^5)^(1/2)*1i)/e^3)*(-e^5)^(1/2)*7i)/(2*a^2*d*e^5)
```

3.34. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^2}\,dx$

3.35 $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx$

3.35.1 Optimal result

Integrand size = 25, antiderivative size = 164

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = -\frac{e^{5/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} - \frac{5e^2\sqrt{e\cot(c+dx)}}{8a^3d(1+\cot(c+dx))} + \frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2}$$

output
```
-1/8*e^(5/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d+1/4*e^(5/2)*arctan
h(1/2*(e^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d*2^(
1/2)-5/8*e^2*(e*cot(d*x+c))^(1/2)/a^3/d/(1+cot(d*x+c))+1/4*e^2*(e*cot(d*x+
c))^(1/2)/a/d/(a+a*cot(d*x+c))^2
```

3.35.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.68 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.38

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{e\left(-48\cot^2(c+dx)(e\cot(c+dx))^{3/2}\,\text{Hypergeometric2F1}\left(2,\frac{7}{2},\frac{9}{2},-\cot(c+d\right.\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x])^3,x]
```

output
```
(e*(-48*Cot[c + d*x]^2*(e*Cot[c + d*x])^(3/2)*Hypergeometric2F1[2, 7/2, 9/
2, -Cot[c + d*x]] - 48*Cot[c + d*x]^2*(e*Cot[c + d*x])^(3/2)*Hypergeometri
c2F1[3, 7/2, 9/2, -Cot[c + d*x]] + 7*(24*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x
]]/Sqrt[e]] + 12*(-e^2)^(3/4)*ArcTan[Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] -
6*Sqrt[2]*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 6*S
qrt[2]*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - 12*(-e
^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] - 48*e*Sqrt[e*Cot[c +
 d*x]] + 16*(e*Cot[c + d*x])^(3/2) - 3*Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[
e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]] + 3*Sqrt[2]*e^(3/2)*Log[Sq
rt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot[c + d*x]]])))/(336*a^3*d
)
```

3.35.3 Rubi [A] (warning: unable to verify)

Time = 1.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4048, 27, 3042, 4132, 3042, 4137, 27, 3042, 4015, 221, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a\cot(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4048}$$

$$\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}-\frac{\int -\frac{a^2e^3+5a^2\cot^2(c+dx)e^3-4a^2\cot(c+dx)e^3}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)^2}dx}{4a^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2e^3+5a^2\cot^2(c+dx)e^3-4a^2\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)^2}dx}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2e^3+5a^2\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3+4a^2\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 4132

$$\frac{-\frac{\int \frac{3a^4e^4-5a^4e^4\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{5e^2\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 3042

$$\frac{-\frac{\int \frac{3a^4e^4-5a^4e^4\tan\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a^3e}-\frac{5e^2\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 4137

$$\frac{-\frac{\frac{\int \frac{8\left(a^5e^4-a^5e^4\cot(c+dx)\right)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}-a^4e^4\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{5e^2\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 27

$$\frac{-\frac{\frac{4\int \frac{a^5e^4-a^5e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2}-a^4e^4\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{5e^2\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 3042

$$\frac{-\frac{\frac{4\int \frac{e^4a^5+e^4\tan\left(c+dx+\frac{\pi}{2}\right)a^5}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-a^4e^4\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a^3e}-\frac{5e^2\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 4015

$$\frac{-\frac{-a^4e^4\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{8a^8e^8\int \frac{1}{2a^{10}e^8-\left(e^4a^5+e^4\cot(c+dx)a^5\right)^2\tan(c+dx)}d\frac{e^4a^5+e^4\cot(c+dx)a^5}{\sqrt{e\cot(c+dx)}}}{d}}{2a^3e}-\frac{5e^2\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 221

3.35. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx$

$$\frac{-\frac{-a^4e^4\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4\sqrt{2}a^3e^{7/2}\text{arctanh}\left(\frac{a^5e^4\cot(c+dx)+a^5e^4}{\sqrt{2}a^5e^{7/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{5e^2\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\Big\downarrow$ 4117

$$\frac{-\frac{-\frac{a^4e^4\int \frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4\sqrt{2}a^3e^{7/2}\text{arctanh}\left(\frac{a^5e^4\cot(c+dx)+a^5e^4}{\sqrt{2}a^5e^{7/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{5e^2\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\Big\downarrow$ 27

$$\frac{-\frac{-\frac{a^3e^4\int \frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4\sqrt{2}a^3e^{7/2}\text{arctanh}\left(\frac{a^5e^4\cot(c+dx)+a^5e^4}{\sqrt{2}a^5e^{7/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{5e^2\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\Big\downarrow$ 73

$$\frac{-\frac{\frac{2a^3e^3\int \frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{d}-\frac{4\sqrt{2}a^3e^{7/2}\text{arctanh}\left(\frac{a^5e^4\cot(c+dx)+a^5e^4}{\sqrt{2}a^5e^{7/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{5e^2\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\Big\downarrow$ 216

$$\frac{-\frac{-\frac{2a^3e^{7/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{d}-\frac{4\sqrt{2}a^3e^{7/2}\text{arctanh}\left(\frac{a^5e^4\cot(c+dx)+a^5e^4}{\sqrt{2}a^5e^{7/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{5e^2\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3}+\frac{e^2\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

input
```
Int[(e*Cot[c + d*x])^(5/2)/(a + a*Cot[c + d*x])^3,x]
```

3.35. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx$

output
```
(e^2*Sqrt[e*Cot[c + d*x]])/(4*a*d*(a + a*Cot[c + d*x])^2) + (-1/2*((-2*a^3
*e^(7/2)*ArcTan[Cot[c + d*x]/Sqrt[e]])/d - (4*Sqrt[2]*a^3*e^(7/2)*ArcTanh[
(a^5*e^4 + a^5*e^4*Cot[c + d*x])/(Sqrt[2]*a^5*e^(7/2)*Sqrt[e*Cot[c + d*x]]
)])/d)/(a^3*e) - (5*e^2*Sqrt[e*Cot[c + d*x]])/(d*(1 + Cot[c + d*x])))/(8*a
^3)
```

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4137
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Sim
p[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*T
an[e + f*x], x], x], x] + Simp[(A*b^2 + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan
[e + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{
a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.35. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx$

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(135) = 270$.

Time = 1.23 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.13

method	result
derivativedivides	$-\frac{2e^4\left(\frac{-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{\left(e^2\right)^{\frac{1}{4}}\cdots}\right.\cdots\right.}{8e}\cdots}{\cdots}\right.\cdots}{\cdots}$
default	$-\frac{2e^4\left(\frac{-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{\left(e^2\right)^{\frac{1}{4}}\cdots}\right.\cdots\right.}{8e}\cdots}{\cdots}\right.\cdots}{\cdots}$

input
```
int((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^3*e^4*(1/4/e*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))
+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+1/4/e*((5/4*(e*cot(d*
x+c))^(3/2)+3/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2+1/4/e^(1/2)*arc
tan((e*cot(d*x+c))^(1/2)/e^(1/2))))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.46

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \left[-\frac{4\left(\sqrt{2}e^2\sin(2\,dx+2\,c)+\sqrt{2}e^2\right)\sqrt{-e}\arctan\left(\frac{\left(\sqrt{2}\cos(2\,dx+2\,c)+\sqrt{2}\sin(2\,dx+2\,c)+\right.}{2\,(e\cos(2\,dx+2\,c}\right.}{}\right.$$

$$-\frac{2\left(e^2\sin(2\,dx+2\,c)+e^2\right)\sqrt{e}\arctan\left(\frac{\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{\sqrt{e}}\right)-2\left(\sqrt{2}e^2\sin(2\,dx+2\,c)+\sqrt{2}e^2\right)\sqrt{e}\log\left(-\left(\sqrt{2}\,c\right.\right.}{}$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*(sqrt(2)*e^2*sin(2*d*x + 2*c) + sqrt(2)*e^2)*sqrt(-e)*arctan(1/2
*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt(-e)*
sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/(e*cos(2*d*x + 2*c) + e))
- (e^2*sin(2*d*x + 2*c) + e^2)*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*
d*x + 2*c) - 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*si
n(2*d*x + 2*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) - (3*e^2*co
s(2*d*x + 2*c) - 5*e^2*sin(2*d*x + 2*c) - 3*e^2)*sqrt((e*cos(2*d*x + 2*c)
+ e)/sin(2*d*x + 2*c)))/(a^3*d*sin(2*d*x + 2*c) + a^3*d), -1/16*(2*(e^2*si
n(2*d*x + 2*c) + e^2)*sqrt(e)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d
*x + 2*c))/sqrt(e)) - 2*(sqrt(2)*e^2*sin(2*d*x + 2*c) + sqrt(2)*e^2)*sqrt(
e)*log(-(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) - sqrt(2))*sq
rt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + 2*e*sin(2*d*x + 2*
c) + e) - (3*e^2*cos(2*d*x + 2*c) - 5*e^2*sin(2*d*x + 2*c) - 3*e^2)*sqrt((
e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(a^3*d*sin(2*d*x + 2*c) + a^3*d
)]
```

3.35.6 Sympy [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{\int \frac{(e\cot(c+dx))^{\frac{5}{2}}}{\cot^3(c+dx)+3\cot^2(c+dx)+3\cot(c+dx)+1}\,dx}{a^3}$$

input
```
integrate((e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**3,x)
```

output
```
Integral((e*cot(c + d*x))**(5/2)/(cot(c + d*x)**3 + 3*cot(c + d*x)**2 + 3*
cot(c + d*x) + 1), x)/a**3
```

3.35.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.35.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \int \frac{(e\cot(dx+c))^{\frac{5}{2}}}{(a\cot(dx+c)+a)^3}\,dx$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(5/2)/(a*cot(d*x + c) + a)^3, x)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.94

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{\sqrt{2}\,e^{5/2}\,\mathrm{atanh}\left(\frac{9\sqrt{2}\,e^{33/2}\,\sqrt{e\cot(c+d\,x)}}{32\left(\frac{9\,e^{17}\cot(c+d\,x)}{32}+\frac{9\,e^{17}}{32}\right)}\right)}{4\,a^3\,d} - \frac{e^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{8\,a^3\,d} - \frac{\frac{3\,e^4\,\sqrt{e\cot(c+d\,x)}}{8}+\frac{5\,e^3\,(e\cot(c+d\,x))^{3/2}}{8}}{d\,a^3\,e^2\cot\left(c+d\,x\right)^2+2\,d\,a^3\,e^2\cot\left(c+d\,x\right)+d\,a^3\,e^2}$$

input
```
int((e*cot(c + d*x))^(5/2)/(a + a*cot(c + d*x))^3,x)
```

output
```
(2^(1/2)*e^(5/2)*atanh((9*2^(1/2)*e^(33/2)*(e*cot(c + d*x))^(1/2))/(32*((9
*e^17*cot(c + d*x))/32 + (9*e^17)/32))))/(4*a^3*d) - (e^(5/2)*atan((e*cot(
c + d*x))^(1/2)/e^(1/2)))/(8*a^3*d) - ((3*e^4*(e*cot(c + d*x))^(1/2))/8 +
(5*e^3*(e*cot(c + d*x))^(3/2))/8)/(a^3*d*e^2 + a^3*d*e^2*cot(c + d*x)^2 +
2*a^3*d*e^2*cot(c + d*x))
```

3.35. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+a\cot(c+dx))^3}\,dx$

3.36 $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx$

3.36.1 Optimal result

Integrand size = 25, antiderivative size = 164

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{5e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} - \frac{e\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{e\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)}$$

output
```
5/8*e^(3/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d+1/4*e^(3/2)*arctan(
1/2*(e^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d*2^(1/
2)-1/4*e*(e*cot(d*x+c))^(1/2)/a/d/(a+a*cot(d*x+c))^2+1/8*e*(e*cot(d*x+c))^
(1/2)/d/(a^3+a^3*cot(d*x+c))
```

3.36.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.13 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.23

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = -\frac{e\left(70\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)+\frac{20e\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)}{\sqrt[4]{-e^2}}-10\sqrt{2}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)+10\sqrt{2}\sqrt{e}\,\mathrm{ar}\cdots\right.}{\cdots}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x])^3,x]
```

output
```
-1/80*(e*(70*Sqrt[e]*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] + (20*e*ArcTan[S
qrt[e*Cot[c + d*x]]/(-e^2)^(1/4)])/(-e^2)^(1/4) - 10*Sqrt[2]*Sqrt[e]*ArcTa
n[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] + 10*Sqrt[2]*Sqrt[e]*ArcTan[
1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - (20*e*ArcTanh[Sqrt[e*Cot[c +
 d*x]]/(-e^2)^(1/4)])/(-e^2)^(1/4) - 80*Sqrt[e*Cot[c + d*x]] - (10*Sqrt[e*
Cot[c + d*x]]*(3 + 5*Cot[c + d*x]))/(1 + Cot[c + d*x])^2 + (16*(e*Cot[c +
d*x])^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -Cot[c + d*x]])/e^2 - 5*Sqrt[2]
*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]
] + 5*Sqrt[2]*Sqrt[e]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*
Cot[c + d*x]]]))/(a^3*d)
```

3.36.3 Rubi [A] (warning: unable to verify)

Time = 1.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4050, 27, 3042, 4132, 25, 3042, 4136, 27, 3042, 4015, 218, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a\cot(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4050}$$

$$-\frac{\int \frac{-3a\cot^2(c+dx)e^2+ae^2-4a\cot(c+dx)e^2}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)^2}dx}{4a^2}-\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-3a\cot^2(c+dx)e^2+ae^2-4a\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)^2}dx}{8a^2}-\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-3a\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+ae^2+4a\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{8a^2}-\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 4132

$$-\frac{-\frac{\int -\frac{a^3e^3+a^3\cot^2(c+dx)e^3-8a^3\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{e\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}-\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 25

$$-\frac{\frac{\int \frac{a^3e^3+a^3\cot^2(c+dx)e^3-8a^3\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{e\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}-\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 3042

$$-\frac{\frac{\int \frac{a^3e^3+a^3\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3+8a^3\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a^3e}-\frac{e\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}-\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 4136

$$-\frac{\frac{5a^3e^3\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+\frac{\int -\frac{8\left(e^3a^4+e^3\cot(c+dx)a^4\right)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}}{2a^3e}-\frac{e\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}-\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 27

$$-\frac{\frac{5a^3e^3\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx-\frac{4\int \frac{e^3a^4+e^3\cot(c+dx)a^4}{\sqrt{e\cot(c+dx)}}dx}{a^2}}{2a^3e}-\frac{e\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}-\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 3042

$$-\frac{\frac{5a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4\int \frac{a^4e^3-a^4e^3\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a^3e}-\frac{e\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}-\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 4015

3.36. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx$

$$-\frac{\frac{5a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{8a^6e^6\int \frac{1}{-2e^6a^8-\left(a^4e^3-a^4e^3\cot(c+dx)\right)^2\tan(c+dx)}d\frac{a^4e^3-a^4e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}}{d}}{2a^3e}-\frac{e\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+}}{8a^2}$$

$$\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 218

$$-\frac{\frac{5a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4\sqrt{2}a^2e^{5/2}\arctan\left(\frac{a^4e^3-a^4e^3\cot(c+dx)}{\sqrt{2}a^4e^{5/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{e\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}-$$

$$\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 4117

$$-\frac{\frac{\frac{5a^3e^3\int \frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4\sqrt{2}a^2e^{5/2}\arctan\left(\frac{a^4e^3-a^4e^3\cot(c+dx)}{\sqrt{2}a^4e^{5/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{e\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}-$$

$$\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{5a^2e^3\int \frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4\sqrt{2}a^2e^{5/2}\arctan\left(\frac{a^4e^3-a^4e^3\cot(c+dx)}{\sqrt{2}a^4e^{5/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{e\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}-$$

$$\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 73

$$-\frac{\frac{-\frac{10a^2e^2\int \frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{d}-\frac{4\sqrt{2}a^2e^{5/2}\arctan\left(\frac{a^4e^3-a^4e^3\cot(c+dx)}{\sqrt{2}a^4e^{5/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{e\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}-$$

$$\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$\downarrow$ 216

$$-\frac{\frac{\frac{10a^2e^{5/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{d}-\frac{4\sqrt{2}a^2e^{5/2}\arctan\left(\frac{a^4e^3-a^4e^3\cot(c+dx)}{\sqrt{2}a^4e^{5/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{e\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}-$$

$$\frac{e\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

3.36. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx$

input
```
Int[(e*Cot[c + d*x])^(3/2)/(a + a*Cot[c + d*x])^3,x]
```

output
```
-1/4*(e*Sqrt[e*Cot[c + d*x]])/(a*d*(a + a*Cot[c + d*x])^2) - (((10*a^2*e^(
5/2)*ArcTan[Cot[c + d*x]/Sqrt[e]])/d - (4*Sqrt[2]*a^2*e^(5/2)*ArcTan[(a^4*
e^3 - a^4*e^3*Cot[c + d*x])/(Sqrt[2]*a^4*e^(5/2)*Sqrt[e*Cot[c + d*x]])])/d
)/(2*a^3*e) - (e*Sqrt[e*Cot[c + d*x]])/(d*(a + a*Cot[c + d*x])))/(8*a^2)
```

3.36.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4050
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/((m
 + 1)*(a^2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^
(n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2
*a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[
2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.36. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx$

3.36.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(135) = 270$.

Time = 0.04 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.13

method	result
derivativedivides	$-\frac{2e^4\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2e^4\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$

input
```
int((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^3*e^4*(1/4/e^2*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
)+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*
arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/4/e^2*((1/4*(e*cot
(d*x+c))^(3/2)-1/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2+5/4/e^(1/2)*
arctan((e*cot(d*x+c))^(1/2)/e^(1/2))))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 533, normalized size of antiderivative = 3.25

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \left[\frac{2\left(\sqrt{2}e\sin\left(2\,dx+2\,c\right)+\sqrt{2}e\right)\sqrt{-e}\log\left(-\left(\sqrt{2}\cos\left(2\,dx+2\,c\right)+\sqrt{2}\sin\left(2\,dx\right.\right.\right.}{}\right.$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*e)*sqrt(-e)*log(-(sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) - sqrt(2))*sqrt(-e)*sqrt((e*cos
(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - 2*e*sin(2*d*x + 2*c) + e) + 5*(e*si
n(2*d*x + 2*c) + e)*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c)
+ 2*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2
*c) + e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) + (e*cos(2*d*x + 2*c)
+ e*sin(2*d*x + 2*c) - e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))
/(a^3*d*sin(2*d*x + 2*c) + a^3*d), 1/16*(4*(sqrt(2)*e*sin(2*d*x + 2*c) + s
qrt(2)*e)*sqrt(e)*arctan(-1/2*(sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*
x + 2*c) + sqrt(2))*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
)/(e*cos(2*d*x + 2*c) + e)) + 10*(e*sin(2*d*x + 2*c) + e)*sqrt(e)*arctan(s
qrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) + (e*cos(2*d*x + 2
*c) + e*sin(2*d*x + 2*c) - e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c)))/(a^3*d*sin(2*d*x + 2*c) + a^3*d)]
```

3.36.6 Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{\int \frac{(e\cot(c+dx))^{\frac{3}{2}}}{\cot^3(c+dx)+3\cot^2(c+dx)+3\cot(c+dx)+1}\,dx}{a^3}$$

input
```
integrate((e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**3,x)
```

output
```
Integral((e*cot(c + d*x))**(3/2)/(cot(c + d*x)**3 + 3*cot(c + d*x)**2 + 3*
cot(c + d*x) + 1), x)/a**3
```

3.36.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.36.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{(a\cot(dx+c)+a)^3}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(3/2)/(a*cot(d*x + c) + a)^3, x)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 13.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.09

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx = \frac{5\,e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{8\,a^3\,d} - \frac{\frac{e^3\,\sqrt{e\cot(c+d\,x)}}{8}-\frac{e^2\,(e\cot(c+d\,x))^{3/2}}{8}}{d\,a^3\,e^2\cot\left(c+d\,x\right)^2+2\,d\,a^3\,e^2\cot\left(c+d\,x\right)+d\,a^3\,e^2} - \frac{\sqrt{2}\,e^{3/2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{8\,a^3\,d}$$

input
```
int((e*cot(c + d*x))^(3/2)/(a + a*cot(c + d*x))^3,x)
```

output
```
(5*e^(3/2)*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(8*a^3*d) - ((e^3*(e*cot(
c + d*x))^(1/2))/8 - (e^2*(e*cot(c + d*x))^(3/2))/8)/(a^3*d*e^2 + a^3*d*e^
2*cot(c + d*x)^2 + 2*a^3*d*e^2*cot(c + d*x)) - (2^(1/2)*e^(3/2)*(2*atan((2
^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d
*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/
(8*a^3*d)
```

3.36. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+a\cot(c+dx))^3}\,dx$

3.37 $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx$

3.37.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx = -\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d} - \frac{\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d} + \frac{\sqrt{e\cot(c+dx)}}{4ad(a+a\cot(c+dx))^2} + \frac{3\sqrt{e\cot(c+dx)}}{8d\left(a^3+a^3\cot(c+dx)\right)}$$

output
```
-1/8*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/a^3/d-1/4*arctanh(1/2*(e
^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))*e^(1/2)/a^3/d*2^(
1/2)+1/4*(e*cot(d*x+c))^(1/2)/a/d/(a+a*cot(d*x+c))^2+3/8*(e*cot(d*x+c))^(1
/2)/d/(a^3+a^3*cot(d*x+c))
```

3.37.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.92 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx = -\frac{e\left(-\frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{2a^3\sqrt{e}} - \frac{\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)}{4a^3\sqrt[4]{-e^2}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{4\sqrt{2}a^3\sqrt{e}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{4\sqrt{2}a^3\sqrt{e}} + \frac{\text{arctanh}(\ldots}{4a^3 \ldots}}{\ldots}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^3,x]
```

output
```
-((e*(-1/2*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]]/(a^3*Sqrt[e]) - ArcTan[Sqr
t[e*Cot[c + d*x]]/(-e^2)^(1/4)]/(4*a^3*(-e^2)^(1/4)) - ArcTan[1 - (Sqrt[2]
*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(4*Sqrt[2]*a^3*Sqrt[e]) + ArcTan[1 + (Sqrt
[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(4*Sqrt[2]*a^3*Sqrt[e]) + ArcTanh[Sqrt[
e*Cot[c + d*x]]/(-e^2)^(1/4)]/(4*a^3*(-e^2)^(1/4)) - (-(e*ArcTan[Sqrt[e*Co
t[c + d*x]]/Sqrt[e]]) - e*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]]*Cot[c + d*x
] + Sqrt[e]*Sqrt[e*Cot[c + d*x]])/(2*a^3*Sqrt[e]*(e + e*Cot[c + d*x])) + (
(e*Cot[c + d*x])^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, -Cot[c + d*x]])/(3*a
^3*e^2) - Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]
]]/(8*Sqrt[2]*a^3*Sqrt[e]) + Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*
Sqrt[e*Cot[c + d*x]]]/(8*Sqrt[2]*a^3*Sqrt[e])))/d)
```

3.37.3 Rubi [A] (warning: unable to verify)

Time = 1.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4051, 27, 3042, 4132, 25, 3042, 4137, 27, 3042, 4015, 221, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a\cot(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4051}$$

$$\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}-\frac{\int -\frac{-3ae\cot^2(c+dx)+4ae\cot(c+dx)+ae}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)^2}dx}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-3ae\cot^2(c+dx)+4ae\cot(c+dx)+ae}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)^2}dx}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-3ae\tan\left(c+dx+\frac{\pi}{2}\right)^2-4ae\tan\left(c+dx+\frac{\pi}{2}\right)+ae}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \; 4132$$

$$\frac{\frac{3\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}-\frac{\int -\frac{5a^3e^2-3a^3e^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^3e}}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{5a^3e^2-3a^3e^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^3e}+\frac{3\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{5a^3e^2-3a^3e^2\tan\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a^3e}+\frac{3\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \; 4137$$

$$\frac{\frac{a^3e^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+\frac{\int \frac{8\left(a^4e^2-a^4e^2\cot(c+dx)\right)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}}{2a^3e}+\frac{3\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a^3e^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+\frac{4\int \frac{a^4e^2-a^4e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2}}{2a^3e}+\frac{3\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{a^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4\int \frac{e^2a^4+e^2\tan\left(c+dx+\frac{\pi}{2}\right)a^4}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a^3e}+\frac{3\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \; 4015$$

3.37. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx$

$$\frac{\frac{a^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{8a^6e^4\int \frac{1}{2a^8e^4-\left(e^2a^4+e^2\cot(c+dx)a^4\right)^2\tan(c+dx)}d\frac{e^2a^4+e^2\cot(c+dx)a^4}{\sqrt{e\cot(c+dx)}}}{d}}{2a^3e}+\frac{3\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{a^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4\sqrt{2}a^2e^{3/2}\text{arctanh}\left(\frac{a^4e^2\cot(c+dx)+a^4e^2}{\sqrt{2}a^4e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}+\frac{3\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \; 4117$$

$$\frac{\frac{\frac{a^3e^2\int \frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4\sqrt{2}a^2e^{3/2}\text{arctanh}\left(\frac{a^4e^2\cot(c+dx)+a^4e^2}{\sqrt{2}a^4e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}+\frac{3\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{a^2e^2\int \frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4\sqrt{2}a^2e^{3/2}\text{arctanh}\left(\frac{a^4e^2\cot(c+dx)+a^4e^2}{\sqrt{2}a^4e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}+\frac{3\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{-\frac{2a^2e\int \frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{d}-\frac{4\sqrt{2}a^2e^{3/2}\text{arctanh}\left(\frac{a^4e^2\cot(c+dx)+a^4e^2}{\sqrt{2}a^4e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}+\frac{3\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{2a^2e^{3/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{d}-\frac{4\sqrt{2}a^2e^{3/2}\text{arctanh}\left(\frac{a^4e^2\cot(c+dx)+a^4e^2}{\sqrt{2}a^4e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}+\frac{3\sqrt{e\cot(c+dx)}}{d(a\cot(c+dx)+a)}}{8a^2}+\frac{\sqrt{e\cot(c+dx)}}{4ad(a\cot(c+dx)+a)^2}$$

3.37. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx$

input
```
Int[Sqrt[e*Cot[c + d*x]]/(a + a*Cot[c + d*x])^3,x]
```

output
```
Sqrt[e*Cot[c + d*x]]/(4*a*d*(a + a*Cot[c + d*x])^2) + (((2*a^2*e^(3/2)*Arc
Tan[Cot[c + d*x]/Sqrt[e]])/d - (4*Sqrt[2]*a^2*e^(3/2)*ArcTanh[(a^4*e^2 + a
^4*e^2*Cot[c + d*x])/(Sqrt[2]*a^4*e^(3/2)*Sqrt[e*Cot[c + d*x]])])/d)/(2*a^
3*e) + (3*Sqrt[e*Cot[c + d*x]])/(d*(a + a*Cot[c + d*x])))/(8*a^2)
```

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4051
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(a^2 + b^2
))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c
*(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Int
egerQ[2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4137
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Sim
p[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*T
an[e + f*x], x], x], x] + Simp[(A*b^2 + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan
[e + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{
a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && !GtQ[n, 0] && !LeQ[n, -1]
```

3.37. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx$

3.37.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(132) = 264$.

Time = 0.05 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.17

method	result
derivativedivides	$-\frac{2e^4\left(\frac{\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx\cdots}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e}}{\cdots}\right.}{\cdots}$
default	$-\frac{2e^4\left(\frac{\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx\cdots}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e}}{\cdots}\right.}{\cdots}$

input
```
int((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^3*e^4*(1/4/e^3*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
)-1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*
arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/4/e^3*((3/4*(e*cot
(d*x+c))^(3/2)+5/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2-1/4/e^(1/2)*
arctan((e*cot(d*x+c))^(1/2)/e^(1/2))))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx$$
$$= \left[\frac{4\left(\sqrt{2}\sin\left(2\,dx+2\,c\right)+\sqrt{2}\right)\sqrt{-e}\arctan\left(\frac{\left(\sqrt{2}\cos(2\,dx+2\,c)+\sqrt{2}\sin(2\,dx+2\,c)+\sqrt{2}\right)\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{2\,(e\cos(2\,dx+2\,c)+e)}\right)+\sqrt{-e}(\mathrm{si}}{}\right.$$
$$-\frac{2\sqrt{e}(\sin\left(2\,dx+2\,c\right)+1)\arctan\left(\frac{\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}}{\sqrt{e}}\right)-2\left(\sqrt{2}\sin\left(2\,dx+2\,c\right)+\sqrt{2}\right)\sqrt{e}\log\left(\left(\sqrt{2}\cos\left(2\,d\right.\right.\right.}{}$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/16*(4*(sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt(-e)*arctan(1/2*(sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*sqrt(-e)*sqrt((e*c
os(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/(e*cos(2*d*x + 2*c) + e)) + sqrt(-e
)*(sin(2*d*x + 2*c) + 1)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) - 2*
sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c)
+ e)/(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) - sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c))*(5*cos(2*d*x + 2*c) - 3*sin(2*d*x + 2*c) - 5))/(a^
3*d*sin(2*d*x + 2*c) + a^3*d), -1/16*(2*sqrt(e)*(sin(2*d*x + 2*c) + 1)*arc
tan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) - 2*(sqrt(2)*
sin(2*d*x + 2*c) + sqrt(2))*sqrt(e)*log((sqrt(2)*cos(2*d*x + 2*c) - sqrt(2
)*sin(2*d*x + 2*c) - sqrt(2))*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*
d*x + 2*c)) + 2*e*sin(2*d*x + 2*c) + e) + sqrt((e*cos(2*d*x + 2*c) + e)/si
n(2*d*x + 2*c))*(5*cos(2*d*x + 2*c) - 3*sin(2*d*x + 2*c) - 5))/(a^3*d*sin(
2*d*x + 2*c) + a^3*d)]
```

3.37.6 Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{e\cot(c+dx)}}{\cot^3(c+dx)+3\cot^2(c+dx)+3\cot(c+dx)+1}\,dx}{a^3}$$

input
```
integrate((e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**3,x)
```

output
```
Integral(sqrt(e*cot(c + d*x))/(cot(c + d*x)**3 + 3*cot(c + d*x)**2 + 3*cot
(c + d*x) + 1), x)/a**3
```

3.37.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.37.8 Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{(a\cot(dx+c)+a)^3}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(e*cot(d*x + c))/(a*cot(d*x + c) + a)^3, x)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+a\cot(c+dx))^3}\,dx = \frac{\frac{3\,e\,(e\cot(c+d\,x))^{3/2}}{8}+\frac{5\,e^2\,\sqrt{e\cot(c+d\,x)}}{8}}{d\,a^3\,e^2\cot\left(c+d\,x\right)^2+2\,d\,a^3\,e^2\cot\left(c+d\,x\right)+d\,a^3\,e^2} - \frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{8\,a^3\,d} - \frac{\sqrt{2}\,\sqrt{e}\,\mathrm{atanh}\left(\frac{9\,\sqrt{2}\,e^{17/2}\,\sqrt{e\cot(c+d\,x)}}{32\,\left(\frac{9\,e^9\cot(c+d\,x)}{32}+\frac{9\,e^9}{32}\right)}\right)}{4\,a^3\,d}$$

input
```
int((e*cot(c + d*x))^(1/2)/(a + a*cot(c + d*x))^3,x)
```

output
```
((3*e*(e*cot(c + d*x))^(3/2))/8 + (5*e^2*(e*cot(c + d*x))^(1/2))/8)/(a^3*d
*e^2 + a^3*d*e^2*cot(c + d*x)^2 + 2*a^3*d*e^2*cot(c + d*x)) - (e^(1/2)*ata
n((e*cot(c + d*x))^(1/2)/e^(1/2)))/(8*a^3*d) - (2^(1/2)*e^(1/2)*atanh((9*2
^(1/2)*e^(17/2)*(e*cot(c + d*x))^(1/2))/(32*((9*e^9*cot(c + d*x))/32 + (9*
e^9)/32))))/(4*a^3*d)
```

3.38 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx$

3.38.1 Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx = -\frac{11\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3d\sqrt{e}} - \frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3d\sqrt{e}} - \frac{7\sqrt{e\cot(c+dx)}}{8a^3de(1+\cot(c+dx))} - \frac{\sqrt{e\cot(c+dx)}}{4ade(a+a\cot(c+dx))^2}$$

output
```
-11/8*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d/e^(1/2)-1/4*arctan(1/2*(e
^(1/2)-cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d*2^(1/2)/e^(
1/2)-7/8*(e*cot(d*x+c))^(1/2)/a^3/d/e/(1+cot(d*x+c))-1/4*(e*cot(d*x+c))^(1
/2)/a/d/e/(a+a*cot(d*x+c))^2
```

3.38.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.39 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx =$$
$$-\frac{16e^{3/2}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)+4(-e^2)^{3/4}\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt[4]{-e^2}}\right)+2\sqrt{2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)-}{}$$

input
```
Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^3),x]
```

output
```
-1/16*(16*e^(3/2)*ArcTan[Sqrt[e*Cot[c + d*x]]/Sqrt[e]] + 4*(-e^2)^(3/4)*Ar
cTan[Sqrt[e*Cot[c + d*x]]/(-e^2)^(1/4)] + 2*Sqrt[2]*e^(3/2)*ArcTan[1 - (Sq
rt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - 2*Sqrt[2]*e^(3/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]] - 4*(-e^2)^(3/4)*ArcTanh[Sqrt[e*Cot[c +
d*x]]/(-e^2)^(1/4)] + (8*e*Sqrt[e*Cot[c + d*x]])/(1 + Cot[c + d*x]) + 16*e
*Sqrt[e*Cot[c + d*x]]*Hypergeometric2F1[1/2, 3, 3/2, -Cot[c + d*x]] + Sqrt
[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*
x]]] - Sqrt[2]*e^(3/2)*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e
*Cot[c + d*x]]])/(a^3*d*e^2)
```

3.38.3 Rubi [A] (warning: unable to verify)

Time = 1.34 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4052, 27, 3042, 4132, 25, 3042, 4136, 27, 3042, 4015, 218, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(c+dx)+a)^3\sqrt{e\cot(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4052}$$

$$-\frac{\int -\frac{3e\cot^2(c+dx)a^2+7ea^2-4e\cot(c+dx)a^2}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)^2}dx}{4a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3e\cot^2(c+dx)a^2+7ea^2-4e\cot(c+dx)a^2}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)^2}dx}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3e\tan\left(c+dx+\frac{\pi}{2}\right)^2a^2+7ea^2+4e\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{\int -\frac{7e^2a^4+7e^2\cot^2(c+dx)a^4-8e^2\cot(c+dx)a^4}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{7\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{7e^2a^4+7e^2\cot^2(c+dx)a^4-8e^2\cot(c+dx)a^4}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{7\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{7e^2a^4+7e^2\tan\left(c+dx+\frac{\pi}{2}\right)^2a^4+8e^2\tan\left(c+dx+\frac{\pi}{2}\right)a^4}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a^3e}-\frac{7\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{4136}$$

$$\frac{\frac{11a^4e^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+\frac{\int -\frac{8\left(e^2a^5+e^2\cot(c+dx)a^5\right)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}}{2a^3e}-\frac{7\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{11a^4e^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx-\frac{4\int \frac{e^2a^5+e^2\cot(c+dx)a^5}{\sqrt{e\cot(c+dx)}}dx}{a^2}}{2a^3e}-\frac{7\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

3.38. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx$

$$\frac{\frac{11a^4e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4\int \frac{a^5e^2-a^5e^2\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a^3e}-\frac{7\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \ 4015$$

$$\frac{\frac{11a^4e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{8a^8e^4\int \frac{1}{-2e^4a^{10}-\left(a^5e^2-a^5e^2\cot(c+dx)\right)^2\tan(c+dx)}d\frac{a^5e^2-a^5e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}}{d}}{2a^3e}-\frac{7\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{11a^4e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4\sqrt{2}a^3e^{3/2}\arctan\left(\frac{a^5e^2-a^5e^2\cot(c+dx)}{\sqrt{2}a^5e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{7\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \ 4117$$

$$\frac{\frac{\frac{11a^4e^2\int \frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4\sqrt{2}a^3e^{3/2}\arctan\left(\frac{a^5e^2-a^5e^2\cot(c+dx)}{\sqrt{2}a^5e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{7\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{11a^3e^2\int \frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4\sqrt{2}a^3e^{3/2}\arctan\left(\frac{a^5e^2-a^5e^2\cot(c+dx)}{\sqrt{2}a^5e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{7\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \ 73$$

3.38. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx$

$$\frac{\frac{-\frac{22a^3e\int \frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{d}-\frac{4\sqrt{2}a^3e^{3/2}\arctan\left(\frac{a^5e^2-a^5e^2\cot(c+dx)}{\sqrt{2}a^5e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{7\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{22a^3e^{3/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{d}-\frac{4\sqrt{2}a^3e^{3/2}\arctan\left(\frac{a^5e^2-a^5e^2\cot(c+dx)}{\sqrt{2}a^5e^{3/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{2a^3e}-\frac{7\sqrt{e\cot(c+dx)}}{d(\cot(c+dx)+1)}}{8a^3e}-\frac{\sqrt{e\cot(c+dx)}}{4ade(a\cot(c+dx)+a)^2}$$

input
```
Int[1/(Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^3),x]
```

output
```
-1/4*Sqrt[e*Cot[c + d*x]]/(a*d*e*(a + a*Cot[c + d*x])^2) + (((22*a^3*e^(3/
2)*ArcTan[Cot[c + d*x]/Sqrt[e]])/d - (4*Sqrt[2]*a^3*e^(3/2)*ArcTan[(a^5*e^
2 - a^5*e^2*Cot[c + d*x])/(Sqrt[2]*a^5*e^(3/2)*Sqrt[e*Cot[c + d*x]])])/d)/
(2*a^3*e) - (7*Sqrt[e*Cot[c + d*x]])/(d*(1 + Cot[c + d*x])))/(8*a^3*e)
```

3.38.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.38. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx$

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.38.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(136) = 272$.

Time = 0.06 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.12

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\ldots}}{(e^2)^{\frac{1}{4}}}\ldots\right.\right.}{8e}\ldots}{\ldots}$
default	$-\frac{2e^4\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\ldots}}{(e^2)^{\frac{1}{4}}}\ldots\right.\right.}{8e}\ldots}{\ldots}$

input
```
int(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^3*e^4*(1/4/e^4*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
))-1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2
*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+1/4/e^4*((7/4*(e*co
t(d*x+c))^(3/2)+9/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2+11/4/e^(1/2
)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.05

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx$$

$$= \left[-\frac{2\sqrt{2}\sqrt{-e}(\sin(2\,dx+2\,c)+1)\log\left(-\sqrt{2}\sqrt{-e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)+\sin(2\,dx+2\,c)-1)\right.}{}\right.$$

$$-\frac{4\sqrt{2}\sqrt{e}(\sin(2\,dx+2\,c)+1)\arctan\left(-\frac{\sqrt{2}\sqrt{e}\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}(\cos(2\,dx+2\,c)-\sin(2\,dx+2\,c)+1)}{2\,(e\cos(2\,dx+2\,c)+e)}\right)+22\sqrt{e}(\sin(2\,dx}{16\,(a^3de\sin(2\,dx}$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*sqrt(2)*sqrt(-e)*(sin(2*d*x + 2*c) + 1)*log(-sqrt(2)*sqrt(-e)*sq
rt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*
x + 2*c) - 1) - 2*e*sin(2*d*x + 2*c) + e) + 11*sqrt(-e)*(sin(2*d*x + 2*c)
+ 1)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) + 2*sqrt(-e)*sqrt((e*cos
(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(2*d*x + 2*
c) + sin(2*d*x + 2*c) + 1)) - sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*
c))*(9*cos(2*d*x + 2*c) - 7*sin(2*d*x + 2*c) - 9))/(a^3*d*e*sin(2*d*x + 2*
c) + a^3*d*e), -1/16*(4*sqrt(2)*sqrt(e)*(sin(2*d*x + 2*c) + 1)*arctan(-1/2
*sqrt(2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*
x + 2*c) - sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x + 2*c) + e)) + 22*sqrt(e)*(s
in(2*d*x + 2*c) + 1)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
)/sqrt(e)) - sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(9*cos(2*d*x
+ 2*c) - 7*sin(2*d*x + 2*c) - 9))/(a^3*d*e*sin(2*d*x + 2*c) + a^3*d*e)]
```

3.38.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx$$
$$= \frac{\int \frac{1}{\sqrt{e\cot(c+dx)}\cot^3(c+dx)+3\sqrt{e\cot(c+dx)}\cot^2(c+dx)+3\sqrt{e\cot(c+dx)}\cot(c+dx)+\sqrt{e\cot(c+dx)}}\,dx}{a^3}$$

input
```
integrate(1/(e*cot(d*x+c))**(1/2)/(a+a*cot(d*x+c))**3,x)
```

output
```
Integral(1/(sqrt(e*cot(c + d*x))*cot(c + d*x)**3 + 3*sqrt(e*cot(c + d*x))*
cot(c + d*x)**2 + 3*sqrt(e*cot(c + d*x))*cot(c + d*x) + sqrt(e*cot(c + d*x
))), x)/a**3
```

3.38.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.38.8 Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx = \int \frac{1}{(a\cot(dx+c)+a)^3\sqrt{e\cot(dx+c)}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*cot(d*x + c) + a)^3*sqrt(e*cot(d*x + c))), x)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 13.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^3}\,dx = \frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{8\,a^3\,d\,\sqrt{e}} - \frac{11\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{8\,a^3\,d\,\sqrt{e}} - \frac{\frac{9\,e\,\sqrt{e\cot(c+d\,x)}}{8}+\frac{7\,(e\cot(c+d\,x))^{3/2}}{8}}{d\,a^3\,e^2\cot\left(c+d\,x\right)^2+2\,d\,a^3\,e^2\cot\left(c+d\,x\right)+d\,a^3\,e^2}$$

input
```
int(1/((e*cot(c + d*x))^(1/2)*(a + a*cot(c + d*x))^3),x)
```

output
```
(2^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2))) + 2*atan((2
^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)) + (2^(1/2)*(e*cot(c + d*x))^(3/
2))/(2*e^(3/2)))))/(8*a^3*d*e^(1/2)) - (11*atan((e*cot(c + d*x))^(1/2)/e^(
1/2)))/(8*a^3*d*e^(1/2)) - ((9*e*(e*cot(c + d*x))^(1/2))/8 + (7*(e*cot(c +
 d*x))^(3/2))/8)/(a^3*d*e^2 + a^3*d*e^2*cot(c + d*x)^2 + 2*a^3*d*e^2*cot(c
 + d*x))
```

3.39 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx$

3.39.1 Optimal result

Integrand size = 25, antiderivative size = 189

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \frac{31\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{e}+\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{3/2}} + \frac{27}{8a^3de\sqrt{e\cot(c+dx)}} - \frac{9}{8a^3de\sqrt{e\cot(c+dx)}(1+\cot(c+dx))} - \frac{1}{4ade\sqrt{e\cot(c+dx)}(a+a\cot(c+dx))^2}$$

output
```
31/8*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d/e^(3/2)+1/4*arctanh(1/2*(e
^(1/2)+cot(d*x+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d/e^(3/2)*2^(
1/2)+27/8/a^3/d/e/(e*cot(d*x+c))^(1/2)-9/8/a^3/d/e/(1+cot(d*x+c))/(e*cot(d
*x+c))^(1/2)-1/4/a/d/e/(a+a*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2)
```

3.39.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.72

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \frac{-2\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)\sqrt{e\cot(c+dx)}+2\sqrt{2}\operatorname{arct}}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^3),x]
```

output
```
(-2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]*Sqrt[e*Cot[
c + d*x]] + 2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]*S
qrt[e*Cot[c + d*x]] + 8*Sqrt[e]*Hypergeometric2F1[-1/2, 1, 1/2, -Cot[c + d
*x]] + 16*Sqrt[e]*Hypergeometric2F1[-1/2, 2, 1/2, -Cot[c + d*x]] + 16*Sqrt
[e]*Hypergeometric2F1[-1/2, 3, 1/2, -Cot[c + d*x]] - 8*Sqrt[e]*Hypergeomet
ric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]*Log[S
qrt[e] + Sqrt[e]*Cot[c + d*x] - Sqrt[2]*Sqrt[e*Cot[c + d*x]]] + Sqrt[2]*Sq
rt[e*Cot[c + d*x]]*Log[Sqrt[e] + Sqrt[e]*Cot[c + d*x] + Sqrt[2]*Sqrt[e*Cot
[c + d*x]]])/(16*a^3*d*e^(3/2)*Sqrt[e*Cot[c + d*x]])
```

3.39.3 Rubi [A] (warning: unable to verify)

Time = 1.70 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.760, Rules used = {3042, 4052, 27, 3042, 4132, 25, 3042, 4132, 27, 3042, 4137, 27, 3042, 4015, 221, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(c+dx)+a)^3(e\cot(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4052}$$

$$-\frac{\int -\frac{5e\cot^2(c+dx)a^2+9ea^2-4e\cot(c+dx)a^2}{2(e\cot(c+dx))^{3/2}(\cot(c+dx)a+a)^2}dx}{4a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{5e\cot^2(c+dx)a^2+9ea^2-4e\cot(c+dx)a^2}{(e\cot(c+dx))^{3/2}(\cot(c+dx)a+a)^2}dx}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{5e\tan\left(c+dx+\frac{\pi}{2}\right)^2a^2+9ea^2+4e\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \ 4132$$

$$\frac{-\frac{\int -\frac{27e^2a^4+27e^2\cot^2(c+dx)a^4-8e^2\cot(c+dx)a^4}{(e\cot(c+dx))^{3/2}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{27e^2a^4+27e^2\cot^2(c+dx)a^4-8e^2\cot(c+dx)a^4}{(e\cot(c+dx))^{3/2}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{27e^2a^4+27e^2\tan\left(c+dx+\frac{\pi}{2}\right)^2a^4+8e^2\tan\left(c+dx+\frac{\pi}{2}\right)a^4}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \ 4132$$

$$\frac{\frac{\frac{2\int -\frac{35e^4a^5+27e^4\cot^2(c+dx)a^5}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}+\frac{54a^3e}{d\sqrt{e\cot(c+dx)}}}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{54a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{35e^4a^5+27e^4\cot^2(c+dx)a^5}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \ 3042$$

3.39. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{54a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{35e^4a^5+27e^4\tan\left(c+dx+\frac{\pi}{2}\right)^2a^5}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 4137

$$\frac{\frac{\frac{54a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{31a^5e^4\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+\frac{\int \frac{8\left(a^6e^4-a^6e^4\cot(c+dx)\right)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}}{ae^3}}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{54a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{31a^5e^4\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+\frac{4\int \frac{a^6e^4-a^6e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2}}{ae^3}}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{54a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{31a^5e^4\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4\int \frac{e^4a^6+e^4\tan\left(c+dx+\frac{\pi}{2}\right)a^6}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{ae^3}}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 4015

$$\frac{\frac{\frac{54a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{31a^5e^4\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{8a^{10}e^8\int \frac{1}{2a^{12}e^8-\left(e^4a^6+e^4\cot(c+dx)a^6\right)^2\tan(c+dx)}d\frac{e^4a^6+e^4\cot(c+dx)a^6}{\sqrt{e\cot(c+dx)}}}{d}}{ae^3}}{2a^3e}-\frac{\;}{d(}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 221

3.39. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{54a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{31a^5e^4\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{4\sqrt{2}a^4e^{7/2}\text{arctanh}\left(\frac{a^6e^4\cot(c+dx)+a^6e^4}{\sqrt{2}a^6e^{7/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{ae^3}}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+}}}{8a^3e}$$

$$\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 4117$$

$$\frac{\frac{\frac{54a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{\frac{31a^5e^4\int\frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4\sqrt{2}a^4e^{7/2}\text{arctanh}\left(\frac{a^6e^4\cot(c+dx)+a^6e^4}{\sqrt{2}a^6e^{7/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{ae^3}}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+}}}{8a^3e}$$

$$\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{54a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{\frac{31a^4e^4\int\frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}-\frac{4\sqrt{2}a^4e^{7/2}\text{arctanh}\left(\frac{a^6e^4\cot(c+dx)+a^6e^4}{\sqrt{2}a^6e^{7/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{ae^3}}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+d}}}{8a^3e}$$

$$\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{54a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{-\frac{62a^4e^3\int\frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{d}-\frac{4\sqrt{2}a^4e^{7/2}\text{arctanh}\left(\frac{a^6e^4\cot(c+dx)+a^6e^4}{\sqrt{2}a^6e^{7/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{ae^3}}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}}}{8a^3e}-$$

$$\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{54a^3e}{d\sqrt{e\cot(c+dx)}}-\frac{\frac{62a^4e^{7/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{d}-\frac{4\sqrt{2}a^4e^{7/2}\text{arctanh}\left(\frac{a^6e^4\cot(c+dx)+a^6e^4}{\sqrt{2}a^6e^{7/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{ae^3}}{2a^3e}-\frac{9}{d(\cot(c+dx)+1)\sqrt{e\cot(c+dx)}}}{8a^3e}-$$

$$\frac{1}{4ade(a\cot(c+dx)+a)^2\sqrt{e\cot(c+dx)}}$$

3.39. $\int\frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx$

input
```
Int[1/((e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^3),x]
```

output
```
-1/4*1/(a*d*e*Sqrt[e*Cot[c + d*x]]*(a + a*Cot[c + d*x])^2) + (-9/(d*Sqrt[e
*Cot[c + d*x]]*(1 + Cot[c + d*x])) + (-(((62*a^4*e^(7/2)*ArcTan[Cot[c + d*
x]/Sqrt[e]])/d - (4*Sqrt[2]*a^4*e^(7/2)*ArcTanh[(a^6*e^4 + a^6*e^4*Cot[c +
 d*x])/(Sqrt[2]*a^6*e^(7/2)*Sqrt[e*Cot[c + d*x]])])/d)/(a*e^3)) + (54*a^3*
e)/(d*Sqrt[e*Cot[c + d*x]]))/(2*a^3*e))/(8*a^3*e)
```

3.39.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.39. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx$

rule 4137
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Sim
p[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*T
an[e + f*x], x], x], x] + Simp[(A*b^2 + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan
[e + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{
a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.39.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(156) = 312$.

Time = 0.05 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.93

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots\right.}{\cdots}$
default	$-\frac{2e^4\left(-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots\right.}{\cdots}$

input
```
int(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^3*e^4*(1/4/e^5*(-1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
))+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2
*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/e^5/(e*cot(d*x+c)
)^(1/2)-1/4/e^5*((11/4*(e*cot(d*x+c))^(3/2)+13/4*e*(e*cot(d*x+c))^(1/2))/(
e*cot(d*x+c)+e)^2+31/4/e^(1/2)*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))))
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(156) = 312$.

Time = 0.29 (sec) , antiderivative size = 697, normalized size of antiderivative = 3.69

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \left[-\frac{4\sqrt{2}((\cos(2\,dx+2\,c)+1)\sin(2\,dx+2\,c)+\cos(2\,dx+2\right.$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*sqrt(2)*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2
*c) + 1)*sqrt(-e)*arctan(1/2*sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e
)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x
 + 2*c) + e)) + 31*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x +
2*c) + 1)*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) - 2*sqrt(-
e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(
cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) + (45*cos(2*d*x + 2*c)^2 - (11*c
os(2*d*x + 2*c) + 43)*sin(2*d*x + 2*c) - 45)*sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c)))/(a^3*d*e^2*cos(2*d*x + 2*c) + a^3*d*e^2 + (a^3*d*e^2*c
os(2*d*x + 2*c) + a^3*d*e^2)*sin(2*d*x + 2*c)), 1/16*(2*sqrt(2)*((cos(2*d*
x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + 1)*sqrt(e)*log(-sqrt(2
)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*(cos(2*d*x + 2*c
) - sin(2*d*x + 2*c) - 1) + 2*e*sin(2*d*x + 2*c) + e) + 62*((cos(2*d*x + 2
*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + 1)*sqrt(e)*arctan(sqrt((e*c
os(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e)) - (45*cos(2*d*x + 2*c)^2 -
 (11*cos(2*d*x + 2*c) + 43)*sin(2*d*x + 2*c) - 45)*sqrt((e*cos(2*d*x + 2*c
) + e)/sin(2*d*x + 2*c)))/(a^3*d*e^2*cos(2*d*x + 2*c) + a^3*d*e^2 + (a^3*d
*e^2*cos(2*d*x + 2*c) + a^3*d*e^2)*sin(2*d*x + 2*c))]
```

3.39.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\cot^3(c+dx)+3(e\cot(c+dx))^{\frac{3}{2}}\cot^2(c+dx)+3(e\cot(c+dx))^{\frac{3}{2}}\operatorname{co}}}{a^3}$$

input
```
integrate(1/(e*cot(d*x+c))**(3/2)/(a+a*cot(d*x+c))**3,x)
```

output
```
Integral(1/((e*cot(c + d*x))**(3/2)*cot(c + d*x)**3 + 3*(e*cot(c + d*x))**
(3/2)*cot(c + d*x)**2 + 3*(e*cot(c + d*x))**(3/2)*cot(c + d*x) + (e*cot(c
+ d*x))**(3/2)), x)/a**3
```

3.39.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.39.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \int \frac{1}{(a\cot(dx+c)+a)^3\,(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2)), x)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx = \frac{\frac{27\,e\cot(c+d\,x)^2}{8}+\frac{45\,e\cot(c+d\,x)}{8}+2\,e}{a^3\,d\,(e\cot(c+d\,x))^{5/2}+2\,a^3\,d\,e\,(e\cot(c+d\,x))^{3/2}+a^3\,d\,e^2} + \frac{31\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{8\,a^3\,d\,e^{3/2}} + \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{63504384\,\sqrt{2}\,a^9\,d^3\,e^{15/2}\,\sqrt{e\cot(c+d\,x)}}{63504384\,a^9\,d^3\,e^8+63504384\,a^9\,d^3\,e^8\cot(c+d\,x)}\right)}{4\,a^3\,d\,e^{3/2}}$$

input
```
int(1/((e*cot(c + d*x))^(3/2)*(a + a*cot(c + d*x))^3),x)
```

output
```
(2*e + (45*e*cot(c + d*x))/8 + (27*e*cot(c + d*x)^2)/8)/(a^3*d*(e*cot(c +
d*x))^(5/2) + 2*a^3*d*e*(e*cot(c + d*x))^(3/2) + a^3*d*e^2*(e*cot(c + d*x)
)^(1/2)) + (31*atan((e*cot(c + d*x))^(1/2)/e^(1/2)))/(8*a^3*d*e^(3/2)) + (
2^(1/2)*atanh((63504384*2^(1/2)*a^9*d^3*e^(15/2)*(e*cot(c + d*x))^(1/2))/(
63504384*a^9*d^3*e^8 + 63504384*a^9*d^3*e^8*cot(c + d*x))))/(4*a^3*d*e^(3/
2))
```

3.39. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^3}\,dx$

3.40 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx$

3.40.1 Optimal result

Integrand size = 25, antiderivative size = 215

$$\begin{aligned}\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx &= -\frac{59\arctan\left(\frac{\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{8a^3de^{5/2}} \\ &+ \frac{\arctan\left(\frac{\sqrt{e}-\sqrt{e}\cot(c+dx)}{\sqrt{2}\sqrt{e\cot(c+dx)}}\right)}{2\sqrt{2}a^3de^{5/2}} + \frac{55}{24a^3de(e\cot(c+dx))^{3/2}} \\ &- \frac{63}{8a^3de^2\sqrt{e\cot(c+dx)}} - \frac{11}{8a^3de(e\cot(c+dx))^{3/2}(1+\cot(c+dx))} \\ &- \frac{1}{4ade(e\cot(c+dx))^{3/2}(a+a\cot(c+dx))^2}\end{aligned}$$

output
```
-59/8*arctan((e*cot(d*x+c))^(1/2)/e^(1/2))/a^3/d/e^(5/2)+55/24/a^3/d/e/(e*
cot(d*x+c))^(3/2)-11/8/a^3/d/e/(e*cot(d*x+c))^(3/2)/(1+cot(d*x+c))-1/4/a/d
/e/(e*cot(d*x+c))^(3/2)/(a+a*cot(d*x+c))^2+1/4*arctan(1/2*(e^(1/2)-cot(d*x
+c)*e^(1/2))*2^(1/2)/(e*cot(d*x+c))^(1/2))/a^3/d/e^(5/2)*2^(1/2)-63/8/a^3/
d/e^2/(e*cot(d*x+c))^(1/2)
```

3.40.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.58

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\cot(c+dx)\right)+2\,\text{Hyperge}}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3),x]
```

output
```
(Hypergeometric2F1[-3/2, 1, -1/2, -Cot[c + d*x]] + 2*Hypergeometric2F1[-3/
2, 2, -1/2, -Cot[c + d*x]] + 2*Hypergeometric2F1[-3/2, 3, -1/2, -Cot[c + d
*x]] - Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2] - 3*Cot[c + d*x]*H
ypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2])/(6*a^3*d*e*(e*Cot[c + d*x
])^(3/2))
```

3.40.3 Rubi [A] (warning: unable to verify)

Time = 2.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.13, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 4052, 27, 3042, 4132, 25, 3042, 4132, 27, 3042, 4133, 27, 3042, 4136, 27, 3042, 4015, 218, 4117, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(c+dx)+a)^3(e\cot(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4052}$$

$$-\frac{\int -\frac{7e\cot^2(c+dx)a^2+11ea^2-4e\cot(c+dx)a^2}{2(e\cot(c+dx))^{5/2}(\cot(c+dx)a+a)^2}dx}{4a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7e\cot^2(c+dx)a^2+11ea^2-4e\cot(c+dx)a^2}{(e\cot(c+dx))^{5/2}(\cot(c+dx)a+a)^2}dx}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{7e\tan\left(c+dx+\frac{\pi}{2}\right)^2a^2+11ea^2+4e\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4132$$

$$\frac{-\frac{\int -\frac{55e^2a^4+55e^2\cot^2(c+dx)a^4-8e^2\cot(c+dx)a^4}{(e\cot(c+dx))^{5/2}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{55e^2a^4+55e^2\cot^2(c+dx)a^4-8e^2\cot(c+dx)a^4}{(e\cot(c+dx))^{5/2}(\cot(c+dx)a+a)}dx}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{55e^2a^4+55e^2\tan\left(c+dx+\frac{\pi}{2}\right)^2a^4+8e^2\tan\left(c+dx+\frac{\pi}{2}\right)a^4}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4132$$

$$\frac{\frac{\frac{2\int -\frac{3\left(63e^4a^5+55e^4\cot^2(c+dx)a^5\right)}{2(e\cot(c+dx))^{3/2}(\cot(c+dx)a+a)}dx}{3ae^3}+\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\int \frac{63e^4a^5+55e^4\cot^2(c+dx)a^5}{(e\cot(c+dx))^{3/2}(\cot(c+dx)a+a)}dx}{ae^3}}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

3.40. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\int \frac{63e^4a^5+55e^4\tan\left(c+dx+\frac{\pi}{2}\right)^2a^5}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4133$$

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2\int -\frac{63a^6e^6+63a^6\cot^2(c+dx)e^6+8a^6\cot(c+dx)e^6}{2\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}+\frac{126a^4e^3}{d\sqrt{e\cot(c+dx)}}}{ae^3}}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{126a^4e^3}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{63a^6e^6+63a^6\cot^2(c+dx)e^6+8a^6\cot(c+dx)e^6}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx}{ae^3}}{ae^3}}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{126a^4e^3}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{63a^6e^6+63a^6\tan\left(c+dx+\frac{\pi}{2}\right)^2e^6-8a^6\tan\left(c+dx+\frac{\pi}{2}\right)e^6}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}}{ae^3}}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4136$$

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{126a^4e^3}{d\sqrt{e\cot(c+dx)}}-\frac{59a^6e^6\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+\frac{\int \frac{8\left(e^6a^7+e^6\cot(c+dx)a^7\right)}{\sqrt{e\cot(c+dx)}}dx}{2a^2}}{ae^3}}{ae^3}}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

3.40. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{126a^4e^3}{d\sqrt{e\cot(c+dx)}}-\frac{59a^6e^6\int\frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)a+a)}dx+\frac{4\int\frac{e^6a^7+e^6\cot(c+dx)a^7}{\sqrt{e\cot(c+dx)}}dx}{a^2}}{ae^3}}{ae^3}}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(c+dx))^{3/2}}}{8a^3e}-\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{126a^4e^3}{d\sqrt{e\cot(c+dx)}}-\frac{59a^6e^6\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4\int\frac{a^7e^6-a^7e^6\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{ae^3}}{ae^3}}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(}}{8a^3e}$$
$$\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 4015

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{126a^4e^3}{d\sqrt{e\cot(c+dx)}}-\frac{59a^6e^6\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx-\frac{8a^{12}e^{12}\int\frac{1}{-2e^{12}a^{14}-\left(a^7e^6-a^7e^6\cot(c+dx)\right)^2\tan(c+dx)}d\frac{a^7e}{}}{d}}{ae^3}}{ae^3}}{2a^3e}}{8a^3e}$$
$$\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{126a^4e^3}{d\sqrt{e\cot(c+dx)}}-\frac{59a^6e^6\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{4\sqrt{2}a^5e^{11/2}\arctan\left(\frac{a^7e^6-a^7e^6\cot(c+dx)}{\sqrt{2}a^7e^{11/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{ae^3}}{ae^3}}{2a^3e}-\frac{}{d(\cot(c+}}{8a^3e}$$
$$\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$\Big\downarrow$ 4117

3.40. $\int\frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{126a^4e^3}{d\sqrt{e\cot(c+dx)}}-\frac{\frac{59a^6e^6\int\frac{1}{a\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}+\frac{4\sqrt{2}a^5e^{11/2}\arctan\left(\frac{a^7e^6-a^7e^6\cot(c+dx)}{\sqrt{2}a^7e^{11/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{ae^3}}{ae^3}}{2a^3e}-\frac{\cdots}{d(\cot(c+\cdots}}{8a^3e}$$

$$\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{126a^4e^3}{d\sqrt{e\cot(c+dx)}}-\frac{\frac{59a^5e^6\int\frac{1}{\sqrt{e\cot(c+dx)}(\cot(c+dx)+1)}d(-\cot(c+dx))}{d}+\frac{4\sqrt{2}a^5e^{11/2}\arctan\left(\frac{a^7e^6-a^7e^6\cot(c+dx)}{\sqrt{2}a^7e^{11/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{ae^3}}{ae^3}}{2a^3e}-\frac{\cdots}{d(\cot(c+d\cdots}}{8a^3e}$$

$$\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{126a^4e^3}{d\sqrt{e\cot(c+dx)}}-\frac{\frac{4\sqrt{2}a^5e^{11/2}\arctan\left(\frac{a^7e^6-a^7e^6\cot(c+dx)}{\sqrt{2}a^7e^{11/2}\sqrt{e\cot(c+dx)}}\right)}{d}-\frac{118a^5e^5\int\frac{1}{\frac{\cot^2(c+dx)}{e}+1}d\sqrt{e\cot(c+dx)}}{d}}{ae^3}}{ae^3}}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot\cdots}}{8a^3e}$$

$$\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{\frac{110a^3e}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{126a^4e^3}{d\sqrt{e\cot(c+dx)}}-\frac{\frac{118a^5e^{11/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{e}}\right)}{d}+\frac{4\sqrt{2}a^5e^{11/2}\arctan\left(\frac{a^7e^6-a^7e^6\cot(c+dx)}{\sqrt{2}a^7e^{11/2}\sqrt{e\cot(c+dx)}}\right)}{d}}{ae^3}}{ae^3}}{2a^3e}-\frac{11}{d(\cot(c+dx)+1)(e\cot(c+dx)\cdots}}{8a^3e}$$

$$\frac{1}{4ade(a\cot(c+dx)+a)^2(e\cot(c+dx))^{3/2}}$$

input
```
Int[1/((e*Cot[c + d*x])^(5/2)*(a + a*Cot[c + d*x])^3),x]
```

3.40. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx$

output
```
-1/4*1/(a*d*e*(e*Cot[c + d*x])^(3/2)*(a + a*Cot[c + d*x])^2) + (-11/(d*(e*
Cot[c + d*x])^(3/2)*(1 + Cot[c + d*x])) + ((110*a^3*e)/(3*d*(e*Cot[c + d*x
])^(3/2)) - (-(((118*a^5*e^(11/2)*ArcTan[Cot[c + d*x]/Sqrt[e]])/d + (4*Sqr
t[2]*a^5*e^(11/2)*ArcTan[(a^7*e^6 - a^7*e^6*Cot[c + d*x])/(Sqrt[2]*a^7*e^(
11/2)*Sqrt[e*Cot[c + d*x]])])/d)/(a*e^3)) + (126*a^4*e^3)/(d*Sqrt[e*Cot[c
+ d*x]]))/(a*e^3))/(2*a^3*e))/(8*a^3*e)
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4015
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*c*d + b*x^2), x], x, (c
- d*Tan[e + f*x])/Sqrt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] &&
 EqQ[c^2 - d^2, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4133
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Sim
p[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
 + 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(
m + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m
, -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.40. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx$

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.40.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(178) = 356$.

Time = 0.05 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.76

method	result
derivativedivides	$-\frac{2e^4\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots}{\cdots}$
default	$-\frac{2e^4\left(\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots}{\cdots}$

input
```
int(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d/a^3*e^4*(1/4/e^6*(1/8/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
)+1/8/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*
arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/3/e^5/(e*cot(d*x+c
))^(3/2)+3/e^6/(e*cot(d*x+c))^(1/2)+1/4/e^6*((15/4*(e*cot(d*x+c))^(3/2)+17
/4*e*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)+e)^2+59/4/e^(1/2)*arctan((e*cot(d
*x+c))^(1/2)/e^(1/2))))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 718, normalized size of antiderivative = 3.34

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = \left[-\frac{6\sqrt{2}((\cos(2dx+2c)+1)\sin(2dx+2c)+\cos(2dx+2\right.$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/48*(6*sqrt(2)*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2
*c) + 1)*sqrt(-e)*log(sqrt(2)*sqrt(-e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2
*d*x + 2*c))*(cos(2*d*x + 2*c) + sin(2*d*x + 2*c) - 1) - 2*e*sin(2*d*x + 2
*c) + e) + 177*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)
 + 1)*sqrt(-e)*log((e*cos(2*d*x + 2*c) - e*sin(2*d*x + 2*c) + 2*sqrt(-e)*s
qrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + e)/(cos(
2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)) - (339*cos(2*d*x + 2*c)^2 - 7*(11*co
s(2*d*x + 2*c) + 43)*sin(2*d*x + 2*c) - 32*cos(2*d*x + 2*c) - 307)*sqrt((e
*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(a^3*d*e^3*cos(2*d*x + 2*c) + a^
3*d*e^3 + (a^3*d*e^3*cos(2*d*x + 2*c) + a^3*d*e^3)*sin(2*d*x + 2*c)), 1/48
*(12*sqrt(2)*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) +
 1)*sqrt(e)*arctan(-1/2*sqrt(2)*sqrt(e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(
2*d*x + 2*c))*(cos(2*d*x + 2*c) - sin(2*d*x + 2*c) + 1)/(e*cos(2*d*x + 2*c
) + e)) - 354*((cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)
+ 1)*sqrt(e)*arctan(sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))/sqrt(e
)) + (339*cos(2*d*x + 2*c)^2 - 7*(11*cos(2*d*x + 2*c) + 43)*sin(2*d*x + 2*
c) - 32*cos(2*d*x + 2*c) - 307)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
2*c)))/(a^3*d*e^3*cos(2*d*x + 2*c) + a^3*d*e^3 + (a^3*d*e^3*cos(2*d*x + 2*
c) + a^3*d*e^3)*sin(2*d*x + 2*c))]
```

3.40.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = \frac{\int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\cot^3(c+dx)+3(e\cot(c+dx))^{\frac{5}{2}}\cot^2(c+dx)+3(e\cot(c+dx))^{\frac{5}{2}}\text{co}}}{a^3}$$

input
```
integrate(1/(e*cot(d*x+c))**(5/2)/(a+a*cot(d*x+c))**3,x)
```

output
```
Integral(1/((e*cot(c + d*x))**(5/2)*cot(c + d*x)**3 + 3*(e*cot(c + d*x))**
(5/2)*cot(c + d*x)**2 + 3*(e*cot(c + d*x))**(5/2)*cot(c + d*x) + (e*cot(c
+ d*x))**(5/2)), x)/a**3
```

3.40.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.40.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = \int \frac{1}{(a\cot(dx+c)+a)^3(e\cot(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+a*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(5/2)), x)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.90

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx = -\frac{\frac{63\,e\cot(c+d\,x)^3}{8}+\frac{323\,e\cot(c+d\,x)^2}{24}+\frac{14\,e\cot(c+d\,x)}{3}-\frac{2\,e}{3}}{a^3\,d\,(e\cot(c+d\,x))^{7/2}+2\,a^3\,d\,e\,(e\cot(c+d\,x))^{5/2}+a^3\,d\,e^2\,(e\cot(c+d\,x))^{3/2}} - \frac{59\,\mathrm{atan}\left(\frac{\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{8\,a^3\,d\,e^{5/2}} - \frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e\cot(c+d\,x)}}{2\,\sqrt{e}}+\frac{\sqrt{2}\,(e\cot(c+d\,x))^{3/2}}{2\,e^{3/2}}\right)\right)}{8\,a^3\,d\,e^{5/2}}$$

input
```
int(1/((e*cot(c + d*x))^(5/2)*(a + a*cot(c + d*x))^3),x)
```

output
```
- ((14*e*cot(c + d*x))/3 - (2*e)/3 + (323*e*cot(c + d*x)^2)/24 + (63*e*cot
(c + d*x)^3)/8)/(a^3*d*(e*cot(c + d*x))^(7/2) + 2*a^3*d*e*(e*cot(c + d*x))
^(5/2) + a^3*d*e^2*(e*cot(c + d*x))^(3/2)) - (59*atan((e*cot(c + d*x))^(1/
2)/e^(1/2)))/(8*a^3*d*e^(5/2)) - (2^(1/2)*(2*atan((2^(1/2)*(e*cot(c + d*x)
)^(1/2))/(2*e^(1/2))) + 2*atan((2^(1/2)*(e*cot(c + d*x))^(1/2))/(2*e^(1/2)
) + (2^(1/2)*(e*cot(c + d*x))^(3/2))/(2*e^(3/2)))))/(8*a^3*d*e^(5/2))
```

3.40. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+a\cot(c+dx))^3}\,dx$

3.41 $\int \cot^2(x)\sqrt{1+\cot(x)}\,dx$

3.41.1 Optimal result

Integrand size = 13, antiderivative size = 223

$$\begin{aligned}\int \cot^2(x)\sqrt{1+\cot(x)}\,dx =& -\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\ &+\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\ &-\frac{2}{3}(1+\cot(x))^{3/2}\\ &+\frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}\\ &-\frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}\end{aligned}$$

output
```
-2/3*(1+cot(x))^(3/2)-1/2*arctan((-2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))
/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2*arctan((2*(1+cot(x))^(1/2)+
(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2*ln(1+co
t(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)-1/2
*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(
1/2)
```

3.41.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.31

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx = -i\sqrt{1-i}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) + i\sqrt{1+i}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - \frac{2}{3}(1+\cot(x))^{3/2}$$

input
```
Integrate[Cot[x]^2*Sqrt[1 + Cot[x]],x]
```

output
```
(-I)*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] + I*Sqrt[1 + I]*Arc
Tanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]] - (2*(1 + Cot[x])^(3/2))/3
```

3.41.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4026, 25, 3042, 3966, 483, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(x)\sqrt{\cot(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}\tan\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4026}$$

$$\int -\sqrt{\cot(x)+1}dx - \frac{2}{3}(\cot(x)+1)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$-\int \sqrt{\cot(x)+1}dx - \frac{2}{3}(\cot(x)+1)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$-\int \sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}dx-\frac{2}{3}(\cot(x)+1)^{3/2}$$

$$\Big\downarrow \ 3966$$

$$\int \frac{\sqrt{\cot(x)+1}}{\cot^2(x)+1}d\cot(x)-\frac{2}{3}(\cot(x)+1)^{3/2}$$

$$\Big\downarrow \ 483$$

$$2\int \frac{\cot(x)+1}{(\cot(x)+1)^2-2(\cot(x)+1)+2}d\sqrt{\cot(x)+1}-\frac{2}{3}(\cot(x)+1)^{3/2}$$

$$\Big\downarrow \ 1447$$

$$2\left(\frac{1}{2}\int \frac{\cot(x)+\sqrt{2}+1}{(\cot(x)+1)^2-2(\cot(x)+1)+2}d\sqrt{\cot(x)+1}-\frac{1}{2}\int \frac{-\cot(x)+\sqrt{2}-1}{(\cot(x)+1)^2-2(\cot(x)+1)+2}d\sqrt{\cot(x)+1}\right)-\frac{2}{3}(\cot(x)+1)^{3/2}$$

$$\Big\downarrow \ 1475$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}+\frac{1}{2}\int \frac{1}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+ \cdots}\right.\right. \cdots$$
$$\frac{2}{3}(\cot(x)+1)^{3/2}$$

$$\Big\downarrow \ 1083$$

$$2\left(\frac{1}{2}\left(-\int \frac{1}{-\cot(x)+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}\right)-\int \frac{1}{-\cot(x)+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{\cdots}\right.\right.\right. \cdots$$
$$\frac{2}{3}(\cot(x)+1)^{3/2}$$

$$\Big\downarrow \ 217$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{2}\int \frac{-\cot(x)+\sqrt{2}-1}{(\cot(x)+1)^2-2(\cot(x)+ \cdots}\right. \cdots$$
$$\frac{2}{3}(\cot(x)+1)^{3/2}$$

$$\Big\downarrow \ 1478$$

3.41. $\int \cot^2(x)\sqrt{1+\cot(x)}\,dx$

$$2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{2\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\int -\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\right.$$

$$\frac{2}{3}(\cot(x)+1)^{3/2}$$

$\big\downarrow$ 25

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\int \frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{2}\right.$$

$$\frac{2}{3}(\cot(x)+1)^{3/2}$$

$\big\downarrow$ 1103

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\right.}{2\sqrt{2\left(1+\right.}}\right.\right.$$

$$\frac{2}{3}(\cot(x)+1)^{3/2}$$

input
```
Int[Cot[x]^2*Sqrt[1 + Cot[x]],x]
```

output
```
(-2*(1 + Cot[x])^(3/2))/3 + 2*((ArcTan[(-Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1
+ Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(-1 + Sqrt[2])] + ArcTan[(Sqrt[2
*(1 + Sqrt[2])] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(-1 +
 Sqrt[2])])/2 + (Log[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 +
 Cot[x]]]/(2*Sqrt[2*(1 + Sqrt[2])]) - Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1
 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/(2*Sqrt[2*(1 + Sqrt[2])]))/2)
```

3.41. $\int \cot^2(x)\sqrt{1+\cot(x)}\,dx$

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

3.41.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(160) = 320$.

Time = 0.15 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.60

method	result
derivativedivides	$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{}}{\sqrt{-2+2\sqrt{}}}\right)}{2\sqrt{-2+2\sqrt{2}}}$
default	$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{}}{\sqrt{-2+2\sqrt{}}}\right)}{2\sqrt{-2+2\sqrt{2}}}$

input
```
int(cot(x)^2*(1+cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(1+cot(x))^(3/2)+1/4*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1/2)-
(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/2*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1
/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^
(1/2))-1/4*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2
^(1/2))^(1/2))-1/2*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))
^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))^(1/2)*
2^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/2*2^(1
/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(
1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/4*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(
1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/2*(2+2*2^(1/2))/(-2+2*2^(1/2)
)^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/
2))
```

3.41.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.81

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx$$
$$= \frac{3\sqrt{i-1}\log\left(i\sqrt{i-1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\sin(2x) - 3\sqrt{i-1}\log\left(-i\sqrt{i-1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\sin}{}$$

input
```
integrate(cot(x)^2*(1+cot(x))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(I - 1)*log(I*sqrt(I - 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(
2*x)))*sin(2*x) - 3*sqrt(I - 1)*log(-I*sqrt(I - 1) + sqrt((cos(2*x) + sin(
2*x) + 1)/sin(2*x)))*sin(2*x) - 3*sqrt(-I - 1)*log(I*sqrt(-I - 1) + sqrt((
cos(2*x) + sin(2*x) + 1)/sin(2*x)))*sin(2*x) + 3*sqrt(-I - 1)*log(-I*sqrt(
-I - 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x)))*sin(2*x) - 4*sqrt((cos
(2*x) + sin(2*x) + 1)/sin(2*x))*(cos(2*x) + sin(2*x) + 1))/sin(2*x)
```

3.41.6 Sympy [F]

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx = \int \sqrt{\cot(x)+1}\cot^2(x)\,dx$$

input
```
integrate(cot(x)**2*(1+cot(x))**(1/2),x)
```

output
```
Integral(sqrt(cot(x) + 1)*cot(x)**2, x)
```

3.41.7 Maxima [F]

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx = \int \sqrt{\cot(x)+1}\cot(x)^2\,dx$$

input
```
integrate(cot(x)^2*(1+cot(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cot(x) + 1)*cot(x)^2, x)
```

3.41.8 Giac [F]

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx = \int \sqrt{\cot(x)+1}\cot(x)^2\,dx$$

input
```
integrate(cot(x)^2*(1+cot(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cot(x) + 1)*cot(x)^2, x)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 12.98 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.53

$$\int \cot^2(x)\sqrt{1+\cot(x)}\,dx$$
$$= \operatorname{atanh}\left(4\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}+\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}+2\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)-\frac{2\,(\cot(x)+1)^{3/2}}{3}$$
$$+\operatorname{atanh}\left(4\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}-\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{8}-\frac{1}{8}}-2\sqrt{\frac{\sqrt{2}}{8}-\frac{1}{8}}\right)$$

input
```
int(cot(x)^2*(cot(x) + 1)^(1/2),x)
```

output
```
atanh(4*(cot(x) + 1)^(1/2)*((- 2^(1/2)/8 - 1/8)^(1/2) + (2^(1/2)/8 - 1/8)^
(1/2))^3)*(2*(- 2^(1/2)/8 - 1/8)^(1/2) + 2*(2^(1/2)/8 - 1/8)^(1/2)) - (2*(
cot(x) + 1)^(3/2))/3 + atanh(4*(cot(x) + 1)^(1/2)*((- 2^(1/2)/8 - 1/8)^(1/
2) - (2^(1/2)/8 - 1/8)^(1/2))^3)*(2*(- 2^(1/2)/8 - 1/8)^(1/2) - 2*(2^(1/2)
/8 - 1/8)^(1/2))
```

3.42 $\int \cot(x)\sqrt{1+\cot(x)}\,dx$

3.42.1 Optimal result

Integrand size = 11, antiderivative size = 135

$$\int \cot(x)\sqrt{1+\cot(x)}\,dx = \sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\arctan\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\cot(x)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right)+\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{4+3\sqrt{2}+\left(2+\sqrt{2}\right)\cot(x)}{2\sqrt{7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right)-2\sqrt{1+\cot(x)}$$

output
```
-2*(1+cot(x))^(1/2)+1/2*arctan(1/2*(4+cot(x)*(2-2^(1/2))-3*2^(1/2))/(1+cot
(x))^(1/2)/(-7+5*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)+1/2*arctanh(1/2*(4+3
*2^(1/2)+cot(x)*(2+2^(1/2)))/(1+cot(x))^(1/2)/(7+5*2^(1/2))^(1/2))*(2+2*2^
(1/2))^(1/2)
```

3.42.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \cot(x)\sqrt{1+\cot(x)}\,dx = \sqrt{1-i}\operatorname{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)+\sqrt{1+i}\operatorname{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)-2\sqrt{1+\cot(x)}$$

input
```
Integrate[Cot[x]*Sqrt[1 + Cot[x]],x]
```

output
```
Sqrt[1 - I]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] + Sqrt[1 + I]*ArcTanh[Sq
rt[1 + Cot[x]]/Sqrt[1 + I]] - 2*Sqrt[1 + Cot[x]]
```

3.42.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {3042, 25, 4011, 3042, 4019, 25, 3042, 4018, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)\sqrt{\cot(x)+1}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 25$$

$$-\int \sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 4011$$

$$-\int \frac{1-\cot(x)}{\sqrt{\cot(x)+1}}dx-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \; 3042$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)+1}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \; 4019$$

$$-\frac{\int \frac{\left(2-\sqrt{2}\right)\cot(x)+\sqrt{2}}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}-\left(2+\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\left(2-\sqrt{2}\right)\cot(x)+\sqrt{2}}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}-\left(2+\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{2}-\left(-2-\sqrt{2}\right)\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}-\left(2-\sqrt{2}\right)\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx}{2\sqrt{2}}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{4018}$$

$$\sqrt{2}\left(3-2\sqrt{2}\right)\int \frac{1}{\frac{\left(\left(2-\sqrt{2}\right)\cot(x)-3\sqrt{2}+4\right)^2}{\cot(x)+1}-4\left(7-5\sqrt{2}\right)}d\frac{\left(2-\sqrt{2}\right)\cot(x)-3\sqrt{2}+4}{\sqrt{\cot(x)+1}}+\sqrt{2}\left(3+2\sqrt{2}\right)\int \frac{1}{\frac{\left(\left(2+\sqrt{2}\right)\cot(x)+3\sqrt{2}+4\right)^2}{\cot(x)+1}-4\left(7+5\sqrt{2}\right)}d\left(-\frac{\left(2+\sqrt{2}\right)\cot(x)+3\sqrt{2}+4}{\sqrt{\cot(x)+1}}\right)-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{216}$$

$$\sqrt{2}\left(3+2\sqrt{2}\right)\int \frac{1}{\frac{\left(\left(2+\sqrt{2}\right)\cot(x)+3\sqrt{2}+4\right)^2}{\cot(x)+1}-4\left(7+5\sqrt{2}\right)}d\left(-\frac{\left(2+\sqrt{2}\right)\cot(x)+3\sqrt{2}+4}{\sqrt{\cot(x)+1}}\right)+\frac{\left(3-2\sqrt{2}\right)\arctan\left(\frac{\left(2-\sqrt{2}\right)\cot(x)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\sqrt{\cot(x)+1}}\right)}{\sqrt{2\left(5\sqrt{2}-7\right)}}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{220}$$

$$\frac{\left(3-2\sqrt{2}\right)\arctan\left(\frac{\left(2-\sqrt{2}\right)\cot(x)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\sqrt{\cot(x)+1}}\right)}{\sqrt{2\left(5\sqrt{2}-7\right)}}+\frac{\left(3+2\sqrt{2}\right)\text{arctanh}\left(\frac{\left(2+\sqrt{2}\right)\cot(x)+3\sqrt{2}+4}{2\sqrt{7+5\sqrt{2}}\sqrt{\cot(x)+1}}\right)}{\sqrt{2\left(7+5\sqrt{2}\right)}}-2\sqrt{\cot(x)+1}$$

input
```
Int[Cot[x]*Sqrt[1 + Cot[x]],x]
```

output
```
((3 - 2*Sqrt[2])*ArcTan[(4 - 3*Sqrt[2] + (2 - Sqrt[2])*Cot[x])/(2*Sqrt[-7
+ 5*Sqrt[2]]*Sqrt[1 + Cot[x]])])/Sqrt[2*(-7 + 5*Sqrt[2])] + ((3 + 2*Sqrt[2
])*ArcTanh[(4 + 3*Sqrt[2] + (2 + Sqrt[2])*Cot[x])/(2*Sqrt[7 + 5*Sqrt[2]]*S
qrt[1 + Cot[x]])])/Sqrt[2*(7 + 5*Sqrt[2])] - 2*Sqrt[1 + Cot[x]]
```

3.42. $\int \cot(x)\sqrt{1+\cot(x)}\,dx$

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4018
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*b*c*d - 4*a*d^2
+ x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0
] && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

rule 4019
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Simp[1/(2*q)   Int[(
a*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
 x], x] - Simp[1/(2*q)   Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f
*x])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d -
 b*(c^2 - d^2), 0] && NiceSqrtQ[a^2 + b^2]
```

3.42.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

method	result
derivativedivides	$-2\sqrt{1+\cot(x)}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}+\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+}}{\sqrt{-2+2\sqrt{2}}}\right.}{\sqrt{-2+2\sqrt{2}}}$
default	$-2\sqrt{1+\cot(x)}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}+\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+}}{\sqrt{-2+2\sqrt{2}}}\right.}{\sqrt{-2+2\sqrt{2}}}$

input
```
int(cot(x)*(1+cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(1+cot(x))^(1/2)-1/4*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))
^(1/2)*(2+2*2^(1/2))^(1/2))+(2^(1/2)-1)/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+
cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/4*(2+2*2^(1/2))
^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-(1-2^(1/2
))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-
2+2*2^(1/2))^(1/2))
```

3.42.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \cot(x)\sqrt{1+\cot(x)}\,dx =& \frac{1}{2}\sqrt{i+1}\log\left(\sqrt{i+1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\\ &-\frac{1}{2}\sqrt{i+1}\log\left(-\sqrt{i+1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\\ &+\frac{1}{2}\sqrt{-i+1}\log\left(\sqrt{-i+1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\\ &-\frac{1}{2}\sqrt{-i+1}\log\left(-\sqrt{-i+1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\\ &-2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\end{aligned}$$

input
```
integrate(cot(x)*(1+cot(x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(I + 1)*log(sqrt(I + 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))
) - 1/2*sqrt(I + 1)*log(-sqrt(I + 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(
2*x))) + 1/2*sqrt(-I + 1)*log(sqrt(-I + 1) + sqrt((cos(2*x) + sin(2*x) + 1
)/sin(2*x))) - 1/2*sqrt(-I + 1)*log(-sqrt(-I + 1) + sqrt((cos(2*x) + sin(2
*x) + 1)/sin(2*x))) - 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))
```

3.42.6 Sympy [F]

$$\int \cot(x)\sqrt{1+\cot(x)}\,dx = \int \sqrt{\cot(x)+1}\cot(x)\,dx$$

input
```
integrate(cot(x)*(1+cot(x))**(1/2),x)
```

output
```
Integral(sqrt(cot(x) + 1)*cot(x), x)
```

3.42.7 Maxima [F]

$$\int \cot(x)\sqrt{1+\cot(x)}\,dx = \int \sqrt{\cot(x)+1}\cot(x)\,dx$$

input
```
integrate(cot(x)*(1+cot(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cot(x) + 1)*cot(x), x)
```

3.42.8 Giac [F]

$$\int \cot(x)\sqrt{1+\cot(x)}\,dx = \int \sqrt{\cot(x)+1}\cot(x)\,dx$$

input
```
integrate(cot(x)*(1+cot(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cot(x) + 1)*cot(x), x)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 12.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.56

$$\int \cot(x)\sqrt{1+\cot(x)}\,dx = \operatorname{atanh}\left(\frac{\sqrt{\cot(x)+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}+\frac{\sqrt{\cot(x)+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}-\frac{\sqrt{2}\sqrt{\cot(x)+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}+\frac{\sqrt{2}\sqrt{\cot(x)+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}+2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right) - \operatorname{atanh}\left(\frac{\sqrt{\cot(x)+1}}{4\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}-\frac{\sqrt{\cot(x)+1}}{4\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}+\frac{\sqrt{2}\sqrt{\cot(x)+1}}{8\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}}+\frac{\sqrt{2}\sqrt{\cot(x)+1}}{8\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}}\right)\left(2\sqrt{\frac{1}{8}-\frac{\sqrt{2}}{8}}-2\sqrt{\frac{\sqrt{2}}{8}+\frac{1}{8}}\right) - 2\sqrt{\cot(x)+1}$$

input
```
int(cot(x)*(cot(x) + 1)^(1/2),x)
```

output
```
atanh((cot(x) + 1)^(1/2)/(4*(1/8 - 2^(1/2)/8)^(1/2)) + (cot(x) + 1)^(1/2)/
(4*(2^(1/2)/8 + 1/8)^(1/2)) - (2^(1/2)*(cot(x) + 1)^(1/2))/(8*(1/8 - 2^(1/
2)/8)^(1/2)) + (2^(1/2)*(cot(x) + 1)^(1/2))/(8*(2^(1/2)/8 + 1/8)^(1/2)))*(
2*(1/8 - 2^(1/2)/8)^(1/2) + 2*(2^(1/2)/8 + 1/8)^(1/2)) - atanh((cot(x) + 1
)^(1/2)/(4*(2^(1/2)/8 + 1/8)^(1/2)) - (cot(x) + 1)^(1/2)/(4*(1/8 - 2^(1/2)
/8)^(1/2)) + (2^(1/2)*(cot(x) + 1)^(1/2))/(8*(1/8 - 2^(1/2)/8)^(1/2)) + (2
^(1/2)*(cot(x) + 1)^(1/2))/(8*(2^(1/2)/8 + 1/8)^(1/2)))*(2*(1/8 - 2^(1/2)/
8)^(1/2) - 2*(2^(1/2)/8 + 1/8)^(1/2)) - 2*(cot(x) + 1)^(1/2)
```

3.43 $\int \cot^2(x)(1+\cot(x))^{3/2}\,dx$

3.43.1 Optimal result

Integrand size = 13, antiderivative size = 139

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = -\sqrt{-1+\sqrt{2}}\arctan\left(\frac{3-2\sqrt{2}+(1-\sqrt{2})\cot(x)}{\sqrt{2(-7+5\sqrt{2})}\sqrt{1+\cot(x)}}\right) - \sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{3+2\sqrt{2}+(1+\sqrt{2})\cot(x)}{\sqrt{2(7+5\sqrt{2})}\sqrt{1+\cot(x)}}\right) + 2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2}$$

output
```
-2/5*(1+cot(x))^(5/2)+2*(1+cot(x))^(1/2)-arctan((3+cot(x)*(1-2^(1/2))-2*2^
(1/2))/(1+cot(x))^(1/2)/(-14+10*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)-arctanh(
(3+2*2^(1/2)+cot(x)*(1+2^(1/2)))/(1+cot(x))^(1/2)/(14+10*2^(1/2))^(1/2))*(
1+2^(1/2))^(1/2)
```

3.43.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}} + 2\sqrt{1+\cot(x)} - \frac{2}{5}(1+\cot(x))^{5/2}$$

input
```
Integrate[Cot[x]^2*(1 + Cot[x])^(3/2),x]
```

output
```
(-2*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]])/Sqrt[1 - I] - (2*ArcTanh[Sqrt[1
 + Cot[x]]/Sqrt[1 + I]])/Sqrt[1 + I] + 2*Sqrt[1 + Cot[x]] - (2*(1 + Cot[x]
)^(5/2))/5
```

3.43.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.21, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.077, Rules used = {3042, 4026, 25, 3042, 3963, 27, 3042, 25, 4019, 25, 3042, 4018, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(x)(\cot(x)+1)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}\tan\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4026}$$

$$\int -(\cot(x)+1)^{3/2}dx-\frac{2}{5}(\cot(x)+1)^{5/2}$$

$$\Big\downarrow \text{25}$$

$$-\int (\cot(x)+1)^{3/2}dx-\frac{2}{5}(\cot(x)+1)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$-\int \left(1-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2}{5}(\cot(x)+1)^{5/2}$$

$$\Big\downarrow \text{3963}$$

$$-\int \frac{2\cot(x)}{\sqrt{\cot(x)+1}}dx-\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{\cot(x)}{\sqrt{\cot(x)+1}}dx-\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{3042}$$

$$-2\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx-\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{25}$$

$$2\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx-\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{4019}$$

$$2\left(\frac{\int -\frac{1-\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}-\frac{\int -\frac{1-\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}\right)-\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{1-\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}-\frac{\int \frac{1-\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}\right)-\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{3042}$$

$$2\left(\frac{\int \frac{1-\left(-1-\sqrt{2}\right)\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx}{2\sqrt{2}}-\frac{\int \frac{1-\left(-1+\sqrt{2}\right)\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx}{2\sqrt{2}}\right)-\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{4018}$$

$$2\left(\frac{\left(3-2\sqrt{2}\right)\int \frac{1}{\frac{\left(\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3\right)^2}{\cot(x)+1}-2\left(7-5\sqrt{2}\right)}d\left(-\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{\cot(x)+1}}\right)}{\sqrt{2}}-\frac{\left(3+2\sqrt{2}\right)\int \frac{1}{\frac{\left(\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3\right)^2}{\cot(x)+1}-2\left(7 \cdots\right.}}{\sqrt{2}} \cdots\right.$$

$$\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{216}$$

3.43. $\int \cot^2(x)(1+\cot(x))^{3/2}\,dx$

$$2\left(-\frac{\left(3+2\sqrt{2}\right)\int \frac{1}{\frac{\left(\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3\right)^2}{\cot(x)+1}-2\left(7+5\sqrt{2}\right)}d\left(-\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{\cot(x)+1}}\right)}{\sqrt{2}}-\frac{\left(3-2\sqrt{2}\right)\arctan\left(\frac{\left(1-\sqrt{2}\right)\cot(x)-2\cdots}{\sqrt{2\left(5\sqrt{2}-7\right)}\sqrt{\cot\cdots}}\right)}{2\sqrt{5\sqrt{2}-7}}\right.$$

$$\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}$$

$\big\downarrow$ 220

$$2\left(-\frac{\left(3-2\sqrt{2}\right)\arctan\left(\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\sqrt{\cot(x)+1}}\right)}{2\sqrt{5\sqrt{2}-7}}-\frac{\left(3+2\sqrt{2}\right)\text{arctanh}\left(\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{\cot(x)+1}}\right)}{2\sqrt{7+5\sqrt{2}}}\right)-$$

$$\frac{2}{5}(\cot(x)+1)^{5/2}+2\sqrt{\cot(x)+1}$$

input
```
Int[Cot[x]^2*(1 + Cot[x])^(3/2),x]
```

output
```
2*(-1/2*((3 - 2*Sqrt[2])*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Cot[x])/(Sq
rt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])])/Sqrt[-7 + 5*Sqrt[2]] - ((3 + 2*
Sqrt[2])*ArcTanh[(3 + 2*Sqrt[2] + (1 + Sqrt[2])*Cot[x])/(Sqrt[2*(7 + 5*Sqr
t[2])]*Sqrt[1 + Cot[x]])])/(2*Sqrt[7 + 5*Sqrt[2]])) + 2*Sqrt[1 + Cot[x]] -
 (2*(1 + Cot[x])^(5/2))/5
```

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3963
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

rule 4018
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*b*c*d - 4*a*d^2
+ x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0
] && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

rule 4019
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Simp[1/(2*q)   Int[(
a*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
 x], x] - Simp[1/(2*q)   Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f
*x])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d -
 b*(c^2 - d^2), 0] && NiceSqrtQ[a^2 + b^2]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

3.43. $\int \cot^2(x)(1+\cot(x))^{3/2}\,dx$

3.43.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.42

method	result
derivativedivides	$-\frac{2(1+\cot(x))^{\frac{5}{2}}}{5}+2\sqrt{1+\cot(x)}-\frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(1-\sqrt{2}\right)\arctan\left(\frac{2\sqrt{}}{}\right)}{\sqrt{-}}\right.}{2}$
default	$-\frac{2(1+\cot(x))^{\frac{5}{2}}}{5}+2\sqrt{1+\cot(x)}-\frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(1-\sqrt{2}\right)\arctan\left(\frac{2\sqrt{}}{}\right)}{\sqrt{-}}\right.}{2}$

input
```
int(cot(x)^2*(1+cot(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(1+cot(x))^(5/2)+2*(1+cot(x))^(1/2)-1/2*2^(1/2)*(-1/2*(2+2*2^(1/2))^(
1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(1-2^(1/2
))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-
2+2*2^(1/2))^(1/2)))-1/2*2^(1/2)*(1/2*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1
/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/
2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

3.43.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.45

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx =$$

$$-\frac{5\sqrt{2i+2}(\cos(2x)-1)\log\left(-(i-1)\sqrt{2i+2}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)-5\sqrt{2i+2}(\cos(2x)-1)\log\left((i\right.}{}$$

input
```
integrate(cot(x)^2*(1+cot(x))^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(5*sqrt(2*I + 2)*(cos(2*x) - 1)*log(-(I - 1)*sqrt(2*I + 2) + 2*sqrt(
(cos(2*x) + sin(2*x) + 1)/sin(2*x))) - 5*sqrt(2*I + 2)*(cos(2*x) - 1)*log(
(I - 1)*sqrt(2*I + 2) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + 5*sq
rt(-2*I + 2)*(cos(2*x) - 1)*log((I + 1)*sqrt(-2*I + 2) + 2*sqrt((cos(2*x)
+ sin(2*x) + 1)/sin(2*x))) - 5*sqrt(-2*I + 2)*(cos(2*x) - 1)*log(-(I + 1)*
sqrt(-2*I + 2) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) - 4*sqrt((cos
(2*x) + sin(2*x) + 1)/sin(2*x))*(5*cos(2*x) + 2*sin(2*x) - 3))/(cos(2*x) -
 1)
```

3.43.6 Sympy [F]

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = \int (\cot(x)+1)^{\frac{3}{2}}\cot^2(x)\,dx$$

input
```
integrate(cot(x)**2*(1+cot(x))**(3/2),x)
```

output
```
Integral((cot(x) + 1)**(3/2)*cot(x)**2, x)
```

3.43.7 Maxima [F(-1)]

Timed out.

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(x)^2*(1+cot(x))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.43.8 Giac [F]

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = \int (\cot(x)+1)^{\frac{3}{2}}\cot(x)^2\,dx$$

input
```
integrate(cot(x)^2*(1+cot(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((cot(x) + 1)^(3/2)*cot(x)^2, x)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 12.86 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.83

$$\int \cot^2(x)(1+\cot(x))^{3/2}\,dx = \operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64} - \frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\,\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}-64}\right)\left(\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,2\mathrm{i}+\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\,2\mathrm{i}\right) - \operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}+64} + \frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\,\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}+64}\right)\left(\sqrt{\frac{1}{4}-\frac{\sqrt{2}}{4}}\,2\mathrm{i}-\sqrt{\frac{\sqrt{2}}{4}+\frac{1}{4}}\,2\mathrm{i}\right) + 2\,\sqrt{\cot(x)+1} - \frac{2\,(\cot(x)+1)^{5/2}}{5}$$

input
```
int(cot(x)^2*(cot(x) + 1)^(3/2),x)
```

output
```
atan((2^(1/2)*(1/4 - 2^(1/2)/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(1/4 -
2^(1/2)/4)^(1/2)*(2^(1/2)/4 + 1/4)^(1/2) - 64) - (2^(1/2)*(2^(1/2)/4 + 1/4
)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(1/4 - 2^(1/2)/4)^(1/2)*(2^(1/2)/4 +
1/4)^(1/2) - 64))*((1/4 - 2^(1/2)/4)^(1/2)*2i + (2^(1/2)/4 + 1/4)^(1/2)*2i
) - atan((2^(1/2)*(1/4 - 2^(1/2)/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(1/
4 - 2^(1/2)/4)^(1/2)*(2^(1/2)/4 + 1/4)^(1/2) + 64) + (2^(1/2)*(2^(1/2)/4 +
 1/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(1/4 - 2^(1/2)/4)^(1/2)*(2^(1/2)/
4 + 1/4)^(1/2) + 64))*((1/4 - 2^(1/2)/4)^(1/2)*2i - (2^(1/2)/4 + 1/4)^(1/2
)*2i) + 2*(cot(x) + 1)^(1/2) - (2*(cot(x) + 1)^(5/2))/5
```

3.43. $\int \cot^2(x)(1+\cot(x))^{3/2}\,dx$

3.44 $\int \cot(x)(1+\cot(x))^{3/2}\,dx$

3.44.1 Optimal result

Integrand size = 11, antiderivative size = 221

$$\begin{aligned}\int \cot(x)(1+\cot(x))^{3/2}\,dx =& -\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\
&+\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)-2\sqrt{1+\cot(x)}\\
&-\frac{2}{3}(1+\cot(x))^{3/2}-\frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}}\\
&+\frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{2\sqrt{1+\sqrt{2}}}\end{aligned}$$

output
```
-2/3*(1+cot(x))^(3/2)-2*(1+cot(x))^(1/2)-1/2*ln(1+cot(x)+2^(1/2)-(1+cot(x)
)^(1/2)*(2+2*2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)+1/2*ln(1+cot(x)+2^(1/2)+(1+
cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)-arctan((-2*(1+cot(x))
^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)+arctan
((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2)
)^(1/2)
```

3.44.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.30

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = (1-i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) + (1+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - \frac{2}{3}\sqrt{1+\cot(x)}(4+\cot(x))$$

input
```
Integrate[Cot[x]*(1 + Cot[x])^(3/2),x]
```

output
```
(1 - I)^(3/2)*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] + (1 + I)^(3/2)*ArcTan
h[Sqrt[1 + Cot[x]]/Sqrt[1 + I]] - (2*Sqrt[1 + Cot[x]]*(4 + Cot[x]))/3
```

3.44.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.364$, Rules used = {3042, 25, 4011, 3042, 4011, 27, 3042, 3966, 484, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(x)(\cot(x)+1)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\left(1-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(1-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4011}$$

$$-\int (1-\cot(x))\sqrt{\cot(x)+1}dx - \frac{2}{3}(\cot(x)+1)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$-\int \sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}\left(\tan\left(x+\frac{\pi}{2}\right)+1\right)dx-\frac{2}{3}(\cot(x)+1)^{3/2}$$

$$\Big\downarrow \text{4011}$$

$$-\int \frac{2}{\sqrt{\cot(x)+1}}dx-\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{1}{\sqrt{\cot(x)+1}}dx-\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{3042}$$

$$-2\int \frac{1}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx-\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{3966}$$

$$2\int \frac{1}{\sqrt{\cot(x)+1}\,(\cot^2(x)+1)}d\cot(x)-\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{484}$$

$$4\int \frac{1}{(\cot(x)+1)^2-2(\cot(x)+1)+2}d\sqrt{\cot(x)+1}-\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{1407}$$

$$4\left(\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{4\sqrt{1+\sqrt{2}}}+\frac{\int \frac{\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{4\sqrt{1+\sqrt{2}}}\right)-$$
$$\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{1142}$$

$$4\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}-\frac{1}{2}\int -\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x}}{4\sqrt{1+\sqrt{2}}}\right.$$
$$\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}$$

3.44. $\int \cot(x)(1+\cot(x))^{3/2}\,dx$

$$\Big\downarrow \ 25$$

$$4\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}+\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)}}{4\sqrt{1+\sqrt{2}}}\right.$$

$$\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \ 1083$$

$$4\left(\frac{\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-\cot(x)+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{\cot(x)+1}-\sqrt{2}\right.}{4\sqrt{1+\sqrt{2}}}\right.$$

$$\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \ 217$$

$$4\left(\frac{\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}+\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\frac{1}{2}\int \frac{2\sqrt{\cot}}{\cot(x)+\sqrt{2(}}}{}\right.$$

$$\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \ 1103$$

$$4\left(\frac{\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{2}\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\text{a}}{}\right.$$

$$\frac{2}{3}(\cot(x)+1)^{3/2}-2\sqrt{\cot(x)+1}$$

3.44. $\int \cot(x)(1+\cot(x))^{3/2}\,dx$

input
```
Int[Cot[x]*(1 + Cot[x])^(3/2),x]
```

output
```
-2*Sqrt[1 + Cot[x]] - (2*(1 + Cot[x])^(3/2))/3 + 4*((Sqrt[(1 + Sqrt[2])/(-
1 + Sqrt[2])]*ArcTan[(-Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*
(-1 + Sqrt[2])]] - Log[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1
 + Cot[x]]]/2)/(4*Sqrt[1 + Sqrt[2]]) + (Sqrt[(1 + Sqrt[2])/(-1 + Sqrt[2])]
*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])
]] + Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/2)
/(4*Sqrt[1 + Sqrt[2]]))
```

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 484
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[2*
d   Subst[Int[1/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

3.44.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(159) = 318$.

Time = 0.04 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.05

method	result
derivativedivides	$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\cot(x)}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}-\frac{\sqrt{2+2\sqrt{2}}\,\ln}{}$
default	$-\frac{2(1+\cot(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\cot(x)}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}-\frac{\sqrt{2+2\sqrt{2}}\,\ln}{}$

input
```
int(cot(x)*(1+cot(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(1+cot(x))^(3/2)-2*(1+cot(x))^(1/2)+1/4*(2+2*2^(1/2))^(1/2)*2^(1/2)*l
n(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/2*(2+2*2^(1/2))
^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/2*2^(1/
2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1
/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arcta
n((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+2/(-2+2*2
^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2
))^(1/2))*2^(1/2)-1/4*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1/2)+(1+c
ot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/2*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1
/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/2*2^(1/2)*(2+2*2^(1/2))/(-2+2*
2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/
2))^(1/2))-(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(
2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+2/(-2+2*2^(1/2))^(1/2)*arctan((2
*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
```

3.44.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.86

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = 3\sqrt{2i-2}\log\left(-(i-1)\sqrt{2i-2}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\sin(2x) - 3\sqrt{2i-2}\log\left((i-1)\right.$$

input
```
integrate(cot(x)*(1+cot(x))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(2*I - 2)*log(-(I - 1)*sqrt(2*I - 2) + 2*sqrt((cos(2*x) + sin(2
*x) + 1)/sin(2*x)))*sin(2*x) - 3*sqrt(2*I - 2)*log((I - 1)*sqrt(2*I - 2) +
 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x)))*sin(2*x) + 3*sqrt(-2*I - 2)*l
og((I + 1)*sqrt(-2*I - 2) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x)))*si
n(2*x) - 3*sqrt(-2*I - 2)*log(-(I + 1)*sqrt(-2*I - 2) + 2*sqrt((cos(2*x) +
 sin(2*x) + 1)/sin(2*x)))*sin(2*x) - 4*sqrt((cos(2*x) + sin(2*x) + 1)/sin(
2*x))*(cos(2*x) + 4*sin(2*x) + 1))/sin(2*x)
```

3.44.6 Sympy [F]

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = \int (\cot(x)+1)^{\frac{3}{2}}\cot(x)\,dx$$

input
```
integrate(cot(x)*(1+cot(x))**(3/2),x)
```

output
```
Integral((cot(x) + 1)**(3/2)*cot(x), x)
```

3.44.7 Maxima [F]

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = \int (\cot(x)+1)^{\frac{3}{2}}\cot(x)\,dx$$

input
```
integrate(cot(x)*(1+cot(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((cot(x) + 1)^(3/2)*cot(x), x)
```

3.44.8 Giac [F]

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = \int (\cot(x)+1)^{\frac{3}{2}}\cot(x)\,dx$$

input
```
integrate(cot(x)*(1+cot(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((cot(x) + 1)^(3/2)*cot(x), x)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 12.59 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.15

$$\int \cot(x)(1+\cot(x))^{3/2}\,dx = -\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64} - \frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}\right)\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}+\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}\right) + \operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64} + \frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{\cot(x)+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64}\right)\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}-\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}\right) - 2\sqrt{\cot(x)+1} - \frac{2\,(\cot(x)+1)^{3/2}}{3}$$

input
```
int(cot(x)*(cot(x) + 1)^(3/2),x)
```

output
```
atan((2^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(2^(1
/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) + 64) + (2^(1/2)*(2^(1/2)/4 -
 1/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2
)/4 - 1/4)^(1/2) + 64))*((- 2^(1/2)/4 - 1/4)^(1/2)*2i - (2^(1/2)/4 - 1/4)^
(1/2)*2i) - atan((2^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2)*(cot(x) + 1)^(1/2)*64i
)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64) - (2^(1/2)*
(2^(1/2)/4 - 1/4)^(1/2)*(cot(x) + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/
2)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64))*((- 2^(1/2)/4 - 1/4)^(1/2)*2i + (2^(1/
2)/4 - 1/4)^(1/2)*2i) - 2*(cot(x) + 1)^(1/2) - (2*(cot(x) + 1)^(3/2))/3
```

3.45 $\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx$

3.45.1 Optimal result

Integrand size = 13, antiderivative size = 214

$$\begin{aligned}\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx =& -\frac{1}{2}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\ &+\frac{1}{2}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\ &-2\sqrt{1+\cot(x)}\\ &-\frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}}\\ &+\frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{1+\sqrt{2}}}\end{aligned}$$

output
```
-2*(1+cot(x))^(1/2)-1/4*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))
^(1/2))/(1+2^(1/2))^(1/2)+1/4*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^
(1/2))^(1/2))/(1+2^(1/2))^(1/2)-1/2*arctan((-2*(1+cot(x))^(1/2)+(2+2*2^(1/
2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)+1/2*arctan((2*(1+cot(x)
)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)
```

3.45.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.31

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = \frac{1}{2}(1-i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) + \frac{1}{2}(1+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - 2\sqrt{1+\cot(x)}$$

input
```
Integrate[Cot[x]^2/Sqrt[1 + Cot[x]],x]
```

output
```
((1 - I)^(3/2)*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]])/2 + ((1 + I)^(3/2)*A
rcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]])/2 - 2*Sqrt[1 + Cot[x]]
```

3.45.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {3042, 4026, 25, 3042, 3966, 484, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(x)}{\sqrt{\cot(x)+1}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan\left(x+\frac{\pi}{2}\right)^2}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 4026$$

$$\int -\frac{1}{\sqrt{\cot(x)+1}}dx - 2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \ 25$$

$$-\int \frac{1}{\sqrt{\cot(x)+1}}dx - 2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{1}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{3966}$$

$$\int \frac{1}{\sqrt{\cot(x)+1}\left(\cot^2(x)+1\right)}d\cot(x)-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{484}$$

$$2\int \frac{1}{(\cot(x)+1)^2-2(\cot(x)+1)+2}d\sqrt{\cot(x)+1}-2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{1407}$$

$$2\left(\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{4\sqrt{1+\sqrt{2}}}+\frac{\int \frac{\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{4\sqrt{1+\sqrt{2}}}\right)-$$
$$2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{1142}$$

$$2\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}-\frac{1}{2}\int -\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x}}{4\sqrt{1+\sqrt{2}}}\right.$$
$$2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}+\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)}}{4\sqrt{1+\sqrt{2}}}\right.$$
$$2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{1083}$$

3.45. $\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx$

$$2\left(\frac{\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-\cot(x)+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{\cot(x)+1}-\sqrt{2\right.}{4\sqrt{1+\sqrt{2}}}\right.$$

$$2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}+\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\frac{1}{2}\int \frac{2\sqrt{\cot}}{\cot(x)+\sqrt{2(}}}{}\right.$$

$$2\sqrt{\cot(x)+1}$$

$$\Big\downarrow \text{1103}$$

$$2\left(\frac{\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{2}\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\text{a}}{}\right.$$

$$2\sqrt{\cot(x)+1}$$

input
```
Int[Cot[x]^2/Sqrt[1 + Cot[x]],x]
```

output
```
-2*Sqrt[1 + Cot[x]] + 2*((Sqrt[(1 + Sqrt[2])/(-1 + Sqrt[2])]*ArcTan[(-Sqrt
[2*(1 + Sqrt[2])] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]] - Log[1 +
Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/2)/(4*Sqrt[1 +
Sqrt[2]]) + (Sqrt[(1 + Sqrt[2])/(-1 + Sqrt[2])]*ArcTan[(Sqrt[2*(1 + Sqrt[2
])] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]] + Log[1 + Sqrt[2] + Cot[
x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/2)/(4*Sqrt[1 + Sqrt[2]]))
```

3.45. $\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx$

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 484
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[2*
d   Subst[Int[1/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

3.45.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(152) = 304$.

Time = 0.04 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.07

method	result
derivativedivides	$-2\sqrt{1+\cot(x)}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\right.}{8}$
default	$-2\sqrt{1+\cot(x)}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{4}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\right.}{8}$

input
```
int(cot(x)^2/(1+cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(1+cot(x))^(1/2)-1/4*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))
^(1/2)*(2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^
(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+
2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(
1/2))^(1/2))-1/2*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(
1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/(-2+2*2^(1/2))^(1/2)*arc
tan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
+1/4*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2)
)^(1/2))-1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1
/2)*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*ar
ctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/2*(2
+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^
(1/2))/(-2+2*2^(1/2))^(1/2))+1/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(
1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
```

3.45.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.83

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = \frac{1}{4}\sqrt{2}\sqrt{i-1}\log\left(-(i-1)\sqrt{2}\sqrt{i-1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) - \frac{1}{4}\sqrt{2}\sqrt{i-1}\log\left((i-1)\sqrt{2}\sqrt{i-1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) + \frac{1}{4}\sqrt{2}\sqrt{-i-1}\log\left((i+1)\sqrt{2}\sqrt{-i-1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) - \frac{1}{4}\sqrt{2}\sqrt{-i-1}\log\left(-(i+1)\sqrt{2}\sqrt{-i-1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right) - 2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}$$

input
```
integrate(cot(x)^2/(1+cot(x))^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*sqrt(I - 1)*log(-(I - 1)*sqrt(2)*sqrt(I - 1) + 2*sqrt((cos(2*x
) + sin(2*x) + 1)/sin(2*x))) - 1/4*sqrt(2)*sqrt(I - 1)*log((I - 1)*sqrt(2)
*sqrt(I - 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + 1/4*sqrt(2)*s
qrt(-I - 1)*log((I + 1)*sqrt(2)*sqrt(-I - 1) + 2*sqrt((cos(2*x) + sin(2*x)
 + 1)/sin(2*x))) - 1/4*sqrt(2)*sqrt(-I - 1)*log(-(I + 1)*sqrt(2)*sqrt(-I -
 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) - 2*sqrt((cos(2*x) + sin
(2*x) + 1)/sin(2*x))
```

3.45.6 Sympy [F]

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = \int \frac{\cot^2(x)}{\sqrt{\cot(x)+1}}\,dx$$

input
```
integrate(cot(x)**2/(1+cot(x))**(1/2),x)
```

output
```
Integral(cot(x)**2/sqrt(cot(x) + 1), x)
```

3.45.7 Maxima [F]

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = \int \frac{\cot(x)^2}{\sqrt{\cot(x)+1}}\,dx$$

input
```
integrate(cot(x)^2/(1+cot(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(x)^2/sqrt(cot(x) + 1), x)
```

3.45.8 Giac [F]

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = \int \frac{\cot(x)^2}{\sqrt{\cot(x)+1}}\,dx$$

input
```
integrate(cot(x)^2/(1+cot(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(x)^2/sqrt(cot(x) + 1), x)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.11

$$\int \frac{\cot^2(x)}{\sqrt{1+\cot(x)}}\,dx = \operatorname{atanh}\left(\frac{16\sqrt{2}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}-8} - \frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}-8}\right)\left(2\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}} + 2\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\right) - \operatorname{atanh}\left(\frac{16\sqrt{2}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}+8} + \frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}}+8}\right)\left(2\sqrt{-\frac{\sqrt{2}}{16}-\frac{1}{16}} - 2\sqrt{\frac{\sqrt{2}}{16}-\frac{1}{16}}\right) - 2\sqrt{\cot(x)+1}$$

input
```
int(cot(x)^2/(cot(x) + 1)^(1/2),x)
```

output
```
atanh((16*2^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(2^
(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) - 8) - (16*2^(1/2)*(2^(
1/2)/16 - 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(2^(1/2)/16 - 1/16)^(1/2)*(
- 2^(1/2)/16 - 1/16)^(1/2) - 8))*(2*(- 2^(1/2)/16 - 1/16)^(1/2) + 2*(2^(1/
2)/16 - 1/16)^(1/2)) - atanh((16*2^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2)*(cot(
x) + 1)^(1/2))/(128*(2^(1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2)
+ 8) + (16*2^(1/2)*(2^(1/2)/16 - 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(2^(
1/2)/16 - 1/16)^(1/2)*(- 2^(1/2)/16 - 1/16)^(1/2) + 8))*(2*(- 2^(1/2)/16 -
 1/16)^(1/2) - 2*(2^(1/2)/16 - 1/16)^(1/2)) - 2*(cot(x) + 1)^(1/2)
```

3.46 $\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx$

3.46.1 Optimal result

Integrand size = 11, antiderivative size = 121

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \frac{1}{2}\sqrt{-1+\sqrt{2}}\arctan\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right) + \frac{1}{2}\sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{3+2\sqrt{2}+\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right)$$

output
```
1/2*arctan((3+cot(x)*(1-2^(1/2))-2*2^(1/2))/(1+cot(x))^(1/2)/(-14+10*2^(1/
2))^(1/2))*(2^(1/2)-1)^(1/2)+1/2*arctanh((3+2*2^(1/2)+cot(x)*(1+2^(1/2)))/
(1+cot(x))^(1/2)/(14+10*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)
```

3.46.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right)}{\sqrt{1-i}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right)}{\sqrt{1+i}}$$

input
```
Integrate[Cot[x]/Sqrt[1 + Cot[x]],x]
```

output
```
ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]]/Sqrt[1 - I] + ArcTanh[Sqrt[1 + Cot[x
]]/Sqrt[1 + I]]/Sqrt[1 + I]
```

3.46.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3042, 25, 4019, 25, 3042, 4018, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sqrt{\cot(x)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4019}$$

$$\frac{\int -\frac{1-\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}-\frac{\int -\frac{1-\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{1-\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}-\frac{\int \frac{1-\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1-\left(-1+\sqrt{2}\right)\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx}{2\sqrt{2}}-\frac{\int \frac{1-\left(-1-\sqrt{2}\right)\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx}{2\sqrt{2}}$$

$$\Big\downarrow \text{4018}$$

$$\frac{(3+2\sqrt{2})\int \frac{1}{\frac{\left(\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3\right)^2}{\cot(x)+1}-2\left(7+5\sqrt{2}\right)}d\left(-\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{\cot(x)+1}}\right)}{\sqrt{2}}-\frac{(3-2\sqrt{2})\int \frac{1}{\frac{\left(\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3\right)^2}{\cot(x)+1}-2\left(7-5\sqrt{2}\right)}d\left(-\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{\cot(x)+1}}\right)}{\sqrt{2}}$$

$$\Big\downarrow \ 216$$

$$\frac{(3+2\sqrt{2})\int \frac{1}{\frac{\left(\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3\right)^2}{\cot(x)+1}-2\left(7+5\sqrt{2}\right)}d\left(-\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{\cot(x)+1}}\right)}{\sqrt{2}}+\frac{(3-2\sqrt{2})\arctan\left(\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\sqrt{\cot(x)+1}}\right)}{2\sqrt{5\sqrt{2}-7}}$$

$$\Big\downarrow \ 220$$

$$\frac{(3-2\sqrt{2})\arctan\left(\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\sqrt{\cot(x)+1}}\right)}{2\sqrt{5\sqrt{2}-7}}+\frac{(3+2\sqrt{2})\text{arctanh}\left(\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{\cot(x)+1}}\right)}{2\sqrt{7+5\sqrt{2}}}$$

input
```
Int[Cot[x]/Sqrt[1 + Cot[x]],x]
```

output
```
((3 - 2*Sqrt[2])*ArcTan[(3 - 2*Sqrt[2] + (1 - Sqrt[2])*Cot[x])/(Sqrt[2*(-7
 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])])/(2*Sqrt[-7 + 5*Sqrt[2]]) + ((3 + 2*Sqrt
[2])*ArcTanh[(3 + 2*Sqrt[2] + (1 + Sqrt[2])*Cot[x])/(Sqrt[2*(7 + 5*Sqrt[2]
)]*Sqrt[1 + Cot[x]])])/(2*Sqrt[7 + 5*Sqrt[2]])
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.46. $\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx$

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4018
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*b*c*d - 4*a*d^2
+ x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0
] && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

rule 4019
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Simp[1/(2*q)   Int[(
a*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
 x], x] - Simp[1/(2*q)   Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f
*x])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d -
 b*(c^2 - d^2), 0] && NiceSqrtQ[a^2 + b^2]
```

3.46.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(85) = 170$.

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(1-\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}\right)}{4}+\sqrt{2}\left(\sqrt{2+2\sqrt{2}}\,\ln\left(\right.\right.$
default	$\frac{\sqrt{2}\left(-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(1-\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}\right)}{4}+\sqrt{2}\left(\sqrt{2+2\sqrt{2}}\,\ln\left(\right.\right.$

input
```
int(cot(x)/(1+cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*(-1/2*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)
*(2+2*2^(1/2))^(1/2))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(
x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))+1/4*2^(1/2)*(1/2*(2+
2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))
+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2
))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

3.46.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.30

$$\begin{aligned}\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx &= \frac{1}{4}\sqrt{2}\sqrt{i+1}\log\left(-(i-1)\sqrt{2}\sqrt{i+1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\\ &\quad-\frac{1}{4}\sqrt{2}\sqrt{i+1}\log\left((i-1)\sqrt{2}\sqrt{i+1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\\ &\quad+\frac{1}{4}\sqrt{2}\sqrt{-i+1}\log\left((i+1)\sqrt{2}\sqrt{-i+1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\\ &\quad-\frac{1}{4}\sqrt{2}\sqrt{-i+1}\log\left(-(i+1)\sqrt{2}\sqrt{-i+1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)\end{aligned}$$

input
```
integrate(cot(x)/(1+cot(x))^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*sqrt(I + 1)*log(-(I - 1)*sqrt(2)*sqrt(I + 1) + 2*sqrt((cos(2*x
) + sin(2*x) + 1)/sin(2*x))) - 1/4*sqrt(2)*sqrt(I + 1)*log((I - 1)*sqrt(2)
*sqrt(I + 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + 1/4*sqrt(2)*s
qrt(-I + 1)*log((I + 1)*sqrt(2)*sqrt(-I + 1) + 2*sqrt((cos(2*x) + sin(2*x)
 + 1)/sin(2*x))) - 1/4*sqrt(2)*sqrt(-I + 1)*log(-(I + 1)*sqrt(2)*sqrt(-I +
 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x)))
```

3.46.6 Sympy [F]

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \int \frac{\cot{(x)}}{\sqrt{\cot{(x)}+1}}\,dx$$

input
```
integrate(cot(x)/(1+cot(x))**(1/2),x)
```

output
```
Integral(cot(x)/sqrt(cot(x) + 1), x)
```

3.46.7 Maxima [F]

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \int \frac{\cot{(x)}}{\sqrt{\cot{(x)}+1}}\,dx$$

input
```
integrate(cot(x)/(1+cot(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(x)/sqrt(cot(x) + 1), x)
```

3.46.8 Giac [F]

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \int \frac{\cot{(x)}}{\sqrt{\cot{(x)}+1}}\,dx$$

input
```
integrate(cot(x)/(1+cot(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(x)/sqrt(cot(x) + 1), x)
```

3.46.9 Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.90

$$\int \frac{\cot(x)}{\sqrt{1+\cot(x)}}\,dx = \operatorname{atanh}\left(\frac{16\sqrt{2}\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}-8} - \frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}-8}\right)\left(2\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}+2\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}\right) - \operatorname{atanh}\left(\frac{16\sqrt{2}\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}+8} + \frac{16\sqrt{2}\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}\sqrt{\cot(x)+1}}{128\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}+8}\right)\left(2\sqrt{\frac{1}{16}-\frac{\sqrt{2}}{16}}-2\sqrt{\frac{\sqrt{2}}{16}+\frac{1}{16}}\right)$$

input
```
int(cot(x)/(cot(x) + 1)^(1/2),x)
```

output
```
atanh((16*2^(1/2)*(1/16 - 2^(1/2)/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16
 - 2^(1/2)/16)^(1/2)*(2^(1/2)/16 + 1/16)^(1/2) - 8) - (16*2^(1/2)*(2^(1/2)
/16 + 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16 - 2^(1/2)/16)^(1/2)*(2^(1
/2)/16 + 1/16)^(1/2) - 8))*(2*(1/16 - 2^(1/2)/16)^(1/2) + 2*(2^(1/2)/16 +
1/16)^(1/2)) - atanh((16*2^(1/2)*(1/16 - 2^(1/2)/16)^(1/2)*(cot(x) + 1)^(1
/2))/(128*(1/16 - 2^(1/2)/16)^(1/2)*(2^(1/2)/16 + 1/16)^(1/2) + 8) + (16*2
^(1/2)*(2^(1/2)/16 + 1/16)^(1/2)*(cot(x) + 1)^(1/2))/(128*(1/16 - 2^(1/2)/
16)^(1/2)*(2^(1/2)/16 + 1/16)^(1/2) + 8))*(2*(1/16 - 2^(1/2)/16)^(1/2) - 2
*(2^(1/2)/16 + 1/16)^(1/2))
```

3.47 $\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx$

3.47.1 Optimal result

Integrand size = 13, antiderivative size = 139

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}\left(-1+\sqrt{2}\right)}\arctan\left(\frac{4-3\sqrt{2}+\left(2-\sqrt{2}\right)\cot(x)}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{4+3\sqrt{2}+\left(2+\sqrt{2}\right)\cot(x)}{2\sqrt{7+5\sqrt{2}}\sqrt{1+\cot(x)}}\right) + \frac{1}{\sqrt{1+\cot(x)}}$$

output
```
1/(1+cot(x))^(1/2)+1/4*arctan(1/2*(4+cot(x)*(2-2^(1/2))-3*2^(1/2))/(1+cot(
x))^(1/2)/(-7+5*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)+1/4*arctanh(1/2*(4+3*
2^(1/2)+cot(x)*(2+2^(1/2)))/(1+cot(x))^(1/2)/(7+5*2^(1/2))^(1/2))*(2+2*2^(
1/2))^(1/2)
```

3.47.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.45

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \frac{4-(1+i)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\left(\frac{1}{2}-\frac{i}{2}\right)(1+\cot(x))\right)-(1-i)\,\text{Hypergeo}}{2\sqrt{1+\cot(x)}}$$

input
```
Integrate[Cot[x]^2/(1 + Cot[x])^(3/2),x]
```

output
```
(4 - (1 + I)*Hypergeometric2F1[-1/2, 1, 1/2, (1/2 - I/2)*(1 + Cot[x])] - (
1 - I)*Hypergeometric2F1[-1/2, 1, 1/2, (1/2 + I/2)*(1 + Cot[x])])/(2*Sqrt[
1 + Cot[x]])
```

3.47.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 4025, 25, 3042, 4019, 25, 3042, 4018, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(x)}{(\cot(x)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(x+\frac{\pi}{2}\right)^2}{\left(1-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4025}$$

$$\frac{1}{2}\int -\frac{1-\cot(x)}{\sqrt{\cot(x)+1}}dx+\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{\sqrt{\cot(x)+1}}-\frac{1}{2}\int \frac{1-\cot(x)}{\sqrt{\cot(x)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{\sqrt{\cot(x)+1}}-\frac{1}{2}\int \frac{\tan\left(x+\frac{\pi}{2}\right)+1}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4019}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\left(2+\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}-\frac{\int \frac{\left(2-\sqrt{2}\right)\cot(x)+\sqrt{2}}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}\right)+\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\left(2-\sqrt{2}\right)\cot(x)+\sqrt{2}}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}-\left(2+\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}\right)+\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\left(-2-\sqrt{2}\right)\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}-\left(2-\sqrt{2}\right)\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx}{2\sqrt{2}}\right)+\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{4018}$$

$$\frac{1}{2}\left(\sqrt{2}\left(3-2\sqrt{2}\right)\int \frac{1}{\frac{\left(\left(2-\sqrt{2}\right)\cot(x)-3\sqrt{2}+4\right)^2}{\cot(x)+1}-4\left(7-5\sqrt{2}\right)}d\frac{\left(2-\sqrt{2}\right)\cot(x)-3\sqrt{2}+4}{\sqrt{\cot(x)+1}}+\sqrt{2}\left(3+2\sqrt{2}\right)\int \frac{}{\left(\left(2+\right.\right.}\right.$$
$$\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\sqrt{2}\left(3+2\sqrt{2}\right)\int \frac{1}{\frac{\left(\left(2+\sqrt{2}\right)\cot(x)+3\sqrt{2}+4\right)^2}{\cot(x)+1}-4\left(7+5\sqrt{2}\right)}d\left(-\frac{\left(2+\sqrt{2}\right)\cot(x)+3\sqrt{2}+4}{\sqrt{\cot(x)+1}}\right)+\frac{\left(3-2\sqrt{2}\right)\text{arcta}}{\sqrt{2}}\right.$$
$$\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\frac{\left(3-2\sqrt{2}\right)\arctan\left(\frac{\left(2-\sqrt{2}\right)\cot(x)-3\sqrt{2}+4}{2\sqrt{5\sqrt{2}-7}\sqrt{\cot(x)+1}}\right)}{\sqrt{2\left(5\sqrt{2}-7\right)}}+\frac{\left(3+2\sqrt{2}\right)\text{arctanh}\left(\frac{\left(2+\sqrt{2}\right)\cot(x)+3\sqrt{2}+4}{2\sqrt{7+5\sqrt{2}}\sqrt{\cot(x)+1}}\right)}{\sqrt{2\left(7+5\sqrt{2}\right)}}\right)+$$
$$\frac{1}{\sqrt{\cot(x)+1}}$$

input
```
Int[Cot[x]^2/(1 + Cot[x])^(3/2),x]
```

3.47. $\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx$

output
```
(((3 - 2*Sqrt[2])*ArcTan[(4 - 3*Sqrt[2] + (2 - Sqrt[2])*Cot[x])/(2*Sqrt[-7
 + 5*Sqrt[2]]*Sqrt[1 + Cot[x]])])/Sqrt[2*(-7 + 5*Sqrt[2])] + ((3 + 2*Sqrt[
2])*ArcTanh[(4 + 3*Sqrt[2] + (2 + Sqrt[2])*Cot[x])/(2*Sqrt[7 + 5*Sqrt[2]]*
Sqrt[1 + Cot[x]])])/Sqrt[2*(7 + 5*Sqrt[2])])/2 + 1/Sqrt[1 + Cot[x]]
```

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4018
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*b*c*d - 4*a*d^2
+ x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0
] && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

rule 4019
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Simp[1/(2*q)   Int[(
a*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
 x], x] - Simp[1/(2*q)   Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f
*x])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d -
 b*(c^2 - d^2), 0] && NiceSqrtQ[a^2 + b^2]
```

rule 4025
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e
+ f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
 && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

3.47.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{1}{\sqrt{1+\cot(x)}}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{2\sqrt{-2+2\sqrt{2}}}+$
default	$\frac{1}{\sqrt{1+\cot(x)}}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{2\sqrt{-2+2\sqrt{2}}}+$

input
```
int(cot(x)^2/(1+cot(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(1+cot(x))^(1/2)-1/8*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^
(1/2)*(2+2*2^(1/2))^(1/2))+1/2*(2^(1/2)-1)/(-2+2*2^(1/2))^(1/2)*arctan((2*
(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/
2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/2*(1
-2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1
/2))/(-2+2*2^(1/2))^(1/2))
```

3.47.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.42

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \frac{\sqrt{i+1}(\cos(2x)+\sin(2x)+1)\log\left(\sqrt{i+1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)-\sqrt{i+1}(\cos(2$$

input
```
integrate(cot(x)^2/(1+cot(x))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(I + 1)*(cos(2*x) + sin(2*x) + 1)*log(sqrt(I + 1) + sqrt((cos(2*x
) + sin(2*x) + 1)/sin(2*x))) - sqrt(I + 1)*(cos(2*x) + sin(2*x) + 1)*log(-
sqrt(I + 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + sqrt(-I + 1)*(co
s(2*x) + sin(2*x) + 1)*log(sqrt(-I + 1) + sqrt((cos(2*x) + sin(2*x) + 1)/s
in(2*x))) - sqrt(-I + 1)*(cos(2*x) + sin(2*x) + 1)*log(-sqrt(-I + 1) + sqr
t((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + 4*sqrt((cos(2*x) + sin(2*x) + 1)/
sin(2*x))*sin(2*x))/(cos(2*x) + sin(2*x) + 1)
```

3.47.6 Sympy [F]

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \int \frac{\cot^2{(x)}}{(\cot{(x)}+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)**2/(1+cot(x))**(3/2),x)
```

output
```
Integral(cot(x)**2/(cot(x) + 1)**(3/2), x)
```

3.47.7 Maxima [F]

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \int \frac{\cot{(x)}^2}{(\cot{(x)}+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)^2/(1+cot(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(x)^2/(cot(x) + 1)^(3/2), x)
```

3.47.8 Giac [F]

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \int \frac{\cot(x)^2}{(\cot(x)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)^2/(1+cot(x))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(x)^2/(cot(x) + 1)^(3/2), x)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.50

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{3/2}}\,dx = \frac{1}{\sqrt{\cot(x)+1}} - \operatorname{atanh}\left(\frac{\sqrt{\cot(x)+1}}{8\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}} - \frac{\sqrt{\cot(x)+1}}{8\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}} + \frac{\sqrt{2}\sqrt{\cot(x)+1}}{16\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}} + \frac{\sqrt{2}\sqrt{\cot(x)+1}}{16\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}}\right)\left(2\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}} - 2\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}\right) + \operatorname{atanh}\left(\frac{\sqrt{\cot(x)+1}}{8\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}} + \frac{\sqrt{\cot(x)+1}}{8\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}} - \frac{\sqrt{2}\sqrt{\cot(x)+1}}{16\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}}} + \frac{\sqrt{2}\sqrt{\cot(x)+1}}{16\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}}\right)\left(2\sqrt{\frac{1}{32}-\frac{\sqrt{2}}{32}} + 2\sqrt{\frac{\sqrt{2}}{32}+\frac{1}{32}}\right)$$

input
```
int(cot(x)^2/(cot(x) + 1)^(3/2),x)
```

output
```
1/(cot(x) + 1)^(1/2) - atanh((cot(x) + 1)^(1/2)/(8*(2^(1/2)/32 + 1/32)^(1/
2)) - (cot(x) + 1)^(1/2)/(8*(1/32 - 2^(1/2)/32)^(1/2)) + (2^(1/2)*(cot(x)
+ 1)^(1/2))/(16*(1/32 - 2^(1/2)/32)^(1/2)) + (2^(1/2)*(cot(x) + 1)^(1/2))/
(16*(2^(1/2)/32 + 1/32)^(1/2)))*(2*(1/32 - 2^(1/2)/32)^(1/2) - 2*(2^(1/2)/
32 + 1/32)^(1/2)) + atanh((cot(x) + 1)^(1/2)/(8*(1/32 - 2^(1/2)/32)^(1/2))
 + (cot(x) + 1)^(1/2)/(8*(2^(1/2)/32 + 1/32)^(1/2)) - (2^(1/2)*(cot(x) + 1
)^(1/2))/(16*(1/32 - 2^(1/2)/32)^(1/2)) + (2^(1/2)*(cot(x) + 1)^(1/2))/(16
*(2^(1/2)/32 + 1/32)^(1/2)))*(2*(1/32 - 2^(1/2)/32)^(1/2) + 2*(2^(1/2)/32
+ 1/32)^(1/2))
```

3.48 $\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx$

3.48.1 Optimal result

Integrand size = 11, antiderivative size = 226

$$\begin{aligned}\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx &= \frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\ &-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\ &-\frac{1}{\sqrt{1+\cot(x)}}-\frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}\\ &+\frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}\end{aligned}$$

output
```
-1/(1+cot(x))^(1/2)+1/4*arctan((-2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(
-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)-1/4*arctan((2*(1+cot(x))^(1/2)+(2
+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)-1/4*ln(1+cot(
x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)+1/4*l
n(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/
2)
```

3.48.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.31

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx = \frac{1}{2}i\sqrt{1-i}\operatorname{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) - \frac{1}{2}i\sqrt{1+i}\operatorname{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - \frac{1}{\sqrt{1+\cot(x)}}$$

input
```
Integrate[Cot[x]/(1 + Cot[x])^(3/2),x]
```

output
```
(I/2)*Sqrt[1 - I]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]] - (I/2)*Sqrt[1 + I
]*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]] - 1/Sqrt[1 + Cot[x]]
```

3.48.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.273$, Rules used = {3042, 25, 4012, 25, 3042, 3966, 483, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{(\cot(x)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\left(1-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\left(1-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$-\frac{1}{2}\int -\sqrt{\cot(x)+1}dx - \frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \sqrt{\cot(x)+1}dx-\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}dx-\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{3966}$$

$$-\frac{1}{2}\int \frac{\sqrt{\cot(x)+1}}{\cot^2(x)+1}d\cot(x)-\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{483}$$

$$-\int \frac{\cot(x)+1}{(\cot(x)+1)^2-2(\cot(x)+1)+2}d\sqrt{\cot(x)+1}-\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\int \frac{-\cot(x)+\sqrt{2}-1}{(\cot(x)+1)^2-2(\cot(x)+1)+2}d\sqrt{\cot(x)+1}-\frac{1}{2}\int \frac{\cot(x)+\sqrt{2}+1}{(\cot(x)+1)^2-2(\cot(x)+1)+2}d\sqrt{\cot(x)+1}-\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}-\frac{1}{2}\int \frac{1}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\right.$$
$$\frac{1}{2}\int \frac{-\cot(x)+\sqrt{2}-1}{(\cot(x)+1)^2-2(\cot(x)+1)+2}d\sqrt{\cot(x)+1}-\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\int \frac{1}{-\cot(x)+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}\right)+\int \frac{1}{-\cot(x)+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{\cot(x\right.\right.$$
$$\frac{1}{2}\int \frac{-\cot(x)+\sqrt{2}-1}{(\cot(x)+1)^2-2(\cot(x)+1)+2}d\sqrt{\cot(x)+1}-\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{217}$$

3.48. $\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx$

$$\frac{1}{2}\int \frac{-\cot(x)+\sqrt{2}-1}{(\cot(x)+1)^2-2(\cot(x)+1)+2}d\sqrt{\cot(x)+1}+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\int -\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{2\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\int \frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{2}\left(-\frac{\arctan\left(\frac{2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{\sqrt{\cot(x)+1}}$$

$$\Big\downarrow \text{1103}$$

3.48. $\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}-\frac{\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{\sqrt{\cot(x)+1}}+\frac{1}{2}\left(\frac{\log\left(\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)$$

input
```
Int[Cot[x]/(1 + Cot[x])^(3/2),x]
```

output
```
(-(ArcTan[(-Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[
2])]]/Sqrt[2*(-1 + Sqrt[2])]) - ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 +
 Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(-1 + Sqrt[2])])/2 - 1/Sqrt[1 + C
ot[x]] + (-1/2*Log[1 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + C
ot[x]]]/Sqrt[2*(1 + Sqrt[2])] + Log[1 + Sqrt[2] + Cot[x] + Sqrt[2*(1 + Sqr
t[2])]*Sqrt[1 + Cot[x]]]/(2*Sqrt[2*(1 + Sqrt[2])]))/2
```

3.48.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

3.48. $\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx$

3.48.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(160) = 320$.

Time = 0.04 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.58

method	result
derivativedivides	$-\frac{1}{\sqrt{1+\cot(x)}}-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{8}-\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2-}}{\sqrt{-2+2\sqrt{2}}}\right.}{4\sqrt{-2+2\sqrt{2}}}$
default	$-\frac{1}{\sqrt{1+\cot(x)}}-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{8}-\frac{\sqrt{2}\left(2+2\sqrt{2}\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{2-}}{\sqrt{-2+2\sqrt{2}}}\right.}{4\sqrt{-2+2\sqrt{2}}}$

input
```
int(cot(x)/(1+cot(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(1+cot(x))^(1/2)-1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1/2)-(1
+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2
))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1
/2))+1/8*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(
1/2))^(1/2))+1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(
1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^
(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/4*2^(1/2
)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/
2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/
2)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^
(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)
)
```

3.48.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.89

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx =$$
$$-\frac{\sqrt{i-1}(\cos(2x)+\sin(2x)+1)\log\left(i\sqrt{i-1}+\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)-\sqrt{i-1}(\cos(2x)+\sin(2x)+1)\log}{}$$

input
```
integrate(cot(x)/(1+cot(x))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(I - 1)*(cos(2*x) + sin(2*x) + 1)*log(I*sqrt(I - 1) + sqrt((cos(
2*x) + sin(2*x) + 1)/sin(2*x))) - sqrt(I - 1)*(cos(2*x) + sin(2*x) + 1)*lo
g(-I*sqrt(I - 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) - sqrt(-I - 1
)*(cos(2*x) + sin(2*x) + 1)*log(I*sqrt(-I - 1) + sqrt((cos(2*x) + sin(2*x)
 + 1)/sin(2*x))) + sqrt(-I - 1)*(cos(2*x) + sin(2*x) + 1)*log(-I*sqrt(-I -
 1) + sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + 4*sqrt((cos(2*x) + sin(2
*x) + 1)/sin(2*x))*sin(2*x))/(cos(2*x) + sin(2*x) + 1)
```

3.48.6 Sympy [F]

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx = \int \frac{\cot(x)}{(\cot(x)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)/(1+cot(x))**(3/2),x)
```

output
```
Integral(cot(x)/(cot(x) + 1)**(3/2), x)
```

3.48.7 Maxima [F]

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx = \int \frac{\cot(x)}{(\cot(x)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)/(1+cot(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(x)/(cot(x) + 1)^(3/2), x)
```

3.48.8 Giac [F]

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx = \int \frac{\cot(x)}{(\cot(x)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(x)/(1+cot(x))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(x)/(cot(x) + 1)^(3/2), x)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 12.84 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.54

$$\int \frac{\cot(x)}{(1+\cot(x))^{3/2}}\,dx = -\mathrm{atanh}\left(32\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)-\frac{1}{\sqrt{\cot(x)+1}}-\mathrm{atanh}\left(32\sqrt{\cot(x)+1}\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)$$

input
```
int(cot(x)/(cot(x) + 1)^(3/2),x)
```

output
```
- atanh(32*(cot(x) + 1)^(1/2)*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 -
 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1
/2)) - 1/(cot(x) + 1)^(1/2) - atanh(32*(cot(x) + 1)^(1/2)*((- 2^(1/2)/32 -
 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32 - 1/32)^(1/2
) - 2*(2^(1/2)/32 - 1/32)^(1/2))
```

3.49 $\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx$

3.49.1 Optimal result

Integrand size = 13, antiderivative size = 143

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \frac{1}{4}\sqrt{-1+\sqrt{2}}\arctan\left(\frac{3-2\sqrt{2}+\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{2\left(-7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right) + \frac{1}{4}\sqrt{1+\sqrt{2}}\operatorname{arctanh}\left(\frac{3+2\sqrt{2}+\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{1+\cot(x)}}\right) + \frac{1}{3(1+\cot(x))^{3/2}} - \frac{1}{\sqrt{1+\cot(x)}}$$

output
```
1/3/(1+cot(x))^(3/2)-1/(1+cot(x))^(1/2)+1/4*arctan((3+cot(x)*(1-2^(1/2))-2
*2^(1/2))/(1+cot(x))^(1/2)/(-14+10*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)+1/4*a
rctanh((3+2*2^(1/2)+cot(x)*(1+2^(1/2)))/(1+cot(x))^(1/2)/(14+10*2^(1/2))^(
1/2))*(1+2^(1/2))^(1/2)
```

3.49.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.43

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \frac{4-(1+i)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\left(\frac{1}{2}-\frac{i}{2}\right)(1+\cot(x))\right)-(1-i)\,\text{Hyperge}}{6(1+\cot(x))^{3/2}}$$

input
```
Integrate[Cot[x]^2/(1 + Cot[x])^(5/2),x]
```

output
```
(4 - (1 + I)*Hypergeometric2F1[-3/2, 1, -1/2, (1/2 - I/2)*(1 + Cot[x])] -
(1 - I)*Hypergeometric2F1[-3/2, 1, -1/2, (1/2 + I/2)*(1 + Cot[x])])/(6*(1
+ Cot[x])^(3/2))
```

3.49.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.077, Rules used = {3042, 4025, 25, 3042, 4012, 27, 3042, 25, 4019, 25, 3042, 4018, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(x)}{(\cot(x)+1)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(x+\frac{\pi}{2}\right)^2}{\left(1-\tan\left(x+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4025}$$

$$\frac{1}{2}\int -\frac{1-\cot(x)}{(\cot(x)+1)^{3/2}}dx+\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3(\cot(x)+1)^{3/2}}-\frac{1}{2}\int \frac{1-\cot(x)}{(\cot(x)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3(\cot(x)+1)^{3/2}}-\frac{1}{2}\int \frac{\tan\left(x+\frac{\pi}{2}\right)+1}{\left(1-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int -\frac{2\cot(x)}{\sqrt{\cot(x)+1}}dx-\frac{2}{\sqrt{\cot(x)+1}}\right)+\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\int \frac{\cot(x)}{\sqrt{\cot(x)+1}}dx-\frac{2}{\sqrt{\cot(x)+1}}\right)+\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx-\frac{2}{\sqrt{\cot(x)+1}}\right)+\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx-\frac{2}{\sqrt{\cot(x)+1}}\right)+\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \text{4019}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{1-\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}+\frac{\int -\frac{1-\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}-\frac{2}{\sqrt{\cot(x)+1}}\right)+\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{1-\left(1-\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}-\frac{\int \frac{1-\left(1+\sqrt{2}\right)\cot(x)}{\sqrt{\cot(x)+1}}dx}{2\sqrt{2}}-\frac{2}{\sqrt{\cot(x)+1}}\right)+\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{\int \frac{1-\left(-1-\sqrt{2}\right)\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx}{2\sqrt{2}}+\frac{\int \frac{1-\left(-1+\sqrt{2}\right)\tan\left(x+\frac{\pi}{2}\right)}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx}{2\sqrt{2}}-\frac{2}{\sqrt{\cot(x)+1}}\right)+\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \text{4018}$$

$$\frac{1}{2}\left(-\frac{\left(3-2\sqrt{2}\right)\int \frac{1}{\frac{\left(\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3\right)^2}{\cot(x)+1}-2\left(7-5\sqrt{2}\right)}d\left(-\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{\cot(x)+1}}\right)}{\sqrt{2}}+\frac{\left(3+2\sqrt{2}\right)\int \frac{1}{\frac{\left(\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3\right)^2}{\cot(x)+1}-2\cdots}\cdots}{\sqrt{\cdots}}\right.$$

$$\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \text{216}$$

3.49. $\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx$

$$\frac{1}{2}\left(\frac{(3+2\sqrt{2})\int \frac{1}{\frac{\left(\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3\right)^2}{\cot(x)+1}-2\left(7+5\sqrt{2}\right)}d\left(-\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{\cot(x)+1}}\right)}{\sqrt{2}}+\frac{(3-2\sqrt{2})\arctan\left(\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{}}{\sqrt{2\left(5\sqrt{2}-7\right)}\sqrt{\cot(x}}\right.}{2\sqrt{5\sqrt{2}-7}}\right.$$

$$\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \ 220$$

$$\frac{1}{2}\left(\frac{(3-2\sqrt{2})\arctan\left(\frac{\left(1-\sqrt{2}\right)\cot(x)-2\sqrt{2}+3}{\sqrt{2\left(5\sqrt{2}-7\right)}\sqrt{\cot(x)+1}}\right)}{2\sqrt{5\sqrt{2}-7}}+\frac{(3+2\sqrt{2})\,\text{arctanh}\left(\frac{\left(1+\sqrt{2}\right)\cot(x)+2\sqrt{2}+3}{\sqrt{2\left(7+5\sqrt{2}\right)}\sqrt{\cot(x)+1}}\right)}{2\sqrt{7+5\sqrt{2}}}-\frac{2}{\sqrt{\cot(x)+1}}\right)+$$

$$\frac{1}{3(\cot(x)+1)^{3/2}}$$

input
```
Int[Cot[x]^2/(1 + Cot[x])^(5/2),x]
```

output
```
1/(3*(1 + Cot[x])^(3/2)) + (((3 - 2*Sqrt[2])*ArcTan[(3 - 2*Sqrt[2] + (1 -
Sqrt[2])*Cot[x])/(Sqrt[2*(-7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])])/(2*Sqrt[-7
+ 5*Sqrt[2]]) + ((3 + 2*Sqrt[2])*ArcTanh[(3 + 2*Sqrt[2] + (1 + Sqrt[2])*Co
t[x])/(Sqrt[2*(7 + 5*Sqrt[2])]*Sqrt[1 + Cot[x]])])/(2*Sqrt[7 + 5*Sqrt[2]])
 - 2/Sqrt[1 + Cot[x]])/2
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4018
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*b*c*d - 4*a*d^2
+ x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0
] && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

rule 4019
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Simp[1/(2*q)   Int[(
a*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
 x], x] - Simp[1/(2*q)   Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f
*x])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d -
 b*(c^2 - d^2), 0] && NiceSqrtQ[a^2 + b^2]
```

rule 4025
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e
+ f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
 && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

3.49. $\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx$

3.49.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{1}{3(1+\cot(x))^{\frac{3}{2}}}-\frac{1}{\sqrt{1+\cot(x)}}-\frac{\sqrt{2}\left(\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{}}{\sqrt{-2+2\sqrt{}}}\right.}{\sqrt{-2+2\sqrt{2}}}\right.}{8}$
default	$\frac{1}{3(1+\cot(x))^{\frac{3}{2}}}-\frac{1}{\sqrt{1+\cot(x)}}-\frac{\sqrt{2}\left(\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{2\left(\sqrt{2}-1\right)\arctan\left(\frac{2\sqrt{1+\cot(x)}-\sqrt{}}{\sqrt{-2+2\sqrt{}}}\right.}{\sqrt{-2+2\sqrt{2}}}\right.}{8}$

input
```
int(cot(x)^2/(1+cot(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(1+cot(x))^(3/2)-1/(1+cot(x))^(1/2)-1/8*2^(1/2)*(1/2*(2+2*2^(1/2))^(1/
2)*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(2^(1/2)-1)
/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+
2*2^(1/2))^(1/2)))+1/8*2^(1/2)*(1/2*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2
)+(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+2*(1-2^(1/2))/(-2+2*2^(1/2))^(1/2)
*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

3.49.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.66

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \frac{3\sqrt{i+1}\left(\sqrt{2}\sin(2x)+\sqrt{2}\right)\log\left(-(i-1)\sqrt{2}\sqrt{i+1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)-3\sqrt{}}{}$$

input
```
integrate(cot(x)^2/(1+cot(x))^(5/2),x, algorithm="fricas")
```

output
```
1/24*(3*sqrt(I + 1)*(sqrt(2)*sin(2*x) + sqrt(2))*log(-(I - 1)*sqrt(2)*sqrt
(I + 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) - 3*sqrt(I + 1)*(sqr
t(2)*sin(2*x) + sqrt(2))*log((I - 1)*sqrt(2)*sqrt(I + 1) + 2*sqrt((cos(2*x
) + sin(2*x) + 1)/sin(2*x))) + 3*sqrt(-I + 1)*(sqrt(2)*sin(2*x) + sqrt(2))
*log((I + 1)*sqrt(2)*sqrt(-I + 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2
*x))) - 3*sqrt(-I + 1)*(sqrt(2)*sin(2*x) + sqrt(2))*log(-(I + 1)*sqrt(2)*s
qrt(-I + 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) + 4*sqrt((cos(2*
x) + sin(2*x) + 1)/sin(2*x))*(2*cos(2*x) - 3*sin(2*x) - 2))/(sin(2*x) + 1)
```

3.49.6 Sympy [F]

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \int \frac{\cot^2(x)}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(x)**2/(1+cot(x))**(5/2),x)
```

output
```
Integral(cot(x)**2/(cot(x) + 1)**(5/2), x)
```

3.49.7 Maxima [F]

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \int \frac{\cot(x)^2}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(x)^2/(1+cot(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cot(x)^2/(cot(x) + 1)^(5/2), x)
```

3.49.8 Giac [F]

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \int \frac{\cot(x)^2}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(x)^2/(1+cot(x))^(5/2),x, algorithm="giac")
```

output
```
integrate(cot(x)^2/(cot(x) + 1)^(5/2), x)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.69

$$\int \frac{\cot^2(x)}{(1+\cot(x))^{5/2}}\,dx = \operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1} - \frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}-1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}+2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right) - \operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}+1} + \frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}+1}\right)\left(2\sqrt{\frac{1}{64}-\frac{\sqrt{2}}{64}}-2\sqrt{\frac{\sqrt{2}}{64}+\frac{1}{64}}\right) - \frac{\cot(x)+\frac{2}{3}}{(\cot(x)+1)^{3/2}}$$

input
```
int(cot(x)^2/(cot(x) + 1)^(5/2),x)
```

output
```
atanh((4*2^(1/2)*(1/64 - 2^(1/2)/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(1/64 -
 2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2) - 1) - (4*2^(1/2)*(2^(1/2)/64
 + 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/
64 + 1/64)^(1/2) - 1))*(2*(1/64 - 2^(1/2)/64)^(1/2) + 2*(2^(1/2)/64 + 1/64
)^(1/2)) - atanh((4*2^(1/2)*(1/64 - 2^(1/2)/64)^(1/2)*(cot(x) + 1)^(1/2))/
(64*(1/64 - 2^(1/2)/64)^(1/2)*(2^(1/2)/64 + 1/64)^(1/2) + 1) + (4*2^(1/2)*
(2^(1/2)/64 + 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(1/64 - 2^(1/2)/64)^(1/2
)*(2^(1/2)/64 + 1/64)^(1/2) + 1))*(2*(1/64 - 2^(1/2)/64)^(1/2) - 2*(2^(1/2
)/64 + 1/64)^(1/2)) - (cot(x) + 2/3)/(cot(x) + 1)^(3/2)
```

3.50 $\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx$

3.50.1 Optimal result

Integrand size = 11, antiderivative size = 216

$$\begin{aligned}\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx &= \frac{1}{4}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\
&-\frac{1}{4}\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2\sqrt{1+\cot(x)}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)\\
&-\frac{1}{3(1+\cot(x))^{3/2}}+\frac{\log\left(1+\sqrt{2}+\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{8\sqrt{1+\sqrt{2}}}\\
&-\frac{\log\left(1+\sqrt{2}+\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{1+\cot(x)}\right)}{8\sqrt{1+\sqrt{2}}}\end{aligned}$$

output
```
-1/3/(1+cot(x))^(3/2)+1/8*ln(1+cot(x)+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2
))^(1/2))/(1+2^(1/2))^(1/2)-1/8*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2)*(2+2*
2^(1/2))^(1/2))/(1+2^(1/2))^(1/2)+1/4*arctan((-2*(1+cot(x))^(1/2)+(2+2*2^(
1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)-1/4*arctan((2*(1+cot(
x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)
```

3.50.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.32

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx = -\frac{1}{4}(1-i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1-i}}\right) - \frac{1}{4}(1+i)^{3/2}\text{arctanh}\left(\frac{\sqrt{1+\cot(x)}}{\sqrt{1+i}}\right) - \frac{1}{3(1+\cot(x))^{3/2}}$$

input
```
Integrate[Cot[x]/(1 + Cot[x])^(5/2),x]
```

output
```
-1/4*((1 - I)^(3/2)*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 - I]]) - ((1 + I)^(3/2
)*ArcTanh[Sqrt[1 + Cot[x]]/Sqrt[1 + I]])/4 - 1/(3*(1 + Cot[x])^(3/2))
```

3.50.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.182, Rules used = {3042, 25, 4012, 25, 3042, 3966, 484, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{(\cot(x)+1)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\tan\left(x+\frac{\pi}{2}\right)}{\left(1-\tan\left(x+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\tan\left(x+\frac{\pi}{2}\right)}{\left(1-\tan\left(x+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4012}$$

$$-\frac{1}{2}\int -\frac{1}{\sqrt{\cot(x)+1}}dx - \frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\cot(x)+1}}dx-\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)}}dx-\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \ 3966$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{\cot(x)+1}\left(\cot^2(x)+1\right)}d\cot(x)-\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \ 484$$

$$-\int \frac{1}{(\cot(x)+1)^2-2(\cot(x)+1)+2}d\sqrt{\cot(x)+1}-\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \ 1407$$

$$-\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{4\sqrt{1+\sqrt{2}}}-\frac{\int \frac{\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{4\sqrt{1+\sqrt{2}}}-\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \ 1142$$

$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}-\frac{1}{2}\int -\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)}}{4\sqrt{1+\sqrt{2}}}$$
$$\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}+\frac{1}{2}\int \frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{4\sqrt{1+\sqrt{2}}}$$
$$\frac{1}{3(\cot(x)+1)^{3/2}}$$

$$\Big\downarrow \ 25$$

3.50. $\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx$

$$-\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}+\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+}}{4\sqrt{1+\sqrt{2}}}$$

$$\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\int \frac{1}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}+\frac{1}{2}\int \frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}}{4\sqrt{1+\sqrt{2}}}$$

$$\frac{1}{3(\cot(x)+1)^{3/2}}$$

$\downarrow$ 1083

$$-\frac{\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-\cot(x)+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{\cot(x)+1}-\sqrt{2\left(\right.}\right.}{4\sqrt{1+\sqrt{2}}}$$

$$\frac{\frac{1}{2}\int \frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}\int \frac{1}{-\cot(x)+2\left(1-\sqrt{2}\right)-1}d\left(2\sqrt{\cot(x)+1}+\sqrt{2\left(1\right.}\right.}{4\sqrt{1+\sqrt{2}}}$$

$$\frac{1}{3(\cot(x)+1)^{3/2}}$$

$\downarrow$ 217

$$-\frac{\frac{1}{2}\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{\cot(x)+1}}{\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}+\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{4\sqrt{1+\sqrt{2}}}-$$

$$\frac{\frac{1}{2}\int \frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1}d\sqrt{\cot(x)+1}+\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{4\sqrt{1+\sqrt{2}}}-$$

$$\frac{1}{3(\cot(x)+1)^{3/2}}$$

$\downarrow$ 1103

3.50. $\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx$

$$-\frac{\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{\cot(x)+1}-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{2}\log\left(\cot(x)-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}-\frac{\sqrt{\frac{1+\sqrt{2}}{\sqrt{2}-1}}\arctan\left(\frac{2\sqrt{\cot(x)+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\log\left(\cot(x)+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{\cot(x)+1}+\sqrt{2}+1\right)}{4\sqrt{1+\sqrt{2}}}-\frac{1}{3(\cot(x)+1)^{3/2}}$$

input
```
Int[Cot[x]/(1 + Cot[x])^(5/2),x]
```

output
```
-1/3*1/(1 + Cot[x])^(3/2) - (Sqrt[(1 + Sqrt[2])/(-1 + Sqrt[2])]*ArcTan[(-S
qrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]] - Log[1
 + Sqrt[2] + Cot[x] - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/2)/(4*Sqrt[1
 + Sqrt[2]]) - (Sqrt[(1 + Sqrt[2])/(-1 + Sqrt[2])]*ArcTan[(Sqrt[2*(1 + Sqr
t[2])] + 2*Sqrt[1 + Cot[x]])/Sqrt[2*(-1 + Sqrt[2])]] + Log[1 + Sqrt[2] + C
ot[x] + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + Cot[x]]]/2)/(4*Sqrt[1 + Sqrt[2]])
```

3.50.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 484
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[2*
d   Subst[Int[1/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

3.50. $\int \frac{\cot(x)}{(1+\cot(x))^{5/2}} \, dx$

3.50.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(152) = 304.

Time = 0.03 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.06

method	result
derivativedivides	$-\frac{1}{3(1+\cot(x))^{\frac{3}{2}}}-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{16}+\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+c\right.}}{8}$
default	$-\frac{1}{3(1+\cot(x))^{\frac{3}{2}}}-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+\cot(x)}\,\sqrt{2+2\sqrt{2}}\right)}{16}+\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(1+\cot(x)+\sqrt{2}-\sqrt{1+c\right.}}{8}$

input
```
int(cot(x)/(1+cot(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(1+cot(x))^(3/2)-1/16*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1/2)
-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)
+2^(1/2)-(1+cot(x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/8*2^(1/2)*(2+2*2^(1/2))/(
-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*
2^(1/2))^(1/2))+1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x)
)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/2/(-2+2*2^(1/2))^(1/2
)*arctan((2*(1+cot(x))^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^
(1/2)+1/16*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(x))^(1/2
)*(2+2*2^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)*ln(1+cot(x)+2^(1/2)+(1+cot(
x))^(1/2)*(2+2*2^(1/2))^(1/2))-1/8*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1
/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+
1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+cot(x))^(1/2)+(2+2*2^(
1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/2/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+c
ot(x))^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)
```

3.50.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.06

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx =$$

$$-\frac{3\sqrt{i-1}\left(\sqrt{2}\sin(2x)+\sqrt{2}\right)\log\left(-(i-1)\,\sqrt{2}\sqrt{i-1}+2\sqrt{\frac{\cos(2x)+\sin(2x)+1}{\sin(2x)}}\right)-3\sqrt{i-1}\left(\sqrt{2}\sin(2x)+\sqrt{\right.}}{}$$

input
```
integrate(cot(x)/(1+cot(x))^(5/2),x, algorithm="fricas")
```

output
```
-1/24*(3*sqrt(I - 1)*(sqrt(2)*sin(2*x) + sqrt(2))*log(-(I - 1)*sqrt(2)*sqr
t(I - 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) - 3*sqrt(I - 1)*(sq
rt(2)*sin(2*x) + sqrt(2))*log((I - 1)*sqrt(2)*sqrt(I - 1) + 2*sqrt((cos(2*
x) + sin(2*x) + 1)/sin(2*x))) + 3*sqrt(-I - 1)*(sqrt(2)*sin(2*x) + sqrt(2)
)*log((I + 1)*sqrt(2)*sqrt(-I - 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(
2*x))) - 3*sqrt(-I - 1)*(sqrt(2)*sin(2*x) + sqrt(2))*log(-(I + 1)*sqrt(2)*
sqrt(-I - 1) + 2*sqrt((cos(2*x) + sin(2*x) + 1)/sin(2*x))) - 4*sqrt((cos(2
*x) + sin(2*x) + 1)/sin(2*x))*(cos(2*x) - 1))/(sin(2*x) + 1)
```

3.50.6 Sympy [F]

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx = \int \frac{\cot(x)}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(x)/(1+cot(x))**(5/2),x)
```

output
```
Integral(cot(x)/(cot(x) + 1)**(5/2), x)
```

3.50.7 Maxima [F]

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx = \int \frac{\cot(x)}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(x)/(1+cot(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cot(x)/(cot(x) + 1)^(5/2), x)
```

3.50.8 Giac [F]

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx = \int \frac{\cot(x)}{(\cot(x)+1)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(x)/(1+cot(x))^(5/2),x, algorithm="giac")
```

output
```
integrate(cot(x)/(cot(x) + 1)^(5/2), x)
```

3.50.9 Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.10

$$\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx = \operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1} + \frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right) - \operatorname{atanh}\left(\frac{4\sqrt{2}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1} - \frac{4\sqrt{2}\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{\cot(x)+1}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right) - \frac{1}{3(\cot(x)+1)^{3/2}}$$

input
```
int(cot(x)/(cot(x) + 1)^(5/2),x)
```

output
```
atanh((4*2^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(2^(1
/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) + 1) + (4*2^(1/2)*(2^(1/2
)/64 - 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^
(1/2)/64 - 1/64)^(1/2) + 1))*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*(2^(1/2)/6
4 - 1/64)^(1/2)) - atanh((4*2^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2)*(cot(x) +
1)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) - 1) -
 (4*2^(1/2)*(2^(1/2)/64 - 1/64)^(1/2)*(cot(x) + 1)^(1/2))/(64*(2^(1/2)/64
- 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) - 1))*(2*(- 2^(1/2)/64 - 1/64)^(
1/2) + 2*(2^(1/2)/64 - 1/64)^(1/2)) - 1/(3*(cot(x) + 1)^(3/2))
```

3.50. $\int \frac{\cot(x)}{(1+\cot(x))^{5/2}}\,dx$

3.51 $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx$

3.51.1 Optimal result

Integrand size = 23, antiderivative size = 247

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx = -\frac{(a+b)e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} + \frac{(a+b)e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{2ae\sqrt{e\cot(c+dx)}}{d} - \frac{2b(e\cot(c+dx))^{3/2}}{3d} - \frac{(a-b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} + \frac{(a-b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}$$

output
```
-2/3*b*(e*cot(d*x+c))^(3/2)/d-1/2*(a+b)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*
x+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/2*(a+b)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d
*x+c))^(1/2)/e^(1/2))/d*2^(1/2)-1/4*(a-b)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^
(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)+1/4*(a-b)*e^(3/2)*ln(e^(1/2)
+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)-2*a*e*(e*cot(d
*x+c))^(1/2)/d
```

3.51.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.28

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx =$$
$$-\frac{2e\sqrt{e\cot(c+dx)}\left(b\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\tan^2(c+dx)\right)+3a\,\text{Hypergeometric2F1}\right.}{3d}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x]),x]
```

output
```
(-2*e*Sqrt[e*Cot[c + d*x]]*(b*Cot[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4,
 -Tan[c + d*x]^2] + 3*a*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2]))
/(3*d)
```

3.51.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3042, 4011, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{ 4011}$$
$$\int \sqrt{e\cot(c+dx)}(ae\cot(c+dx)-be)dx-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-be-a\tan\left(c+dx+\frac{\pi}{2}\right)e\right)dx-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$
$$\Big\downarrow \text{ 4011}$$

$$\int \frac{-ae^2-b\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx-\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

$\big\downarrow$ 3042

$$\int \frac{be^2\tan\left(c+dx+\frac{\pi}{2}\right)-ae^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

$\big\downarrow$ 4017

$$\frac{2\int \frac{e^2(ae+b\cot(c+dx)e)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

$\big\downarrow$ 27

$$\frac{2e^2\int \frac{ae+b\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

$\big\downarrow$ 1482

$$\frac{2e^2\left(\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

$\big\downarrow$ 1476

$$\frac{2e^2\left(\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\right.\right.}{d}$$
$$\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

$\big\downarrow$ 1082

$$\frac{2e^2\left(\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}$$
$$\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

$\big\downarrow$ 217

3.51. $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx$

$$\frac{2e^2\left(\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d}$$

$$\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

$\downarrow$ 1479

$$\frac{2e^2\left(\frac{1}{2}(a-b)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+\right.}{d}$$

$$\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

$\downarrow$ 25

$$\frac{2e^2\left(\frac{1}{2}(a-b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\right.\right.}{d}$$

$$\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

$\downarrow$ 27

$$\frac{2e^2\left(\frac{1}{2}(a-b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\right.\right.}{d}$$

$$\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

$\downarrow$ 1103

$$\frac{2e^2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right.\right.}{d}$$

$$\frac{2ae\sqrt{e\cot(c+dx)}}{d}-\frac{2b(e\cot(c+dx))^{3/2}}{3d}$$

input
```
Int[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x]),x]
```

3.51. $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx$

output
```
(-2*a*e*Sqrt[e*Cot[c + d*x]])/d - (2*b*(e*Cot[c + d*x])^(3/2))/(3*d) + (2*
e^2*(((a + b)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[
2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]
*Sqrt[e])))/2 + ((a - b)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sq
rt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*S
qrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/d
```

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

3.51.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.23

method	result
parts	$-\frac{2ae\left(\sqrt{e\cot(dx+c)}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan}{8}\right.}{d}$
derivativedivides	$\frac{-\frac{2b(e\cot(dx+c))^{\frac{3}{2}}}{3}-2ae\sqrt{e\cot(dx+c)}+2e^2\left(\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e}}{(e}\right.\right.}{8e}\right.}{}$
default	$\frac{-\frac{2b(e\cot(dx+c))^{\frac{3}{2}}}{3}-2ae\sqrt{e\cot(dx+c)}+2e^2\left(\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e}}{(e}\right.\right.}{8e}\right.}{}$

input
```
int((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*a/d*e*((e*cot(d*x+c))^(1/2)-1/8*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)+1)))+b/d*(-2/3*(e*cot(d*x+c))^(3/2)+1/4*e^2/(e^2)^(1/4)*2^(1/2)*(ln((e
*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d
*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1
/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)+1)))
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(194) = 388$.

Time = 0.28 (sec) , antiderivative size = 843, normalized size of antiderivative = 3.41

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx =$$

$$-\frac{3d\sqrt{-\frac{2abe^3+\sqrt{-\frac{(a^4-2a^2b^2+b^4)e^6}{d^4}}d^2}{d^2}}\log\left(-(a^4-b^4)e^4\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}+\left((a^3-ab^2)de^3+\sqrt{-\frac{(a^4-2a^2b^2+b^4)e^6}{d^4}}\right.\right.}{}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(3*d*sqrt(-(2*a*b*e^3 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d
^2)*log(-(a^4 - b^4)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) +
 ((a^3 - a*b^2)*d*e^3 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*b*d^3)*sqrt
(-(2*a*b*e^3 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d^2))*sin(2*d*x
 + 2*c) - 3*d*sqrt(-(2*a*b*e^3 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^
2)/d^2)*log(-(a^4 - b^4)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c
)) - ((a^3 - a*b^2)*d*e^3 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*b*d^3)*
sqrt(-(2*a*b*e^3 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d^2))*sin(2
*d*x + 2*c) + 3*d*sqrt(-(2*a*b*e^3 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4
)*d^2)/d^2)*log(-(a^4 - b^4)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
 2*c)) + ((a^3 - a*b^2)*d*e^3 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*b*d
^3)*sqrt(-(2*a*b*e^3 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d^2))*s
in(2*d*x + 2*c) - 3*d*sqrt(-(2*a*b*e^3 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6
/d^4)*d^2)/d^2)*log(-(a^4 - b^4)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d
*x + 2*c)) - ((a^3 - a*b^2)*d*e^3 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)
*b*d^3)*sqrt(-(2*a*b*e^3 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/d^4)*d^2)/d^2
))*sin(2*d*x + 2*c) + 4*(b*e*cos(2*d*x + 2*c) + 3*a*e*sin(2*d*x + 2*c) + b
*e)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*sin(2*d*x + 2*c))
```

3.51.6 Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx = \int (e\cot{(c+dx)})^{\frac{3}{2}}\,(a+b\cot{(c+dx)})\,\,dx$$

input
```
integrate((e*cot(d*x+c))**(3/2)*(a+b*cot(d*x+c)),x)
```

output
```
Integral((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x)), x)
```

3.51.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.51.8 Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx = \int (b\cot(dx+c)+a)(e\cot(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)*(e*cot(d*x + c))^(3/2), x)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 13.86 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.62

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx = \frac{(-1)^{1/4}\,b\,e^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d} - \frac{2\,a\,e\,\sqrt{e\cot\left(c+d\,x\right)}}{d} - \frac{2\,b\,\left(e\cot(c+d\,x)\right)^{3/2}}{3\,d} - \frac{(-1)^{1/4}\,b\,e^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d} - \frac{(-1)^{1/4}\,a\,e^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d} - \frac{(-1)^{1/4}\,a\,e^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d}$$

input
```
int((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x)),x)
```

output
```
((-1)^(1/4)*b*e^(3/2)*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)))/d
 - (2*a*e*(e*cot(c + d*x))^(1/2))/d - ((-1)^(1/4)*a*e^(3/2)*atan(((-1)^(1/
4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*a*e^(3/2)*atanh(((
-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - (2*b*(e*cot(c + d*x))^(
3/2))/(3*d) - ((-1)^(1/4)*b*e^(3/2)*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/
2))/e^(1/2)))/d
```

3.51. $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))\,dx$

3.52 $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx$

3.52.1 Optimal result

Integrand size = 23, antiderivative size = 226

$$\begin{aligned}\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx &= \frac{(a-b)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} - \frac{(a-b)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\ &\quad - \frac{2b\sqrt{e\cot(c+dx)}}{d} - \frac{(a+b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\\ &\quad + \frac{(a+b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\end{aligned}$$

output
```
1/2*(a-b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)
-1/2*(a-b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2
)-1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^
(1/2)/d*2^(1/2)+1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x
+c))^(1/2))*e^(1/2)/d*2^(1/2)-2*b*(e*cot(d*x+c))^(1/2)/d
```

3.52.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.69

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx =$$

$$-\frac{\sqrt{e\cot(c+dx)}\left(8b\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\tan^2(c+dx)\right)+\sqrt{2}a\left(2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x]),x]
```

output
```
-1/4*(Sqrt[e*Cot[c + d*x]]*(8*b*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d
*x]^2] + Sqrt[2]*a*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
 + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c
+ d*x]]))/d
```

3.52.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{ae\cot(c+dx)-be}{\sqrt{e\cot(c+dx)}}dx-\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-be-a\tan\left(c+dx+\frac{\pi}{2}\right)e}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{ 4017}$$

$$\frac{2\int \frac{e(be-ae\cot(c+dx))}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2e\int \frac{be-ae\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{ 1482}$$

$$\frac{2e\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{2e\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int\right.\right.}{d}$$

$$\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2e\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}$$

$$\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2e\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d}-$$

$$\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{ 1479}$$

3.52. $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx$

$$\frac{2e\left(\frac{1}{2}(a+b)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b}{d}$$

$$\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

$\downarrow$ 25

$$\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\text{ar}}{-}\right.}{d}$$

$$\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

$\downarrow$ 27

$$\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\text{ar}}{-}\right.}{d}$$

$$\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

$\downarrow$ 1103

$$\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(e\cot(c+dx)-\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\,c}}{\sqrt{}}\right.}{\sqrt{2}\sqrt{}}\right.}{d}$$

$$\frac{2b\sqrt{e\cot(c+dx)}}{d}$$

input
```
Int[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x]),x]
```

output
```
(-2*b*Sqrt[e*Cot[c + d*x]])/d + (2*e*(-1/2*((a - b)*(-(ArcTan[1 - (Sqrt[2]
*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*S
qrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e]))) + ((a + b)*(-1/2*Log[e +
 e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e])
+ Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2
]*Sqrt[e])))/2))/d
```

3.52. $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx$

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

3.52.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.27

method	result
parts	$-\frac{ae\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\right.\right.}{4d(e^2)^{\frac{1}{4}}}$
derivativedivides	$-2\sqrt{e\cot(dx+c)}\,b-2e\left(-\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{ar}\right.}{8e}\right.$
default	$-2\sqrt{e\cot(dx+c)}\,b-2e\left(-\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{ar}\right.}{8e}\right.$

input
```
int((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/d*e/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+b/d*(-2*(e*cot(d
*x+c))^(1/2)+1/4*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(177) = 354$.

Time = 0.27 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.23

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx = \frac{d\sqrt{\frac{2\,abe+d^2\sqrt{-\frac{\left(a^4-2\,a^2b^2+b^4\right)e^2}{d^4}}}{d^2}}\log\left(-(a^4-b^4)e\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(ad^3\sqrt{-\frac{(a^4-2\,a^2b^2+b^4)e^2}{d^4}}-(a^2b-b^3)de\right)\right.}{}$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(d*sqrt((2*a*b*e + d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4))/d^2)*lo
g(-(a^4 - b^4)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (a*d^3*
sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4) - (a^2*b - b^3)*d*e)*sqrt((2*a*b*e
+ d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4))/d^2)) - d*sqrt((2*a*b*e + d^
2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4))/d^2)*log(-(a^4 - b^4)*e*sqrt((e*
cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (a*d^3*sqrt(-(a^4 - 2*a^2*b^2 +
b^4)*e^2/d^4) - (a^2*b - b^3)*d*e)*sqrt((2*a*b*e + d^2*sqrt(-(a^4 - 2*a^2*
b^2 + b^4)*e^2/d^4))/d^2)) - d*sqrt((2*a*b*e - d^2*sqrt(-(a^4 - 2*a^2*b^2
+ b^4)*e^2/d^4))/d^2)*log(-(a^4 - b^4)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c)) + (a*d^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4) + (a^2*b -
b^3)*d*e)*sqrt((2*a*b*e - d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4))/d^2)
) + d*sqrt((2*a*b*e - d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4))/d^2)*log
(-(a^4 - b^4)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (a*d^3*s
qrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4) + (a^2*b - b^3)*d*e)*sqrt((2*a*b*e -
 d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/d^4))/d^2)) - 4*b*sqrt((e*cos(2*d*x
 + 2*c) + e)/sin(2*d*x + 2*c)))/d
```

3.52.6 Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx = \int \sqrt{e\cot{(c+dx)}}(a+b\cot{(c+dx)})\;dx$$

input
```
integrate((e*cot(d*x+c))**(1/2)*(a+b*cot(d*x+c)),x)
```

output
```
Integral(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x)), x)
```

3.52.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.52.8 Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx = \int (b\cot{(dx+c)}+a)\sqrt{e\cot{(dx+c)}}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)*sqrt(e*cot(d*x + c)), x)
```

3.52.9 Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.57

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))\,dx$$
$$= -\frac{2\,b\,\sqrt{e\cot\left(c+d\,x\right)}}{d} - \frac{{\left(-1\right)}^{1/4}\,a\,\sqrt{e}\,\left(\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)-\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\right)}{d} - \frac{{\left(-1\right)}^{1/4}\,b\,\sqrt{e}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,1{}\mathrm{i}}{d} - \frac{{\left(-1\right)}^{1/4}\,b\,\sqrt{e}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{e\cot\left(c+d\,x\right)}}{\sqrt{e}}\right)\,1{}\mathrm{i}}{d}$$

input
```
int((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x)),x)
```

output
```
- (2*b*(e*cot(c + d*x))^(1/2))/d - ((-1)^(1/4)*b*e^(1/2)*atan(((-1)^(1/4)*
(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*b*e^(1/2)*atanh(((-1)
^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/d - ((-1)^(1/4)*a*e^(1/2)*(ata
n(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2)) - atanh(((-1)^(1/4)*(e*cot(
c + d*x))^(1/2))/e^(1/2))))/d
```

3.53 $\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

3.53.1 Optimal result

Integrand size = 23, antiderivative size = 208

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx = \frac{(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{(a+b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} + \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}$$

output
```
1/2*(a+b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)
-1/2*(a+b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2
)+1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*
2^(1/2)/e^(1/2)-1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x
+c))^(1/2))/d*2^(1/2)/e^(1/2)
```

3.53.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.80

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$$
$$= \frac{3\sqrt{2}b\left(-2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\tan(c+d}\right.\right.}{12d\sqrt{}}$$

input
```
Integrate[(a + b*Cot[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

output
```
(3*Sqrt[2]*b*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) + 8*a*Hypergeome
tric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(3/2))/(12*d*Sqrt[e*Cot
[c + d*x]]*Sqrt[Tan[c + d*x]])
```

3.53.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4017}$$

$$\frac{2\int -\frac{ae+b\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{ae+b\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}$$

$$\downarrow \text{1482}$$

$$\frac{2\left(-\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}$$

$$\downarrow \text{1476}$$

$$\frac{2\left(-\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\ldots\right.\right.}{d}$$

$$\downarrow \text{1082}$$

$$\frac{2\left(-\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\ldots}{}\right)}{\sqrt{2}\sqrt{e}\ldots}\right.\right.}{d}$$

$$\downarrow \text{217}$$

$$\frac{2\left(-\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d}$$

$$\downarrow \text{1479}$$

$$\frac{2\left(-\frac{1}{2}(a-b)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+\ldots\right.}{d}$$

$$\downarrow \text{25}$$

$$\frac{2\left(-\frac{1}{2}(a-b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(\ldots\right.\right.}{d}$$

$$\downarrow \text{27}$$

$$\frac{2\left(-\frac{1}{2}(a-b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(\ldots\right.\right.}{d}$$

3.53. $\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

$\big\downarrow$ 1103

$$\frac{2\left(-\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right.\right.}{d}$$

input
```
Int[(a + b*Cot[c + d*x])/Sqrt[e*Cot[c + d*x]],x]
```

output
```
(2*(-1/2*((a + b)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(S
qrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqr
t[2]*Sqrt[e]))) - ((a - b)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*
Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]
*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/d
```

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

3.53.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.31

method	result
derivativedivides	$-\frac{a\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+\right.}{4e}$
default	$-\frac{a\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+\right.}{4e}$
parts	$-\frac{a\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(}{\left(e^{2}\right)^{\frac{1}{4}}}\right.}{4de}$

input
```
int((a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4*b/(e^2)^(
1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1
/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(161) = 322$.

Time = 0.27 (sec) , antiderivative size = 721, normalized size of antiderivative = 3.47

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$$
$$= \frac{1}{2}\sqrt{-\frac{d^2e\sqrt{-\frac{a^4-2a^2b^2+b^4}{d^4e^2}}+2ab}{d^2e}}\log\left(-(a^4-b^4)\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}\right.$$
$$\left.+\left(bd^3e^2\sqrt{-\frac{a^4-2a^2b^2+b^4}{d^4e^2}}+(a^3-ab^2)de\right)\sqrt{-\frac{d^2e\sqrt{-\frac{a^4-2a^2b^2+b^4}{d^4e^2}}+2ab}{d^2e}}\right)$$
$$-\frac{1}{2}\sqrt{-\frac{d^2e\sqrt{-\frac{a^4-2a^2b^2+b^4}{d^4e^2}}+2ab}{d^2e}}\log\left(-(a^4-b^4)\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}\right.$$
$$\left.-\left(bd^3e^2\sqrt{-\frac{a^4-2a^2b^2+b^4}{d^4e^2}}+(a^3-ab^2)de\right)\sqrt{-\frac{d^2e\sqrt{-\frac{a^4-2a^2b^2+b^4}{d^4e^2}}+2ab}{d^2e}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{d^2e\sqrt{-\frac{a^4-2a^2b^2+b^4}{d^4e^2}}-2ab}{d^2e}}\log\left(-(a^4-b^4)\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}\right.$$
$$\left.+\left(bd^3e^2\sqrt{-\frac{a^4-2a^2b^2+b^4}{d^4e^2}}-(a^3-ab^2)de\right)\sqrt{\frac{d^2e\sqrt{-\frac{a^4-2a^2b^2+b^4}{d^4e^2}}-2ab}{d^2e}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{d^2e\sqrt{-\frac{a^4-2a^2b^2+b^4}{d^4e^2}}-2ab}{d^2e}}\log\left(-(a^4-b^4)\sqrt{\frac{e\cos(2dx+2c)+e}{\sin(2dx+2c)}}\right.$$
$$\left.-\left(bd^3e^2\sqrt{-\frac{a^4-2a^2b^2+b^4}{d^4e^2}}-(a^3-ab^2)de\right)\sqrt{\frac{d^2e\sqrt{-\frac{a^4-2a^2b^2+b^4}{d^4e^2}}-2ab}{d^2e}}\right)$$

input
```
integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

3.53. $\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx$

output
```
1/2*sqrt(-(d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) + 2*a*b)/(d^2*e)
)*log(-(a^4 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (b*d^
3*e^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) + (a^3 - a*b^2)*d*e)*sqrt(-
(d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) + 2*a*b)/(d^2*e))) - 1/2*s
qrt(-(d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) + 2*a*b)/(d^2*e))*log
(-(a^4 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (b*d^3*e^2
*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) + (a^3 - a*b^2)*d*e)*sqrt(-(d^2*
e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) + 2*a*b)/(d^2*e))) - 1/2*sqrt((
d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) - 2*a*b)/(d^2*e))*log(-(a^4
 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (b*d^3*e^2*sqrt(
-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) - (a^3 - a*b^2)*d*e)*sqrt((d^2*e*sqrt(
-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) - 2*a*b)/(d^2*e))) + 1/2*sqrt((d^2*e*s
qrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^2)) - 2*a*b)/(d^2*e))*log(-(a^4 - b^4)
*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (b*d^3*e^2*sqrt(-(a^4 -
 2*a^2*b^2 + b^4)/(d^4*e^2)) - (a^3 - a*b^2)*d*e)*sqrt((d^2*e*sqrt(-(a^4 -
 2*a^2*b^2 + b^4)/(d^4*e^2)) - 2*a*b)/(d^2*e)))
```

3.53.6 Sympy [F]

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx=\int \frac{a+b\cot\left(c+dx\right)}{\sqrt{e\cot\left(c+dx\right)}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))**(1/2),x)
```

output
```
Integral((a + b*cot(c + d*x))/sqrt(e*cot(c + d*x)), x)
```

3.53.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.53.8 Giac [F]

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx=\int \frac{b\cot\left(dx+c\right)+a}{\sqrt{e\cot\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)/sqrt(e*cot(d*x + c)), x)
```

3.53.9 Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.57

$$\int \frac{a+b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}\,dx=\frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,\sqrt{e}}-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,\sqrt{e}}+\frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d\,\sqrt{e}}+\frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d\,\sqrt{e}}$$

input
```
int((a + b*cot(c + d*x))/(e*cot(c + d*x))^(1/2),x)
```

output
```
((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/(d*e^(
1/2)) + ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1
i)/(d*e^(1/2)) - ((-1)^(1/4)*b*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^
(1/2)))/(d*e^(1/2)) + ((-1)^(1/4)*b*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/
2))/e^(1/2)))/(d*e^(1/2))
```

3.54 $\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

3.54.1 Optimal result

Integrand size = 23, antiderivative size = 229

$$\begin{aligned}\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx &= -\frac{(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\ &+ \frac{(a-b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} + \frac{2a}{de\sqrt{e\cot(c+dx)}} \\ &+ \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\ &- \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}\end{aligned}$$

output
```
-1/2*(a-b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2
)+1/2*(a-b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/
2)+1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d
/e^(3/2)*2^(1/2)-1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*
x+c))^(1/2))/d/e^(3/2)*2^(1/2)+2*a/d/e/(e*cot(d*x+c))^(1/2)
```

3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.85

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{3a\left(2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\right.}{}$$

input
```
Integrate[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

output
```
(3*a*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*Sqrt[2]*ArcTan[
1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]] + 8*Sqrt[Tan[c + d*x]]) + 8*b*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c
+ d*x]^2]*Tan[c + d*x]^(3/2))/(12*d*(e*Cot[c + d*x])^(3/2)*Tan[c + d*x]^(3
/2))
```

3.54.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3042, 4012, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4012}$$

$$\frac{\int \frac{be-ae\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{2a}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{be+a\tan\left(c+dx+\frac{\pi}{2}\right)e}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}+\frac{2a}{de\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 4017

$$\frac{2\int -\frac{e(be-ae\cot(c+dx))}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}+\frac{2a}{de\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 25

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{2\int \frac{e(be-ae\cot(c+dx))}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}$$

$\downarrow$ 27

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{2\int \frac{be-ae\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de}$$

$\downarrow$ 1482

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{de}$$

$\downarrow$ 1476

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \right.\right.}{de}$$

$\downarrow$ 1082

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{e}}{}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{de}$$

$\downarrow$ 217

3.54. $\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a+b)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{de}$$

$\big\downarrow$ 1479

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a+b)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\ldots}{de}$$

$\big\downarrow$ 25

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\text{arc}\ldots}{\ldots}\right.}{de}$$

$\big\downarrow$ 27

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\text{arc}\ldots}{\ldots}\right.}{de}$$

$\big\downarrow$ 1103

$$\frac{2a}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(e\cot(c+dx)-\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot\ldots}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.}{de}$$

input
```
Int[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(3/2),x]
```

3.54. $\int\frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

output
```
(2*a)/(d*e*Sqrt[e*Cot[c + d*x]]) - (2*(-1/2*((a - b)*(-(ArcTan[1 - (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*
Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e]))) + ((a + b)*(-1/2*Log[e
+ e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e])
 + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[
2]*Sqrt[e])))/2))/(d*e)
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.54. $\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

3.54. $\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx$

3.54.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.29

method	result
derivativedivides	$-\frac{2\left(\frac{b\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2\left(\frac{b\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
parts	$-\frac{2ae\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+\right.\right.}{8e^2\left(e^2\right)^{\frac{1}{4}}}\right.}{d}$

input
```
int((a+b*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/e*(1/8*b/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8*a/(e
^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/
2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-
2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+2*a/e/(e*cot(d*x+c))^(1/2))
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(180) = 360$.

Time = 0.29 (sec) , antiderivative size = 890, normalized size of antiderivative = 3.89

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx =$$

$$-\frac{(de^2\cos(2\,dx+2\,c)+de^2)\sqrt{\frac{d^2e^3\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^6}}+2\,ab}{d^2e^3}}\log\left(-(a^4-b^4)\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(ad^3e^5\sqrt{-\frac{a^4-2\,a^2b^2-}{d^4e^6}}\right.\right.}{}$$

input
```
integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt((d^2*e^3*sqrt(-(a^4 - 2*a^2*b^
2 + b^4)/(d^4*e^6)) + 2*a*b)/(d^2*e^3))*log(-(a^4 - b^4)*sqrt((e*cos(2*d*x
 + 2*c) + e)/sin(2*d*x + 2*c)) + (a*d^3*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/
(d^4*e^6)) - (a^2*b - b^3)*d*e^2)*sqrt((d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b
^4)/(d^4*e^6)) + 2*a*b)/(d^2*e^3))) - (d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqr
t((d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6)) + 2*a*b)/(d^2*e^3))*lo
g(-(a^4 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (a*d^3*e^
5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6)) - (a^2*b - b^3)*d*e^2)*sqrt((d^
2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6)) + 2*a*b)/(d^2*e^3))) - (d*e
^2*cos(2*d*x + 2*c) + d*e^2)*sqrt(-(d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/
(d^4*e^6)) - 2*a*b)/(d^2*e^3))*log(-(a^4 - b^4)*sqrt((e*cos(2*d*x + 2*c) +
 e)/sin(2*d*x + 2*c)) + (a*d^3*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6)
) + (a^2*b - b^3)*d*e^2)*sqrt(-(d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4
*e^6)) - 2*a*b)/(d^2*e^3))) + (d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt(-(d^2*
e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6)) - 2*a*b)/(d^2*e^3))*log(-(a^4
 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (a*d^3*e^5*sqrt(
-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6)) + (a^2*b - b^3)*d*e^2)*sqrt(-(d^2*e^3*
sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^6)) - 2*a*b)/(d^2*e^3))) - 4*a*sqrt((
e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c))/(d*e^2*cos(2*d
*x + 2*c) + d*e^2)
```

3.54.6 Sympy [F]

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*cot(c + d*x))/(e*cot(c + d*x))**(3/2), x)
```

3.54.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.54.8 Giac [F]

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{b\cot(dx+c)+a}{(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)/(e*cot(d*x + c))^(3/2), x)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.60

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2\,a}{d\,e\,\sqrt{e\cot\left(c+d\,x\right)}} + \frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,e^{3/2}} - \frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,e^{3/2}} + \frac{(-1)^{1/4}\,b\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d\,e^{3/2}} + \frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d\,e^{3/2}}$$

input
```
int((a + b*cot(c + d*x))/(e*cot(c + d*x))^(3/2),x)
```

output
```
(2*a)/(d*e*(e*cot(c + d*x))^(1/2)) + ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot
(c + d*x))^(1/2))/e^(1/2)))/(d*e^(3/2)) - ((-1)^(1/4)*a*atanh(((-1)^(1/4)*
(e*cot(c + d*x))^(1/2))/e^(1/2)))/(d*e^(3/2)) + ((-1)^(1/4)*b*atan(((-1)^(
1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/(d*e^(3/2)) + ((-1)^(1/4)*b*atan
h(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/(d*e^(3/2))
```

3.55 $\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx$

3.55.1 Optimal result

Integrand size = 23, antiderivative size = 252

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = -\frac{(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{(a+b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} + \frac{2a}{3de(e\cot(c+dx))^{3/2}} + \frac{2b}{de^2\sqrt{e\cot(c+dx)}} - \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} + \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}$$

output
```
2/3*a/d/e/(e*cot(d*x+c))^(3/2)-1/2*(a+b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(
1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/2*(a+b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^
(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2
^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)+1/4*(a-b)*ln(e^(1/2)+cot(d*
x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)+2*b/d/e^2/(e*
cot(d*x+c))^(1/2)
```

3.55.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.75 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.78

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = \frac{3b\left(2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\right.}{}$$

input
```
Integrate[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(5/2),x]
```

output
```
(3*b*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*Sqrt[2]*ArcTan[
1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]] + 8*Sqrt[Tan[c + d*x]]) - 8*a*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -
Tan[c + d*x]^2])*Tan[c + d*x]^(3/2))/(12*d*(e*Cot[c + d*x])^(5/2)*Tan[c +
d*x]^(5/2))
```

3.55.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.97, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4012, 3042, 4012, 25, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int \frac{be-ae\cot(c+dx)}{(e\cot(c+dx))^{3/2}}dx}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{be+a\tan\left(c+dx+\frac{\pi}{2}\right)e}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\int -\frac{ae^2+b\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{2b}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2b}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{ae^2+b\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx}{e^2}}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2b}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{ae^2-be^2\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\frac{2b}{d\sqrt{e\cot(c+dx)}}-\frac{2\int -\frac{e^2(ae+b\cot(c+dx)e)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2\int \frac{e^2(ae+b\cot(c+dx)e)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}+\frac{2b}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{ae+b\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}+\frac{2b}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{1482}$$

$$\frac{\frac{2\left(\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}+\frac{2b}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{1476}$$

3.55. $\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx$

$$\frac{\frac{2\left(\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}\right.\right.}{d}}{e^2}$$

$$\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{2\left(\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d}+}{e^2}$$

$$\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2\left(\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d}+\frac{2b}{d\sqrt{e\cot(c+dx)}}}{e^2}+$$

$$\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{2\left(\frac{1}{2}(a-b)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}}{e^2}$$

$$\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2\left(\frac{1}{2}(a-b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}}{e^2}$$

$$\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

3.55. $\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx$

$$\frac{2\left(\frac{1}{2}(a-b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.}{d}}{e^2}$$

$$\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 1103$$

$$\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(e\cot(c+dx)-\sqrt{2}\sqrt{e}\sqrt{e\right.}{2\sqrt{2}\sqrt{e}}\right.}{d}}{e^2}$$

$$\frac{2a}{3de(e\cot(c+dx))^{3/2}}$$

input
```
Int[(a + b*Cot[c + d*x])/(e*Cot[c + d*x])^(5/2),x]
```

output
```
(2*a)/(3*d*e*(e*Cot[c + d*x])^(3/2)) + ((2*b)/(d*Sqrt[e*Cot[c + d*x]]) + (
2*(((a + b)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]
*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*S
qrt[e])))/2 + ((a - b)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt
[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqr
t[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/d)/e^2
```

3.55.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

3.55. $\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx$

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

3.55.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.23

method	result
derivativedivides	$-\frac{2\left(-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx-}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
default	$-\frac{2\left(-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx-}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}\right.}{}$
parts	$-\frac{2ae\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e^4}\right.}{d}$

input
```
int((a+b*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/e^2*(-1/8*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/8*b
/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arcta
n(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))+2/3*a/e/(e*cot(d*x+c))^(3
/2)+2*b/e^2/(e*cot(d*x+c))^(1/2))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. 2(199) = 398.

Time = 0.27 (sec) , antiderivative size = 905, normalized size of antiderivative = 3.59

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx =$$

$$-\frac{3\left(de^3\cos\left(2\,dx+2\,c\right)+de^3\right)\sqrt{-\frac{d^2e^5\sqrt{-\frac{a^4-2\,a^2b^2+b^4}{d^4e^{10}}}+2\,ab}{d^2e^5}}\log\left(-(a^4-b^4)\sqrt{\frac{e\cos(2\,dx+2\,c)+e}{\sin(2\,dx+2\,c)}}+\left(bd^3e^8\sqrt{-\frac{a^4-2\,a}{d^4}}\right.\right.}{}$$

input
```
integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^4 - 2*a^2
*b^2 + b^4)/(d^4*e^10)) + 2*a*b)/(d^2*e^5))*log(-(a^4 - b^4)*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (b*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2 + b
^4)/(d^4*e^10)) + (a^3 - a*b^2)*d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^4 - 2*a^2*b
^2 + b^4)/(d^4*e^10)) + 2*a*b)/(d^2*e^5))) - 3*(d*e^3*cos(2*d*x + 2*c) + d
*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^10)) + 2*a*b)/(d
^2*e^5))*log(-(a^4 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))
- (b*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^10)) + (a^3 - a*b^2)*d*e
^3)*sqrt(-(d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^10)) + 2*a*b)/(d^2
*e^5))) - 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt((d^2*e^5*sqrt(-(a^4 - 2*
a^2*b^2 + b^4)/(d^4*e^10)) - 2*a*b)/(d^2*e^5))*log(-(a^4 - b^4)*sqrt((e*co
s(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (b*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2
+ b^4)/(d^4*e^10)) - (a^3 - a*b^2)*d*e^3)*sqrt((d^2*e^5*sqrt(-(a^4 - 2*a^2
*b^2 + b^4)/(d^4*e^10)) - 2*a*b)/(d^2*e^5))) + 3*(d*e^3*cos(2*d*x + 2*c) +
 d*e^3)*sqrt((d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^10)) - 2*a*b)/(
d^2*e^5))*log(-(a^4 - b^4)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))
 - (b*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^10)) - (a^3 - a*b^2)*d*
e^3)*sqrt((d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/(d^4*e^10)) - 2*a*b)/(d^2
*e^5))) + 4*(a*cos(2*d*x + 2*c) - 3*b*sin(2*d*x + 2*c) - a)*sqrt((e*cos(2*
d*x + 2*c) + e)/sin(2*d*x + 2*c)))/(d*e^3*cos(2*d*x + 2*c) + d*e^3)
```

3.55.6 Sympy [F]

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx=\int \frac{a+b\cot{(c+dx)}}{(e\cot{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*cot(c + d*x))/(e*cot(c + d*x))**(5/2), x)
```

3.55.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.55.8 Giac [F]

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx=\int \frac{b\cot{(dx+c)}+a}{(e\cot{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)/(e*cot(d*x + c))^(5/2), x)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.63

$$\int \frac{a+b\cot(c+dx)}{(e\cot(c+dx))^{5/2}}\,dx = \frac{2\,a}{3\,d\,e\,(e\cot\left(c+d\,x\right))^{3/2}} + \frac{2\,b}{d\,e^2\,\sqrt{e\cot\left(c+d\,x\right)}} + \frac{(-1)^{1/4}\,b\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,e^{5/2}} - \frac{(-1)^{1/4}\,b\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)}{d\,e^{5/2}} - \frac{(-1)^{1/4}\,a\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d\,e^{5/2}} - \frac{(-1)^{1/4}\,a\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{e\cot(c+d\,x)}}{\sqrt{e}}\right)\,1\mathrm{i}}{d\,e^{5/2}}$$

input
```
int((a + b*cot(c + d*x))/(e*cot(c + d*x))^(5/2),x)
```

output
```
(2*a)/(3*d*e*(e*cot(c + d*x))^(3/2)) + (2*b)/(d*e^2*(e*cot(c + d*x))^(1/2)
) - ((-1)^(1/4)*a*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2))*1i)/(d
*e^(5/2)) - ((-1)^(1/4)*a*atanh(((-1)^(1/4)*(e*cot(c + d*x))^(1/2))/e^(1/2
))*1i)/(d*e^(5/2)) + ((-1)^(1/4)*b*atan(((-1)^(1/4)*(e*cot(c + d*x))^(1/2)
)/e^(1/2)))/(d*e^(5/2)) - ((-1)^(1/4)*b*atanh(((-1)^(1/4)*(e*cot(c + d*x))
^(1/2))/e^(1/2)))/(d*e^(5/2))
```

3.56 $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx$

3.56.1 Optimal result

Integrand size = 25, antiderivative size = 317

$$\begin{aligned}\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = &-\frac{(a^2+2ab-b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\ &+\frac{(a^2+2ab-b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\ &-\frac{2(a^2-b^2)\,e\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}\\ &-\frac{(a^2-2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\\ &+\frac{(a^2-2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\end{aligned}$$

output
```
-4/3*a*b*(e*cot(d*x+c))^(3/2)/d-2/5*b^2*(e*cot(d*x+c))^(5/2)/d/e-1/2*(a^2+
2*a*b-b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2
)+1/2*(a^2+2*a*b-b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2
))/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(
1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)+1/4*(a^2-2*a*b-b^2)*e^(3/2)*ln(e^(1/2
)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)-2*(a^2-b^2)*e
*(e*cot(d*x+c))^(1/2)/d
```

3.56.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.71

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx =$$
$$-\frac{(e\cot(c+dx))^{3/2}\left(\frac{2}{5}b^2\cot^{\frac{5}{2}}(c+dx)-\frac{4}{3}ab\cot^{\frac{3}{2}}(c+dx)\left(-1+\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx\right.\right.\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^2,x]
```

output
```
-(((e*Cot[c + d*x])^(3/2)*((2*b^2*Cot[c + d*x]^(5/2))/5 - (4*a*b*Cot[c + d
*x]^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/3 + ((a^
2 - b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*Arc
Tan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1
 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]] + Cot[c + d*x]]))/4))/(d*Cot[c + d*x]^(3/2)))
```

3.56.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.92, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4026, 3042, 4011, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4026}$$

$$\int (e\cot(c+dx))^{3/2}\left(a^2+2b\cot(c+dx)a-b^2\right)dx-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a^2-2b\tan\left(c+dx+\frac{\pi}{2}\right)a-b^2\right)dx-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{4011}$$

$$\int \sqrt{e\cot(c+dx)}\left(\left(a^2-b^2\right)e\cot(c+dx)-2abe\right)dx-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-2abe-\left(a^2-b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)e\right)dx-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{-\left(\left(a^2-b^2\right)e^2\right)-2ab\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx-\frac{2e\left(a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{2abe^2\tan\left(c+dx+\frac{\pi}{2}\right)-\left(a^2-b^2\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2e\left(a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{4017}$$

$$\frac{2\int \frac{e^2\left(\left(a^2-b^2\right)e+2ab\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2e\left(a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{27}$$

$$\frac{2e^2\int \frac{\left(a^2-b^2\right)e+2ab\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2e\left(a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$$\Big\downarrow \text{1482}$$

3.56. $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx$

$$\frac{2e^2\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2+2ab-b^2\right)\int\frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-$$
$$\frac{2e\left(a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$\downarrow$ 1476

$$\frac{2e^2\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\right.}{d}$$
$$\frac{2e\left(a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$\downarrow$ 1082

$$\frac{2e^2\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\int\frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\right.\right.}{d}$$
$$\frac{2e\left(a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$\downarrow$ 217

$$\frac{2e^2\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\right.}{}\right.\right.}{d}$$
$$\frac{2e\left(a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$\downarrow$ 1479

$$\frac{2e^2\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)\right.}{d}$$
$$\frac{2e\left(a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$\downarrow$ 25

3.56. $\int(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx$

$$\frac{2e^2\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(\ldots}{d}$$

$$\frac{2e\left(a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$\Big\downarrow$ 27

$$\frac{2e^2\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}(\ldots}{d}$$

$$\frac{2e\left(a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

$\Big\downarrow$ 1103

$$\frac{2e^2\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{\ldots}\right.}{2\sqrt{\ldots}}\right.\ldots}{d}$$

$$\frac{2e\left(a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{4ab(e\cot(c+dx))^{3/2}}{3d}-\frac{2b^2(e\cot(c+dx))^{5/2}}{5de}$$

input
```
Int[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^2,x]
```

output
```
(-2*(a^2 - b^2)*e*Sqrt[e*Cot[c + d*x]])/d - (4*a*b*(e*Cot[c + d*x])^(3/2))
/(3*d) - (2*b^2*(e*Cot[c + d*x])^(5/2))/(5*d*e) + (2*e^2*(((a^2 + 2*a*b -
b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e
])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])
))/2 + ((a^2 - 2*a*b - b^2)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]
*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2
]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/d
```

3.56. $\int(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx$

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

3.56.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.14

method	result
derivativedivides	$-\frac{2\left(\frac{b^2(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2aeb(e\cot(dx+c))^{\frac{3}{2}}}{3}+a^2e^2\sqrt{e\cot(dx+c)}-\sqrt{e\cot(dx+c)}\,b^2e^2-e^3\left(\frac{\left(a^2e-b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot}{e\cot}\right.\right.}{}\right.\right.}{}$
default	$-\frac{2\left(\frac{b^2(e\cot(dx+c))^{\frac{5}{2}}}{5}+\frac{2aeb(e\cot(dx+c))^{\frac{3}{2}}}{3}+a^2e^2\sqrt{e\cot(dx+c)}-\sqrt{e\cot(dx+c)}\,b^2e^2-e^3\left(\frac{\left(a^2e-b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot}{e\cot}\right.\right.}{}\right.\right.}{}$
parts	$-\frac{2a^2e\left(\sqrt{e\cot(dx+c)}-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arcta}\right.}{8}\right.}{d}$

input
```
int((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d/e*(1/5*b^2*(e*cot(d*x+c))^(5/2)+2/3*a*e*b*(e*cot(d*x+c))^(3/2)+a^2*e^
2*(e*cot(d*x+c))^(1/2)-(e*cot(d*x+c))^(1/2)*b^2*e^2-e^3*(1/8*(a^2*e-b^2*e)
*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/4*a*b/(e^2)^(1/4)*2^
(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2
*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1313 vs. $2(260) = 520$.

Time = 0.30 (sec) , antiderivative size = 1313, normalized size of antiderivative = 4.14

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/30*(15*(d*cos(2*d*x + 2*c) - d)*sqrt(-(4*(a^3*b - a*b^3)*e^3 + sqrt(-(a^
8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^6/d^4)*d^2)/d^2)*log((a^
8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*e^4*sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c)) + (2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2
*b^6 + b^8)*e^6/d^4)*a*b*d^3 + (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e^3)*
sqrt(-(4*(a^3*b - a*b^3)*e^3 + sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a
^2*b^6 + b^8)*e^6/d^4)*d^2)/d^2)) - 15*(d*cos(2*d*x + 2*c) - d)*sqrt(-(4*(
a^3*b - a*b^3)*e^3 + sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b
^8)*e^6/d^4)*d^2)/d^2)*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8
)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (2*sqrt(-(a^8 - 12
*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^6/d^4)*a*b*d^3 + (a^6 - 7*a^4*
b^2 + 7*a^2*b^4 - b^6)*d*e^3)*sqrt(-(4*(a^3*b - a*b^3)*e^3 + sqrt(-(a^8 -
12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^6/d^4)*d^2)/d^2)) - 15*(d*co
s(2*d*x + 2*c) - d)*sqrt(-(4*(a^3*b - a*b^3)*e^3 - sqrt(-(a^8 - 12*a^6*b^2
 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^6/d^4)*d^2)/d^2)*log((a^8 - 4*a^6*b^2
- 10*a^4*b^4 - 4*a^2*b^6 + b^8)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*
x + 2*c)) + (2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^
6/d^4)*a*b*d^3 - (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e^3)*sqrt(-(4*(a^3*
b - a*b^3)*e^3 - sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*
e^6/d^4)*d^2)/d^2)) + 15*(d*cos(2*d*x + 2*c) - d)*sqrt(-(4*(a^3*b - a*b...
```

3.56.6 Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = \int (e\cot(c+dx))^{\frac{3}{2}}(a+b\cot(c+dx))^2\,dx$$

input
```
integrate((e*cot(d*x+c))**(3/2)*(a+b*cot(d*x+c))**2,x)
```

output
```
Integral((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**2, x)
```

3.56.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.56.8 Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = \int (b\cot(dx+c)+a)^2(e\cot(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^2*(e*cot(d*x + c))^(3/2), x)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 1274, normalized size of antiderivative = 4.02

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))^2,x)
```

output
```
atan((a^4*e^6*(e*cot(c + d*x))^(1/2)*((a*b^3*e^3)/d^2 - (b^4*e^3*1i)/(4*d^
2) - (a^4*e^3*1i)/(4*d^2) - (a^3*b*e^3)/d^2 + (a^2*b^2*e^3*3i)/(2*d^2))^(1
/2)*32i)/((a^6*e^8*16i)/d - (b^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b
*e^8)/d + (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d - (a^4*b^2*e^8*112i)/
d) + (b^4*e^6*(e*cot(c + d*x))^(1/2)*((a*b^3*e^3)/d^2 - (b^4*e^3*1i)/(4*d^
2) - (a^4*e^3*1i)/(4*d^2) - (a^3*b*e^3)/d^2 + (a^2*b^2*e^3*3i)/(2*d^2))^(1
/2)*32i)/((a^6*e^8*16i)/d - (b^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*a^5*b
*e^8)/d + (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d - (a^4*b^2*e^8*112i)/
d) - (a^2*b^2*e^6*(e*cot(c + d*x))^(1/2)*((a*b^3*e^3)/d^2 - (b^4*e^3*1i)/(
4*d^2) - (a^4*e^3*1i)/(4*d^2) - (a^3*b*e^3)/d^2 + (a^2*b^2*e^3*3i)/(2*d^2)
)^(1/2)*192i)/((a^6*e^8*16i)/d - (b^6*e^8*16i)/d + (32*a*b^5*e^8)/d + (32*
a^5*b*e^8)/d + (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*e^8)/d - (a^4*b^2*e^8*1
12i)/d))*(-(a^4*e^3*1i + b^4*e^3*1i - 4*a*b^3*e^3 + 4*a^3*b*e^3 - a^2*b^2*
e^3*6i)/(4*d^2))^(1/2)*2i + atan((a^4*e^6*(e*cot(c + d*x))^(1/2)*((a^4*e^3
*1i)/(4*d^2) + (b^4*e^3*1i)/(4*d^2) + (a*b^3*e^3)/d^2 - (a^3*b*e^3)/d^2 -
(a^2*b^2*e^3*3i)/(2*d^2))^(1/2)*32i)/((b^6*e^8*16i)/d - (a^6*e^8*16i)/d +
(32*a*b^5*e^8)/d + (32*a^5*b*e^8)/d - (a^2*b^4*e^8*112i)/d - (192*a^3*b^3*
e^8)/d + (a^4*b^2*e^8*112i)/d) + (b^4*e^6*(e*cot(c + d*x))^(1/2)*((a^4*e^3
*1i)/(4*d^2) + (b^4*e^3*1i)/(4*d^2) + (a*b^3*e^3)/d^2 - (a^3*b*e^3)/d^2 -
(a^2*b^2*e^3*3i)/(2*d^2))^(1/2)*32i)/((b^6*e^8*16i)/d - (a^6*e^8*16i)/d...
```

3.56. $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2\,dx$

3.57 $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx$

3.57.1 Optimal result

Integrand size = 25, antiderivative size = 288

$$\begin{aligned}\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx &= \frac{(a^2-2ab-b^2)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\ &- \frac{(a^2-2ab-b^2)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d} \\ &- \frac{4ab\sqrt{e\cot(c+dx)}}{d} - \frac{2b^2(e\cot(c+dx))^{3/2}}{3de} \\ &- \frac{(a^2+2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d} \\ &+ \frac{(a^2+2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\end{aligned}$$

output
```
-2/3*b^2*(e*cot(d*x+c))^(3/2)/d/e+1/2*(a^2-2*a*b-b^2)*arctan(1-2^(1/2)*(e*
cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-1/2*(a^2-2*a*b-b^2)*arctan(1+
2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-1/4*(a^2+2*a*b-b^2
)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/d*2^
(1/2)+1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x
+c))^(1/2))*e^(1/2)/d*2^(1/2)-4*a*b*(e*cot(d*x+c))^(1/2)/d
```

3.57.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.76

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx =$$
$$-\frac{\sqrt{e\cot(c+dx)}\Big(4(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx)\right)+b\Big(6\sqrt{2}a\,\text{arctan}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2,x]
```

output
```
-1/6*(Sqrt[e*Cot[c + d*x]]*(4*(a^2 - b^2)*Cot[c + d*x]^(3/2)*Hypergeometri
c2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + b*(6*Sqrt[2]*a*ArcTan[1 - Sqrt[2]*Sqr
t[Cot[c + d*x]]] - 6*Sqrt[2]*a*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24
*a*Sqrt[Cot[c + d*x]] + 4*b*Cot[c + d*x]^(3/2) + 3*Sqrt[2]*a*Log[1 - Sqrt[
2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 3*Sqrt[2]*a*Log[1 + Sqrt[2]*Sqrt[C
ot[c + d*x]] + Cot[c + d*x]])))/(d*Sqrt[Cot[c + d*x]])
```

3.57.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.91, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4026, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4026}$$

$$\int \sqrt{e\cot(c+dx)}\left(a^2+2b\cot(c+dx)a-b^2\right)dx-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-2b\tan\left(c+dx+\frac{\pi}{2}\right)a-b^2\right)dx-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$\big\downarrow$ 4011

$$\int \frac{\left(a^2-b^2\right)e\cot(c+dx)-2abe}{\sqrt{e\cot(c+dx)}}dx-\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$\big\downarrow$ 3042

$$\int \frac{-2abe-\left(a^2-b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)e}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$\big\downarrow$ 4017

$$\frac{2\int \frac{e(2abe-(a^2-b^2)e\cot(c+dx))}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$\big\downarrow$ 27

$$\frac{2e\int \frac{2abe-(a^2-b^2)e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$\big\downarrow$ 1482

$$\frac{2e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$\big\downarrow$ 1476

$$\frac{2e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e}\right.\right.}{d}$$
$$\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$\big\downarrow$ 1082

$$\frac{2e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int}{}\right.\right.}{d}$$
$$\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$\big\downarrow$ 217

3.57. $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx$

$$\frac{2e\left(\frac{1}{2}(a^2+2ab-b^2)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d}$$
$$\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$\downarrow$ 1479

$$\frac{2e\left(\frac{1}{2}(a^2+2ab-b^2)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\dots\right)}{d}$$
$$\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$\downarrow$ 25

$$\frac{2e\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a\dots\right)}{d}$$
$$\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$\downarrow$ 27

$$\frac{2e\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)-\frac{1}{2}(a\dots\right)}{d}$$
$$\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

$\downarrow$ 1103

$$\frac{2e\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(e\cot(c+dx)-\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a^2-2ab-b\dots\right)}{d}$$
$$\frac{4ab\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2(e\cot(c+dx))^{3/2}}{3de}$$

input
```
Int[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2,x]
```

3.57. $\int\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx$

output
```
(-4*a*b*Sqrt[e*Cot[c + d*x]])/d - (2*b^2*(e*Cot[c + d*x])^(3/2))/(3*d*e) +
 (2*e*(-1/2*((a^2 - 2*a*b - b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x
]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]]
)/Sqrt[e]]/(Sqrt[2]*Sqrt[e]))) + ((a^2 + 2*a*b - b^2)*(-1/2*Log[e + e*Cot[
c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e
 + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[
e])))/2))/d
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

3.57. $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx$

3.57.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.11

method	result
derivativedivides	$-\frac{2\left(\frac{b^2(e\cot(dx+c))^{\frac{3}{2}}}{3}+2abe\sqrt{e\cot(dx+c)}+e^2\left(-\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\cdots\right.\right.}{4e}\right.\right.}{\cdots}$
default	$-\frac{2\left(\frac{b^2(e\cot(dx+c))^{\frac{3}{2}}}{3}+2abe\sqrt{e\cot(dx+c)}+e^2\left(-\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\cdots\right.\right.}{4e}\right.\right.}{\cdots}$
parts	$-\frac{a^2e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{e\cot(dx+c\cdots}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{4d(e^2)^{\frac{1}{4}}}$

input
```
int((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d/e*(1/3*b^2*(e*cot(d*x+c))^(3/2)+2*a*b*e*(e*cot(d*x+c))^(1/2)+e^2*(-1/
4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2
^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-
2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^2-b^2)/(e^2)
^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(
1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. $2(235) = 470$.

Time = 0.29 (sec) , antiderivative size = 1230, normalized size of antiderivative = 4.27

$$\int \sqrt{e \cot(c + dx)}(a + b \cot(c + dx))^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(3*d*sqrt((d^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^
8)*e^2/d^4) + 4*(a^3*b - a*b^3)*e)/d^2)*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4
- 4*a^2*b^6 + b^8)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a
^2 - b^2)*d^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^2
/d^4) - 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e)*sqrt((d^2*sqrt(-(a^8 - 12*a^6*b
^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^2/d^4) + 4*(a^3*b - a*b^3)*e)/d^2))*
sin(2*d*x + 2*c) - 3*d*sqrt((d^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12
*a^2*b^6 + b^8)*e^2/d^4) + 4*(a^3*b - a*b^3)*e)/d^2)*log((a^8 - 4*a^6*b^2
- 10*a^4*b^4 - 4*a^2*b^6 + b^8)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x
+ 2*c)) - ((a^2 - b^2)*d^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b
^6 + b^8)*e^2/d^4) - 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e)*sqrt((d^2*sqrt(-(a
^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^2/d^4) + 4*(a^3*b - a*b
^3)*e)/d^2))*sin(2*d*x + 2*c) - 3*d*sqrt(-(d^2*sqrt(-(a^8 - 12*a^6*b^2 + 3
8*a^4*b^4 - 12*a^2*b^6 + b^8)*e^2/d^4) - 4*(a^3*b - a*b^3)*e)/d^2)*log((a^
8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*e*sqrt((e*cos(2*d*x + 2*c) +
 e)/sin(2*d*x + 2*c)) + ((a^2 - b^2)*d^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*
b^4 - 12*a^2*b^6 + b^8)*e^2/d^4) + 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e)*sqrt
(-(d^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^2/d^4) -
 4*(a^3*b - a*b^3)*e)/d^2))*sin(2*d*x + 2*c) + 3*d*sqrt(-(d^2*sqrt(-(a^8 -
 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)*e^2/d^4) - 4*(a^3*b - a*b^...
```

3.57.6 Sympy [F]

$$\int \sqrt{e \cot(c + dx)}(a + b \cot(c + dx))^2 \, dx = \int \sqrt{e \cot (c + dx)}(a + b \cot (c + dx))^2 \, dx$$

input
```
integrate((e*cot(d*x+c))**(1/2)*(a+b*cot(d*x+c))**2,x)
```

output
```
Integral(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))**2, x)
```

3.57.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.57.8 Giac [F]

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx = \int (b\cot(dx+c)+a)^2\sqrt{e\cot(dx+c)}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^2*sqrt(e*cot(d*x + c)), x)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 1157, normalized size of antiderivative = 4.02

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))^2,x)
```

output
```
atan((a^4*e^4*(e*cot(c + d*x))^(1/2)*((a^4*e*1i)/(4*d^2) + (b^4*e*1i)/(4*d
^2) - (a^2*b^2*e*3i)/(2*d^2) - (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*32i)/(
(16*b^6*e^5)/d - (16*a^6*e^5)/d + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d -
(112*a^2*b^4*e^5)/d - (a^3*b^3*e^5*192i)/d + (112*a^4*b^2*e^5)/d) + (b^4*e
^4*(e*cot(c + d*x))^(1/2)*((a^4*e*1i)/(4*d^2) + (b^4*e*1i)/(4*d^2) - (a^2*
b^2*e*3i)/(2*d^2) - (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*32i)/((16*b^6*e^5
)/d - (16*a^6*e^5)/d + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d - (112*a^2*b^
4*e^5)/d - (a^3*b^3*e^5*192i)/d + (112*a^4*b^2*e^5)/d) - (a^2*b^2*e^4*(e*c
ot(c + d*x))^(1/2)*((a^4*e*1i)/(4*d^2) + (b^4*e*1i)/(4*d^2) - (a^2*b^2*e*3
i)/(2*d^2) - (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*192i)/((16*b^6*e^5)/d -
(16*a^6*e^5)/d + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d - (112*a^2*b^4*e^5)
/d - (a^3*b^3*e^5*192i)/d + (112*a^4*b^2*e^5)/d))*((a^4*e*1i + b^4*e*1i -
a^2*b^2*e*6i - 4*a*b^3*e + 4*a^3*b*e)/(4*d^2))^(1/2)*2i - atan((a^4*e^4*(e
*cot(c + d*x))^(1/2)*((a^2*b^2*e*3i)/(2*d^2) - (b^4*e*1i)/(4*d^2) - (a^4*e
*1i)/(4*d^2) - (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*32i)/((16*a^6*e^5)/d -
 (16*b^6*e^5)/d + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d + (112*a^2*b^4*e^5
)/d - (a^3*b^3*e^5*192i)/d - (112*a^4*b^2*e^5)/d) + (b^4*e^4*(e*cot(c + d*
x))^(1/2)*((a^2*b^2*e*3i)/(2*d^2) - (b^4*e*1i)/(4*d^2) - (a^4*e*1i)/(4*d^2
) - (a*b^3*e)/d^2 + (a^3*b*e)/d^2)^(1/2)*32i)/((16*a^6*e^5)/d - (16*b^6*e^
5)/d + (a*b^5*e^5*32i)/d + (a^5*b*e^5*32i)/d + (112*a^2*b^4*e^5)/d - (a...
```

3.57. $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2\,dx$

3.58 $\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

3.58.1 Optimal result

Integrand size = 25, antiderivative size = 267

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \frac{(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{(a^2+2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} - \frac{2b^2\sqrt{e\cot(c+dx)}}{de} + \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} - \frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}$$

output
```
1/2*(a^2+2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/
2)/e^(1/2)-1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/
2))/d*2^(1/2)/e^(1/2)+1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^
(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)/e^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(e^(1/
2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)/e^(1/2)-2*b^
2*(e*cot(d*x+c))^(1/2)/d/e
```

3.58.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.94 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.72

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx =$$
$$-\frac{\sqrt{\cot(c+dx)}\left(2b^2\sqrt{\cot(c+dx)}+\frac{4}{3}ab\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx)\right)-\frac{(a^2}{}\right.}{d\sqrt{e}}$$

input
```
Integrate[(a + b*Cot[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

output
```
-((Sqrt[Cot[c + d*x]]*(2*b^2*Sqrt[Cot[c + d*x]] + (4*a*b*Cot[c + d*x]^(3/2
)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])/3 - ((a^2 - b^2)*(2*Arc
Tan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*
x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2
]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(2*Sqrt[2])))/(d*Sqrt[e*Cot[c + d*x
]]))
```

3.58.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4026, 3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4026}$$

$$\int \frac{a^2+2b\cot(c+dx)a-b^2}{\sqrt{e\cot(c+dx)}}dx-\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a^2-2b\tan\left(c+dx+\frac{\pi}{2}\right)a-b^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \ 4017$$

$$\frac{2\int -\frac{(a^2-b^2)e+2ab\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \ 25$$

$$-\frac{2\int \frac{(a^2-b^2)e+2ab\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \ 1482$$

$$\frac{2\left(-\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \ 1476$$

$$\frac{2\left(-\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\right.\right.}{d}$$

$$\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \ 1082$$

$$\frac{2\left(-\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\right.\right.}{d}$$

$$\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \ 217$$

$$\frac{2\left(-\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\right.}{}\right.\right.}{d}$$

$$\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

3.58. $\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

$\downarrow$ 1479

$$2\left(-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right) \cdots \right. \over d$$
$$\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

$\downarrow$ 25

$$2\left(-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\left(\cdots\right. \over d$$
$$\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

$\downarrow$ 27

$$2\left(-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)-\frac{1}{2}\left(\cdots\right. \over d$$
$$\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

$\downarrow$ 1103

$$2\left(-\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\cdots\right.}{2\sqrt{\cdots}}\right. \over d$$
$$\frac{2b^2\sqrt{e\cot(c+dx)}}{de}$$

input
```
Int[(a + b*Cot[c + d*x])^2/Sqrt[e*Cot[c + d*x]],x]
```

output
```
(-2*b^2*Sqrt[e*Cot[c + d*x]])/(d*e) + (2*(-1/2*((a^2 + 2*a*b - b^2)*(-(Arc
Tan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTa
n[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e]))) - ((a^2
- 2*a*b - b^2)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c
 + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqr
t[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/d
```

3.58. $\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

3.58. $\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4026
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

3.58.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.15

method	result
derivativedivides	$-\frac{2\left(\sqrt{e\cot(dx+c)}\,b^2+e\left(\frac{\left(a^2e-b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e^2}\right.\right.}{}$
default	$-\frac{2\left(\sqrt{e\cot(dx+c)}\,b^2+e\left(\frac{\left(a^2e-b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e^2}\right.\right.}{}$
parts	$-\frac{a^2\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot}}{\left(e^2\right)}\right.\right.}{4de}$

input
```
int((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/e*((e*cot(d*x+c))^(1/2)*b^2+e*(1/8*(a^2*e-b^2*e)*(e^2)^(1/4)/e^2*2^(1
/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)
)/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*a
rctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/4*a*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+
c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)+1))))
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. $2(218) = 436$.

Time = 0.29 (sec) , antiderivative size = 1183, normalized size of antiderivative = 4.43

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(d*e*sqrt(-(4*a^3*b - 4*a*b^3 + d^2*e*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^
4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^2)))/(d^2*e))*log((a^8 - 4*a^6*b^2 - 10*a
^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))
+ (2*a*b*d^3*e^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/
(d^4*e^2)) + (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e)*sqrt(-(4*a^3*b - 4*a
*b^3 + d^2*e*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4
*e^2)))/(d^2*e))) - d*e*sqrt(-(4*a^3*b - 4*a*b^3 + d^2*e*sqrt(-(a^8 - 12*a
^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^2)))/(d^2*e))*log((a^8 - 4*
a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(
2*d*x + 2*c)) - (2*a*b*d^3*e^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a
^2*b^6 + b^8)/(d^4*e^2)) + (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e)*sqrt(-
(4*a^3*b - 4*a*b^3 + d^2*e*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b
^6 + b^8)/(d^4*e^2)))/(d^2*e))) - d*e*sqrt(-(4*a^3*b - 4*a*b^3 - d^2*e*sqr
t(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^2)))/(d^2*e))
*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x +
2*c) + e)/sin(2*d*x + 2*c)) + (2*a*b*d^3*e^2*sqrt(-(a^8 - 12*a^6*b^2 + 38*
a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^2)) - (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^
6)*d*e)*sqrt(-(4*a^3*b - 4*a*b^3 - d^2*e*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*
b^4 - 12*a^2*b^6 + b^8)/(d^4*e^2)))/(d^2*e))) + d*e*sqrt(-(4*a^3*b - 4*a*b
^3 - d^2*e*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^...
```

3.58.6 Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{(a+b\cot{(c+dx)})^2}{\sqrt{e\cot{(c+dx)}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(1/2),x)
```

output
```
Integral((a + b*cot(c + d*x))**2/sqrt(e*cot(c + d*x)), x)
```

3.58.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.58.8 Giac [F]

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \int \frac{(b\cot{(dx+c)}+a)^2}{\sqrt{e\cot{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^2/sqrt(e*cot(d*x + c)), x)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 13.42 (sec) , antiderivative size = 1234, normalized size of antiderivative = 4.62

$$\int \frac{(a+b\cot(c+dx))^2}{\sqrt{e\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(1/2),x)
```

output
```
2*atanh((32*a^4*e^2*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e) - (b^4*1i)/(4*
d^2*e) - (a^4*1i)/(4*d^2*e) - (a^3*b)/(d^2*e) + (a^2*b^2*3i)/(2*d^2*e))^(1
/2))/((a^6*e^2*16i)/d - (b^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2
)/d + (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d - (a^4*b^2*e^2*112i)/d) +
 (32*b^4*e^2*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e) - (b^4*1i)/(4*d^2*e)
- (a^4*1i)/(4*d^2*e) - (a^3*b)/(d^2*e) + (a^2*b^2*3i)/(2*d^2*e))^(1/2))/((
a^6*e^2*16i)/d - (b^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d + (
a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d - (a^4*b^2*e^2*112i)/d) - (192*a
^2*b^2*e^2*(e*cot(c + d*x))^(1/2)*((a*b^3)/(d^2*e) - (b^4*1i)/(4*d^2*e) -
(a^4*1i)/(4*d^2*e) - (a^3*b)/(d^2*e) + (a^2*b^2*3i)/(2*d^2*e))^(1/2))/((a^
6*e^2*16i)/d - (b^6*e^2*16i)/d + (32*a*b^5*e^2)/d + (32*a^5*b*e^2)/d + (a^
2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)/d - (a^4*b^2*e^2*112i)/d))*((a*b^3)/
(d^2*e) - (b^4*1i)/(4*d^2*e) - (a^4*1i)/(4*d^2*e) - (a^3*b)/(d^2*e) + (a^2
*b^2*3i)/(2*d^2*e))^(1/2) + 2*atanh((32*a^4*e^2*(e*cot(c + d*x))^(1/2)*((a
^4*1i)/(4*d^2*e) + (b^4*1i)/(4*d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e)
- (a^2*b^2*3i)/(2*d^2*e))^(1/2))/((b^6*e^2*16i)/d - (a^6*e^2*16i)/d + (32*
a*b^5*e^2)/d + (32*a^5*b*e^2)/d - (a^2*b^4*e^2*112i)/d - (192*a^3*b^3*e^2)
/d + (a^4*b^2*e^2*112i)/d) + (32*b^4*e^2*(e*cot(c + d*x))^(1/2)*((a^4*1i)/
(4*d^2*e) + (b^4*1i)/(4*d^2*e) + (a*b^3)/(d^2*e) - (a^3*b)/(d^2*e) - (a^2*
b^2*3i)/(2*d^2*e))^(1/2))/((b^6*e^2*16i)/d - (a^6*e^2*16i)/d + (32*a*b^...
```

3.59 $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

3.59.1 Optimal result

Integrand size = 25, antiderivative size = 267

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = &-\frac{(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\ &+\frac{(a^2-2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}}+\frac{2a^2}{de\sqrt{e\cot(c+dx)}} \\ &+\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\ &-\frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}\end{aligned}$$

output
```
-1/2*(a^2-2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(3
/2)*2^(1/2)+1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1
/2))/d/e^(3/2)*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2
^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(3/2)*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(e^(1
/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(3/2)*2^(1/2)+2*a
^2/d/e/(e*cot(d*x+c))^(1/2)
```

3.59.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.70

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx =$$
$$-\frac{\cot^{\frac{3}{2}}(c+dx)\left(-\frac{2b^2}{\sqrt{\cot(c+dx)}}-\frac{2(a^2-b^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\cot^2(c+dx)\right)}{\sqrt{\cot(c+dx)}}-\frac{ab\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\arctan\left(\right.\right.}{}\right.}{d(e\cot(c+dx))^{3/2}}$$

input
```
Integrate[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

output
```
-((Cot[c + d*x]^(3/2)*((-2*b^2)/Sqrt[Cot[c + d*x]] - (2*(a^2 - b^2)*Hyperg
eometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2])/Sqrt[Cot[c + d*x]] - (a*b*(2*A
rcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c +
d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/Sqrt[2]))/(d*(e*Cot[c + d*x])^(3/
2)))
```

3.59.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3042, 4025, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{4025}$$
$$\frac{\int \frac{2abe-(a^2-b^2)e\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{2a^2}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{2abe+\left(a^2-b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)e}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}+\frac{2a^2}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 4017$$

$$\frac{2\int -\frac{e\left(2abe-\left(a^2-b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}+\frac{2a^2}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{2\int \frac{e\left(2abe-\left(a^2-b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}$$

$$\Big\downarrow \; 27$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{2\int \frac{2abe-\left(a^2-b^2\right)e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de}$$

$$\Big\downarrow \; 1482$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{de}$$

$$\Big\downarrow \; 1476$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot\right.\right.}{de}$$

$$\Big\downarrow \; 1082$$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int}{}\right.\right.}{de}$$

$$\Big\downarrow \; 217$$

3.59. $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a^2+2ab-b^2)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{}}{}\right.}{\sqrt{2}\sqrt{}}\right.}{de}$$

$\downarrow$ 1479

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a^2+2ab-b^2)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\right.}{de}$$

$\downarrow$ 25

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a^2\right.}{de}$$

$\downarrow$ 27

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)-\frac{1}{2}(a^2\right.}{de}$$

$\downarrow$ 1103

$$\frac{2a^2}{de\sqrt{e\cot(c+dx)}}-\frac{2\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(e\cot(c+dx)-\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a^2-2ab-b^2\right.}{de}$$

input
```
Int[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(3/2),x]
```

3.59. $\int\frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

output
```
(2*a^2)/(d*e*Sqrt[e*Cot[c + d*x]]) - (2*(-1/2*((a^2 - 2*a*b - b^2)*(-(ArcT
an[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan
[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e]))) + ((a^2 +
 2*a*b - b^2)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c
+ d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt
[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/(d*e)
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.59. $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4025
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e
+ f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
 && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

3.59. $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

3.59.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.13

method	result
derivativedivides	$-\frac{2\left(\frac{ab\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx\cdots}}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right)\right)}{4e}\cdots\right)}{\cdots}$
default	$-\frac{2\left(\frac{ab\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx\cdots}}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right)\right)}{4e}\cdots\right)}{\cdots}$
parts	$-\frac{2a^2e\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}\cdots\right)\right)}{8e^2\left(e^2\right)^{\frac{1}{4}}}\cdots\right)}{d}$

input
```
int((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/e*(1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-a^2
+b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(
1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*
arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-a^2/(e*cot(d*x+c))^(1
/2))
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1298 vs. $2(218) = 436$.

Time = 0.31 (sec) , antiderivative size = 1298, normalized size of antiderivative = 4.86

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

output

```
1/2*(4*a^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c
) + (d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt((d^2*e^3*sqrt(-(a^8 - 12*a^6*b^2
 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^6)) + 4*a^3*b - 4*a*b^3)/(d^2*e^3
))*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x
+ 2*c) + e)/sin(2*d*x + 2*c)) + ((a^2 - b^2)*d^3*e^5*sqrt(-(a^8 - 12*a^6*b
^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^6)) - 2*(a^5*b - 6*a^3*b^3 + a*
b^5)*d*e^2)*sqrt((d^2*e^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^
6 + b^8)/(d^4*e^6)) + 4*a^3*b - 4*a*b^3)/(d^2*e^3))) - (d*e^2*cos(2*d*x +
2*c) + d*e^2)*sqrt((d^2*e^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*
b^6 + b^8)/(d^4*e^6)) + 4*a^3*b - 4*a*b^3)/(d^2*e^3))*log((a^8 - 4*a^6*b^2
 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x +
 2*c)) - ((a^2 - b^2)*d^3*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^
2*b^6 + b^8)/(d^4*e^6)) - 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e^2)*sqrt((d^2*e
^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^6)) + 4
*a^3*b - 4*a*b^3)/(d^2*e^3))) - (d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt(-(d^
2*e^3*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^6))
- 4*a^3*b + 4*a*b^3)/(d^2*e^3))*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*
b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^2 - b^2)*
d^3*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^6)
) + 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e^2)*sqrt(-(d^2*e^3*sqrt(-(a^8 - 12...
```

3.59.6 Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{(a+b\cot{(c+dx)})^2}{(e\cot{(c+dx)})^{\frac{3}{2}}}\,dx$$

input

```
integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(3/2),x)
```

output

```
Integral((a + b*cot(c + d*x))**2/(e*cot(c + d*x))**(3/2), x)
```

3.59.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.59.8 Giac [F]

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{(b\cot(dx+c)+a)^2}{(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(3/2), x)
```

3.59.9 Mupad [B] (verification not implemented)

Time = 12.92 (sec) , antiderivative size = 1196, normalized size of antiderivative = 4.48

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx = \frac{2\,a^2}{d\,e\,\sqrt{e\cot\left(c+d\,x\right)}}$$

$$+2\,\mathrm{atanh}\left(\frac{32\,a^4\,d^3\,e^5\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^4\,1\mathrm{i}}{4\,d^2\,e^3}+\frac{b^4\,1\mathrm{i}}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}+\frac{a^3\,b}{d^2\,e^3}-\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2\,e^3}}}{-16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32\mathrm{i}+112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192\mathrm{i}-112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32\mathrm{i}+1}\right.$$

$$+\frac{32\,b^4\,d^3\,e^5\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^4\,1\mathrm{i}}{4\,d^2\,e^3}+\frac{b^4\,1\mathrm{i}}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}+\frac{a^3\,b}{d^2\,e^3}-\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2\,e^3}}}{-16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32\mathrm{i}+112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192\mathrm{i}-112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32\mathrm{i}+16\,b^6\,d^2\,e^4}$$

$$\left.-\frac{192\,a^2\,b^2\,d^3\,e^5\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^4\,1\mathrm{i}}{4\,d^2\,e^3}+\frac{b^4\,1\mathrm{i}}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}+\frac{a^3\,b}{d^2\,e^3}-\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2\,e^3}}}{-16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32\mathrm{i}+112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192\mathrm{i}-112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32\mathrm{i}+16\,b^6\,d^2\,e^4}\right)$$

$$-2\,\mathrm{atanh}\left(\frac{32\,a^4\,d^3\,e^5\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b}{d^2\,e^3}-\frac{b^4\,1\mathrm{i}}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}-\frac{a^4\,1\mathrm{i}}{4\,d^2\,e^3}+\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2\,e^3}}}{16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32\mathrm{i}-112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192\mathrm{i}+112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32\mathrm{i}-16}\right.$$

$$+\frac{32\,b^4\,d^3\,e^5\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b}{d^2\,e^3}-\frac{b^4\,1\mathrm{i}}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}-\frac{a^4\,1\mathrm{i}}{4\,d^2\,e^3}+\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2\,e^3}}}{16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32\mathrm{i}-112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192\mathrm{i}+112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32\mathrm{i}-16\,b^6\,d^2\,e^4}$$

$$\left.-\frac{192\,a^2\,b^2\,d^3\,e^5\,\sqrt{e\cot\left(c+d\,x\right)}\,\sqrt{\frac{a^3\,b}{d^2\,e^3}-\frac{b^4\,1\mathrm{i}}{4\,d^2\,e^3}-\frac{a\,b^3}{d^2\,e^3}-\frac{a^4\,1\mathrm{i}}{4\,d^2\,e^3}+\frac{a^2\,b^2\,3\mathrm{i}}{2\,d^2\,e^3}}}{16\,a^6\,d^2\,e^4+a^5\,b\,d^2\,e^4\,32\mathrm{i}-112\,a^4\,b^2\,d^2\,e^4-a^3\,b^3\,d^2\,e^4\,192\mathrm{i}+112\,a^2\,b^4\,d^2\,e^4+a\,b^5\,d^2\,e^4\,32\mathrm{i}-16\,b^6\,d^2\,e^4}\right)$$

input
```
int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(3/2),x)
```

output
```
2*atanh((32*a^4*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^3) + (b^
4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) + (a^3*b)/(d^2*e^3) - (a^2*b^2*3i)/(
2*d^2*e^3))^(1/2))/(16*b^6*d^2*e^4 - 16*a^6*d^2*e^4 + a*b^5*d^2*e^4*32i +
a^5*b*d^2*e^4*32i - 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i + 112*a^4*b
^2*d^2*e^4) + (32*b^4*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^3)
 + (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) + (a^3*b)/(d^2*e^3) - (a^2*b^2
*3i)/(2*d^2*e^3))^(1/2))/(16*b^6*d^2*e^4 - 16*a^6*d^2*e^4 + a*b^5*d^2*e^4*
32i + a^5*b*d^2*e^4*32i - 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4*192i + 112
*a^4*b^2*d^2*e^4) - (192*a^2*b^2*d^3*e^5*(e*cot(c + d*x))^(1/2)*((a^4*1i)/
(4*d^2*e^3) + (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) + (a^3*b)/(d^2*e^3)
 - (a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*b^6*d^2*e^4 - 16*a^6*d^2*e^4 + a*b
^5*d^2*e^4*32i + a^5*b*d^2*e^4*32i - 112*a^2*b^4*d^2*e^4 - a^3*b^3*d^2*e^4
*192i + 112*a^4*b^2*d^2*e^4))*(((a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b
^2)*1i)/(4*d^2*e^3))^(1/2) - 2*atanh((32*a^4*d^3*e^5*(e*cot(c + d*x))^(1/2
)*((a^3*b)/(d^2*e^3) - (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) - (a^4*1i)
/(4*d^2*e^3) + (a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*a^6*d^2*e^4 - 16*b^6*d
^2*e^4 + a*b^5*d^2*e^4*32i + a^5*b*d^2*e^4*32i + 112*a^2*b^4*d^2*e^4 - a^3
*b^3*d^2*e^4*192i - 112*a^4*b^2*d^2*e^4) + (32*b^4*d^3*e^5*(e*cot(c + d*x)
)^(1/2)*((a^3*b)/(d^2*e^3) - (b^4*1i)/(4*d^2*e^3) - (a*b^3)/(d^2*e^3) - (a
^4*1i)/(4*d^2*e^3) + (a^2*b^2*3i)/(2*d^2*e^3))^(1/2))/(16*a^6*d^2*e^4 -...
```

3.59. $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{3/2}}\,dx$

3.60 $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx$

3.60.1 Optimal result

Integrand size = 25, antiderivative size = 291

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx &= -\frac{(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}\\ &+\frac{(a^2+2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}}\\ &+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}+\frac{4ab}{de^2\sqrt{e\cot(c+dx)}}\\ &-\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}\\ &+\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}\end{aligned}$$

output
```
2/3*a^2/d/e/(e*cot(d*x+c))^(3/2)-1/2*(a^2+2*a*b-b^2)*arctan(1-2^(1/2)*(e*c
ot(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/2*(a^2+2*a*b-b^2)*arctan(1+2
^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)-1/4*(a^2-2*a*b-b^2)
*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(
1/2)+1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+
c))^(1/2))/d/e^(5/2)*2^(1/2)+4*a*b/d/e^2/(e*cot(d*x+c))^(1/2)
```

3.60.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.28

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \frac{2\left((a^2-b^2)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\cot^2(c+dx)\right)+b\left(b+6a\cot(c+c\right.\right.}{3de(e\cot(c+dx))^{3/2}}$$

input
```
Integrate[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(5/2),x]
```

output
```
(2*((a^2 - b^2)*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2] + b*(b +
6*a*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2])))/(3*d*
e*(e*Cot[c + d*x])^(3/2))
```

3.60.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.91, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3042, 4025, 3042, 4012, 25, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \ 4025$$

$$\frac{\int \frac{2abe-(a^2-b^2)e\cot(c+dx)}{(e\cot(c+dx))^{3/2}}dx}{e^2}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{2abe+(a^2-b^2)\tan\left(c+dx+\frac{\pi}{2}\right)e}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{e^2}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\int -\frac{\left(a^2-b^2\right)e^2+2ab\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{4ab}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{4ab}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{\left(a^2-b^2\right)e^2+2ab\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx}{e^2}}{e^2}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{4ab}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{\left(a^2-b^2\right)e^2-2abe^2\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}}{e^2}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\frac{4ab}{d\sqrt{e\cot(c+dx)}}-\frac{2\int -\frac{e^2\left(\left(a^2-b^2\right)e+2ab\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}}{e^2}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2\int \frac{e^2\left(\left(a^2-b^2\right)e+2ab\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}+\frac{4ab}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{\left(a^2-b^2\right)e+2ab\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}+\frac{4ab}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{1482}$$

$$\frac{\frac{2\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}+\frac{4ab}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{1476}$$

3.60. $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx$

$$\frac{\frac{2\left(\frac{1}{2}(a^2-2ab-b^2)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a^2+2ab-b^2)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e+\sqrt{}}\right.\right.}{d}}{e^2}$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$\downarrow$ 1082

$$\frac{\frac{2\left(\frac{1}{2}(a^2-2ab-b^2)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{e\cot(c+}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}}{e^2}$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$\downarrow$ 217

$$\frac{\frac{2\left(\frac{1}{2}(a^2-2ab-b^2)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d}+\frac{4a}{d\sqrt{e\cot}}}{e^2}$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$\downarrow$ 1479

$$\frac{\frac{2\left(\frac{1}{2}(a^2-2ab-b^2)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a^2+2ab-b^2)\left(\right.\right.}{d}}{e^2}$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$\downarrow$ 25

$$\frac{\frac{2\left(\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan}{}\right.\right.}{d}}{e^2}$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$\downarrow$ 27

3.60. $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx$

$$\frac{2\left(\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\text{arctan}\ldots}{\ldots}\right.}{d}}{e^2}$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

$\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(e\cot\ldots\right.}{\ldots}\right.}{d}}{e^2}$$

$$\frac{2a^2}{3de(e\cot(c+dx))^{3/2}}$$

input
```
Int[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(5/2),x]
```

output
```
(2*a^2)/(3*d*e*(e*Cot[c + d*x])^(3/2)) + ((4*a*b)/(d*Sqrt[e*Cot[c + d*x]])
 + (2*(((a^2 + 2*a*b - b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/S
qrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqr
t[e]]/(Sqrt[2]*Sqrt[e])))/2 + ((a^2 - 2*a*b - b^2)*(-1/2*Log[e + e*Cot[c +
 d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e +
e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])
))/2))/d)/e^2
```

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

3.60. $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx$

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4025
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e
+ f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
 && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

3.60.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.12

method	result
derivativedivides	$-\frac{2\left(\frac{\left(-a^2e+b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}}{}\right.\right.}{8e^2}\right.}{}$
default	$-\frac{2\left(\frac{\left(-a^2e+b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}}{}\right.\right.}{8e^2}\right.}{}$
parts	$-\frac{2a^2e\left(-\frac{\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot}}{\left(e^2\right)}\right.\right.}{8e^4}\right.}{d}$

input
```
int((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/e*(1/e*(1/8*(-a^2*e+b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1))-1/4*a*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/3*a^2/(
e*cot(d*x+c))^(3/2)-2*a*b/e/(e*cot(d*x+c))^(1/2))
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1318 vs. $2(238) = 476$.

Time = 0.31 (sec) , antiderivative size = 1318, normalized size of antiderivative = 4.53

$$\int \frac{(a + b\cot(c + dx))^2}{(e\cot(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/6*(3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^8 - 12*a^6
*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^10)) + 4*a^3*b - 4*a*b^3)/(d^
2*e^5))*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + (2*a*b*d^3*e^8*sqrt(-(a^8 - 12*a^6*b^
2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^10)) + (a^6 - 7*a^4*b^2 + 7*a^2*
b^4 - b^6)*d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12
*a^2*b^6 + b^8)/(d^4*e^10)) + 4*a^3*b - 4*a*b^3)/(d^2*e^5))) - 3*(d*e^3*co
s(2*d*x + 2*c) + d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^
4 - 12*a^2*b^6 + b^8)/(d^4*e^10)) + 4*a^3*b - 4*a*b^3)/(d^2*e^5))*log((a^8
 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c)) - (2*a*b*d^3*e^8*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 -
 12*a^2*b^6 + b^8)/(d^4*e^10)) + (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e^3
)*sqrt(-(d^2*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/
(d^4*e^10)) + 4*a^3*b - 4*a*b^3)/(d^2*e^5))) - 3*(d*e^3*cos(2*d*x + 2*c) +
 d*e^3)*sqrt((d^2*e^5*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 +
b^8)/(d^4*e^10)) - 4*a^3*b + 4*a*b^3)/(d^2*e^5))*log((a^8 - 4*a^6*b^2 - 10
*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
) + (2*a*b*d^3*e^8*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8
)/(d^4*e^10)) - (a^6 - 7*a^4*b^2 + 7*a^2*b^4 - b^6)*d*e^3)*sqrt((d^2*e^5*s
qrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^10)) - 4...
```

3.60.6 Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx=\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*cot(c + d*x))**2/(e*cot(c + d*x))**(5/2), x)
```

3.60.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.60.8 Giac [F]

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx=\int \frac{(b\cot(dx+c)+a)^2}{(e\cot(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(5/2), x)
```

3.60.9 Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 1214, normalized size of antiderivative = 4.17

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(5/2),x)
```

output
```
((2*a^2)/3 + 4*a*b*cot(c + d*x))/(d*e*(e*cot(c + d*x))^(3/2)) - 2*atanh((3
2*a^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^5) + (b^4*1i)/(4*d
^2*e^5) + (a*b^3)/(d^2*e^5) - (a^3*b)/(d^2*e^5) - (a^2*b^2*3i)/(2*d^2*e^5)
)^(1/2))/(b^6*d^2*e^6*16i - a^6*d^2*e^6*16i + 32*a*b^5*d^2*e^6 + 32*a^5*b*
d^2*e^6 - a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 + a^4*b^2*d^2*e^6*112
i) + (32*b^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e^5) + (b^4*1
i)/(4*d^2*e^5) + (a*b^3)/(d^2*e^5) - (a^3*b)/(d^2*e^5) - (a^2*b^2*3i)/(2*d
^2*e^5))^(1/2))/(b^6*d^2*e^6*16i - a^6*d^2*e^6*16i + 32*a*b^5*d^2*e^6 + 32
*a^5*b*d^2*e^6 - a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 + a^4*b^2*d^2*
e^6*112i) - (192*a^2*b^2*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a^4*1i)/(4*d^2*e
^5) + (b^4*1i)/(4*d^2*e^5) + (a*b^3)/(d^2*e^5) - (a^3*b)/(d^2*e^5) - (a^2*
b^2*3i)/(2*d^2*e^5))^(1/2))/(b^6*d^2*e^6*16i - a^6*d^2*e^6*16i + 32*a*b^5*
d^2*e^6 + 32*a^5*b*d^2*e^6 - a^2*b^4*d^2*e^6*112i - 192*a^3*b^3*d^2*e^6 +
a^4*b^2*d^2*e^6*112i))*(((a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2)*1i)
/(4*d^2*e^5))^(1/2) - 2*atanh((32*a^4*d^3*e^8*(e*cot(c + d*x))^(1/2)*((a*b
^3)/(d^2*e^5) - (b^4*1i)/(4*d^2*e^5) - (a^4*1i)/(4*d^2*e^5) - (a^3*b)/(d^2
*e^5) + (a^2*b^2*3i)/(2*d^2*e^5))^(1/2))/(a^6*d^2*e^6*16i - b^6*d^2*e^6*16
i + 32*a*b^5*d^2*e^6 + 32*a^5*b*d^2*e^6 + a^2*b^4*d^2*e^6*112i - 192*a^3*b
^3*d^2*e^6 - a^4*b^2*d^2*e^6*112i) + (32*b^4*d^3*e^8*(e*cot(c + d*x))^(1/2
)*((a*b^3)/(d^2*e^5) - (b^4*1i)/(4*d^2*e^5) - (a^4*1i)/(4*d^2*e^5) - (a...
```

3.61 $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

3.61.1 Optimal result

Integrand size = 25, antiderivative size = 322

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx &= \frac{(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} \\ &- \frac{(a^2-2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{2a^2}{5de(e\cot(c+dx))^{5/2}} \\ &+ \frac{4ab}{3de^2(e\cot(c+dx))^{3/2}} - \frac{2(a^2-b^2)}{de^3\sqrt{e\cot(c+dx)}} \\ &- \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}} \\ &+ \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}\end{aligned}$$

output
```
2/5*a^2/d/e/(e*cot(d*x+c))^(5/2)+4/3*a*b/d/e^2/(e*cot(d*x+c))^(3/2)+1/2*(a
^2-2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(
1/2)-1/2*(a^2-2*a*b-b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/
e^(7/2)*2^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*
(e*cot(d*x+c))^(1/2))/d/e^(7/2)*2^(1/2)+1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot
(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(7/2)*2^(1/2)-2*(a^2-b^2
)/d/e^3/(e*cot(d*x+c))^(1/2)
```

3.61.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.26

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \frac{2\left(3(a^2-b^2)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\cot^2(c+dx)\right)+b\left(3b+10a\cot\right.\right.}{15de(e\cot(c+dx))^{5/2}}$$

input
```
Integrate[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(7/2),x]
```

output
```
(2*(3*(a^2 - b^2)*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] + b*(3
*b + 10*a*Cot[c + d*x]*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2])))
/(15*d*e*(e*Cot[c + d*x])^(5/2))
```

3.61.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.93, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.760, Rules used = {3042, 4025, 3042, 4012, 25, 3042, 4012, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4025}$$

$$\frac{\int \frac{2abe-(a^2-b^2)e\cot(c+dx)}{(e\cot(c+dx))^{5/2}}dx}{e^2}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2abe+(a^2-b^2)\tan\left(c+dx+\frac{\pi}{2}\right)e}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{e^2}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\int -\frac{\left(a^2-b^2\right)e^2+2ab\cot(c+dx)e^2}{(e\cot(c+dx))^{3/2}}dx}{e^2}+\frac{4ab}{3d(e\cot(c+dx))^{3/2}}}{e^2}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\int \frac{\left(a^2-b^2\right)e^2+2ab\cot(c+dx)e^2}{(e\cot(c+dx))^{3/2}}dx}{e^2}}{e^2}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\int \frac{\left(a^2-b^2\right)e^2-2abe^2\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{e^2}}{e^2}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{\int \frac{2abe^3-\left(a^2-b^2\right)e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}}{e^2}}{e^2}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{\int \frac{2abe^3+\left(a^2-b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}+\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}}{e^2}}{e^2}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2\int -\frac{e^3\left(2abe-\left(a^2-b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}+\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}}{e^2}}{e^2}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2\int \frac{e^3\left(2abe-\left(a^2-b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}}{e^2}}{e^2}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

3.61. $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e\int\frac{2abe-\left(a^2-b^2\right)e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}}{e^2}}{e^2}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\downarrow$ 1482

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab-b^2\right)\int\frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}}{e^2}}{e^2}+\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\downarrow$ 1476

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{\right.\right.}{d}}{e^2}}{e^2}$$

$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\downarrow$ 1082

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\int\frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d}}{e^2}}{e^2}$$

$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\downarrow$ 217

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\right.}{}\right.\right.}{d}}{e^2}}{e^2}$$

$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\downarrow$ 1479

3.61. $\int\frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}}}{2\sqrt{2}\sqrt{e}}}{d}}{e^2}}{e^2}$$

$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 25

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{}}{2\sqrt{2}\sqrt{e}}}{d}}{e^2}}{e^2}$$

$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{}}{2\sqrt{e}}}{d}}{e^2}}{e^2}$$

$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 1103

$$\frac{\frac{4ab}{3d(e\cot(c+dx))^{3/2}}-\frac{\frac{2e\left(a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(e\cot(c+dx)-\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\left(a^2-\right.}{d}}{e^2}}{e^2}$$

$$\frac{2a^2}{5de(e\cot(c+dx))^{5/2}}$$

input

```
Int[(a + b*Cot[c + d*x])^2/(e*Cot[c + d*x])^(7/2),x]
```

3.61. $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

output
```
(2*a^2)/(5*d*e*(e*Cot[c + d*x])^(5/2)) + ((4*a*b)/(3*d*(e*Cot[c + d*x])^(3
/2)) - ((2*(a^2 - b^2)*e)/(d*Sqrt[e*Cot[c + d*x]]) - (2*e*(-1/2*((a^2 - 2*
a*b - b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*
Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sq
rt[e]))) + ((a^2 + 2*a*b - b^2)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqr
t[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sq
rt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/d)/e^2)/e^2
```

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4025
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[(b*c - a*d)^2*((a + b*Tan[e + f*x])^(m +
 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e
+ f*x])^(m + 1)*Simp[a*c^2 + 2*b*c*d - a*d^2 - (b*c^2 - 2*a*c*d - b*d^2)*Ta
n[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
 && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

3.61. $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

3.61.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{2\left(-\frac{ab\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(\ldots}}{\left(e^{2}\right)^{\frac{1}{4}}}\ldots\right.\right.}{4e}\ldots}{\ldots}$
default	$-\frac{2\left(-\frac{ab\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(\ldots}}{\left(e^{2}\right)^{\frac{1}{4}}}\ldots\right.\right.}{4e}\ldots}{\ldots}$
parts	$-\frac{2a^{2}e\left(-\frac{1}{5e^{2}(e\cot(dx+c))^{\frac{5}{2}}}+\frac{1}{e^{4}\sqrt{e\cot(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(d\ldots}}{\left(e^{2}\right)^{\frac{1}{4}}}\ldots\right.}{8e^{4}\left(e^{2}\right)^{\frac{1}{4}}}\ldots\right.}{d}\ldots$

input
```
int((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/e*(1/e^2*(-1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*c
ot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1
/8*(a^2-b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/5*a^2/(e*co
t(d*x+c))^(5/2)-(-a^2+b^2)/e^2/(e*cot(d*x+c))^(1/2)-2/3*a*b/e/(e*cot(d*x+c
))^(3/2))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1436 vs. $2(265) = 530$.

Time = 0.32 (sec) , antiderivative size = 1436, normalized size of antiderivative = 4.46

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/30*(15*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)*sq
rt((d^2*e^7*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*
e^14)) + 4*a^3*b - 4*a*b^3)/(d^2*e^7))*log((a^8 - 4*a^6*b^2 - 10*a^4*b^4 -
 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^2
- b^2)*d^3*e^11*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(
d^4*e^14)) - 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e^4)*sqrt((d^2*e^7*sqrt(-(a^8
 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^14)) + 4*a^3*b - 4*a
*b^3)/(d^2*e^7))) - 15*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c
) + d*e^4)*sqrt((d^2*e^7*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6
 + b^8)/(d^4*e^14)) + 4*a^3*b - 4*a*b^3)/(d^2*e^7))*log((a^8 - 4*a^6*b^2 -
 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2
*c)) - ((a^2 - b^2)*d^3*e^11*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2
*b^6 + b^8)/(d^4*e^14)) - 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d*e^4)*sqrt((d^2*e
^7*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^14)) +
4*a^3*b - 4*a*b^3)/(d^2*e^7))) - 15*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*co
s(2*d*x + 2*c) + d*e^4)*sqrt(-(d^2*e^7*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^4*b^
4 - 12*a^2*b^6 + b^8)/(d^4*e^14)) - 4*a^3*b + 4*a*b^3)/(d^2*e^7))*log((a^8
 - 4*a^6*b^2 - 10*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt((e*cos(2*d*x + 2*c) + e)
/sin(2*d*x + 2*c)) + ((a^2 - b^2)*d^3*e^11*sqrt(-(a^8 - 12*a^6*b^2 + 38*a^
4*b^4 - 12*a^2*b^6 + b^8)/(d^4*e^14)) + 2*(a^5*b - 6*a^3*b^3 + a*b^5)*d...
```

3.61.6 Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \int \frac{(a+b\cot{(c+dx)})^2}{(e\cot{(c+dx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))**2/(e*cot(d*x+c))**(7/2),x)
```

output
```
Integral((a + b*cot(c + d*x))**2/(e*cot(c + d*x))**(7/2), x)
```

3.61.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.61.8 Giac [F]

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \int \frac{(b\cot(dx+c)+a)^2}{(e\cot(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^2/(e*cot(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^2/(e*cot(d*x + c))^(7/2), x)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 15.09 (sec) , antiderivative size = 1227, normalized size of antiderivative = 3.81

$$\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
int((a + b*cot(c + d*x))^2/(e*cot(c + d*x))^(7/2),x)
```

output
```
2*atanh((32*a^4*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^2*e^7) - (b^4*
1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) - (a^4*1i)/(4*d^2*e^7) + (a^2*b^2*3i)/
(2*d^2*e^7))^(1/2))/(16*a^6*d^2*e^8 - 16*b^6*d^2*e^8 + a*b^5*d^2*e^8*32i +
 a^5*b*d^2*e^8*32i + 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i - 112*a^4*
b^2*d^2*e^8) + (32*b^4*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^3*b)/(d^2*e^7)
- (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) - (a^4*1i)/(4*d^2*e^7) + (a^2*b
^2*3i)/(2*d^2*e^7))^(1/2))/(16*a^6*d^2*e^8 - 16*b^6*d^2*e^8 + a*b^5*d^2*e^
8*32i + a^5*b*d^2*e^8*32i + 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*e^8*192i - 1
12*a^4*b^2*d^2*e^8) - (192*a^2*b^2*d^3*e^11*(e*cot(c + d*x))^(1/2)*((a^3*b
)/(d^2*e^7) - (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) - (a^4*1i)/(4*d^2*e
^7) + (a^2*b^2*3i)/(2*d^2*e^7))^(1/2))/(16*a^6*d^2*e^8 - 16*b^6*d^2*e^8 +
a*b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i + 112*a^2*b^4*d^2*e^8 - a^3*b^3*d^2*
e^8*192i - 112*a^4*b^2*d^2*e^8))*(-((a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a
^2*b^2)*1i)/(4*d^2*e^7))^(1/2) - 2*atanh((32*a^4*d^3*e^11*(e*cot(c + d*x))
^(1/2)*((a^4*1i)/(4*d^2*e^7) + (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*e^7) +
(a^3*b)/(d^2*e^7) - (a^2*b^2*3i)/(2*d^2*e^7))^(1/2))/(16*b^6*d^2*e^8 - 16*
a^6*d^2*e^8 + a*b^5*d^2*e^8*32i + a^5*b*d^2*e^8*32i - 112*a^2*b^4*d^2*e^8
- a^3*b^3*d^2*e^8*192i + 112*a^4*b^2*d^2*e^8) + (32*b^4*d^3*e^11*(e*cot(c
+ d*x))^(1/2)*((a^4*1i)/(4*d^2*e^7) + (b^4*1i)/(4*d^2*e^7) - (a*b^3)/(d^2*
e^7) + (a^3*b)/(d^2*e^7) - (a^2*b^2*3i)/(2*d^2*e^7))^(1/2))/(16*b^6*d^2...
```

3.61. $\int \frac{(a+b\cot(c+dx))^2}{(e\cot(c+dx))^{7/2}}\,dx$

3.62 $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx$

3.62.1 Optimal result

Integrand size = 25, antiderivative size = 372

$$\begin{aligned}\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx =\\
&-\frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\
&+\frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\
&-\frac{2a(a^2-3b^2)\,e\sqrt{e\cot(c+dx)}}{d}-\frac{2b(3a^2-b^2)\,(e\cot(c+dx))^{3/2}}{3d}\\
&-\frac{32ab^2(e\cot(c+dx))^{5/2}}{35de}-\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}\\
&-\frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\\
&+\frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\end{aligned}$$

output
```
-2/3*b*(3*a^2-b^2)*(e*cot(d*x+c))^(3/2)/d-32/35*a*b^2*(e*cot(d*x+c))^(5/2)
/d/e-2/7*b^2*(e*cot(d*x+c))^(5/2)*(a+b*cot(d*x+c))/d/e-1/2*(a-b)*(a^2+4*a*
b+b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d*2^(1/2)+1/
2*(a-b)*(a^2+4*a*b+b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1
/2))/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(
1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*e^(
3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)
-2*a*(a^2-3*b^2)*e*(e*cot(d*x+c))^(1/2)/d
```

3.62.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.98 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.67

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx =$$

$$-\frac{(e\cot(c+dx))^{3/2}\left(\frac{6}{5}ab^2\cot^{\frac{5}{2}}(c+dx)+\frac{2}{7}b^3\cot^{\frac{7}{2}}(c+dx)+\frac{2}{3}b(-3a^2+b^2)\cot^{\frac{3}{2}}(c+dx)\left(-1+\text{Hypergeom}\right.\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3,x]
```

output
```
-(((e*Cot[c + d*x])^(3/2)*((6*a*b^2*Cot[c + d*x]^(5/2))/5 + (2*b^3*Cot[c +
 d*x]^(7/2))/7 + (2*b*(-3*a^2 + b^2)*Cot[c + d*x]^(3/2)*(-1 + Hypergeometr
ic2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/3 + (a*(a^2 - 3*b^2)*(2*Sqrt[2]*ArcT
an[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot
[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c +
d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
 + d*x]]))/4))/(d*Cot[c + d*x]^(3/2)))
```

3.62.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.94, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4049, 27, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{ 4049}$$

$$-\frac{2\int -\frac{1}{2}(e\cot(c+dx))^{3/2}\left(16ab^2e\cot^2(c+dx)+7b\left(3a^2-b^2\right)e\cot(c+dx)+a\left(7a^2-5b^2\right)e\right)dx}{7e}-\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (e\cot(c+dx))^{3/2}\left(16ab^2e\cot^2(c+dx)+7b\left(3a^2-b^2\right)e\cot(c+dx)+a\left(7a^2-5b^2\right)e\right)dx}{7e}-\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(16ab^2e\tan\left(c+dx+\frac{\pi}{2}\right)^2-7b\left(3a^2-b^2\right)e\tan\left(c+dx+\frac{\pi}{2}\right)+a\left(7a^2-5b^2\right)e\right)dx}{7e}-\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{ 4113}$$

$$\frac{\int (e\cot(c+dx))^{3/2}\left(7a\left(a^2-3b^2\right)e+7b\left(3a^2-b^2\right)\cot(c+dx)e\right)dx-\frac{32ab^2(e\cot(c+dx))^{5/2}}{5d}}{7e}-\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(7a\left(a^2-3b^2\right)e-7b\left(3a^2-b^2\right)e\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{32ab^2(e\cot(c+dx))^{5/2}}{5d}}{7e}-\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{ 4011}$$

$$\frac{\int \sqrt{e\cot(c+dx)}\left(7a\left(a^2-3b^2\right)e^2\cot(c+dx)-7b\left(3a^2-b^2\right)e^2\right)dx-\frac{14be\left(3a^2-b^2\right)(e\cot(c+dx))^{3/2}}{3d}-\frac{32ab^2(e\cot(c+dx))}{5d}}{7e}-\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

3.62. $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-7b(3a^2-b^2)e^2-7a(a^2-3b^2)\tan\left(c+dx+\frac{\pi}{2}\right)e^2\right)dx-\frac{14be(3a^2-b^2)(e\cot(c+dx))^{3/2}}{3d}-}{7e}}{}$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{4011}$$

$$\frac{\int \frac{-7a(a^2-3b^2)e^3-7b(3a^2-b^2)\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}}dx-\frac{14ae^2(a^2-3b^2)\sqrt{e\cot(c+dx)}}{d}-\frac{14be(3a^2-b^2)(e\cot(c+dx))^{3/2}}{3d}-\frac{32ab^2(e\cot(c+dx))^{5/2}}{5d}}{7e}$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7b(3a^2-b^2)e^3\tan\left(c+dx+\frac{\pi}{2}\right)-7a(a^2-3b^2)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{14ae^2(a^2-3b^2)\sqrt{e\cot(c+dx)}}{d}-\frac{14be(3a^2-b^2)(e\cot(c+dx))^{3/2}}{3d}-\frac{32ab^2(e\cot(c+dx))^{5}}{5d}}{7e}$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\frac{2\int \frac{7e^3\left(a\left(a^2-3b^2\right)e+b\left(3a^2-b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{14ae^2(a^2-3b^2)\sqrt{e\cot(c+dx)}}{d}-\frac{14be(3a^2-b^2)(e\cot(c+dx))^{3/2}}{3d}-\frac{32ab^2(e\cot(c}{5d}}{7e}$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{14e^3\int \frac{a\left(a^2-3b^2\right)e+b\left(3a^2-b^2\right)\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{14ae^2(a^2-3b^2)\sqrt{e\cot(c+dx)}}{d}-\frac{14be(3a^2-b^2)(e\cot(c+dx))^{3/2}}{3d}-\frac{32ab^2(e\cot(c+}{5d}}{7e}$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{1482}$$

$$\frac{\frac{14e^3\left(\frac{1}{2}(a+b)(a^2-4ab+b^2)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)(a^2+4ab+b^2)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-\frac{14ae^2(a^2-3b^2)\sqrt{e\,c}}{d}}{7e}$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

3.62. $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx$

$$\Big\downarrow \text{1476}$$

$$\frac{14e^3\left(\frac{1}{2}(a+b)(a^2-4ab+b^2)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)(a^2+4ab+b^2)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \cdots\right.\right.}{d}$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{1082}$$

$$\frac{14e^3\left(\frac{1}{2}(a+b)(a^2-4ab+b^2)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)(a^2+4ab+b^2)\left(\frac{\int\frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e\cot(c+dx)-1}\cdots}{\cdots}\right.\right.}{d}$$

$$7e$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{217}$$

$$\frac{14e^3\left(\frac{1}{2}(a+b)(a^2-4ab+b^2)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)(a^2+4ab+b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\cdots\right.}{d}$$

$$7e$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{1479}$$

$$\frac{14e^3\left(\frac{1}{2}(a+b)(a^2-4ab+b^2)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-b)(\cdots\right.}{d}$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{25}$$

$$\frac{14e^3\left(\frac{1}{2}(a+b)(a^2-4ab+b^2)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-b)(a^2+4\cdots\right.}{d}$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$$\Big\downarrow \text{27}$$

3.62. $\int(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx$

$$\frac{\frac{14e^3\left(\frac{1}{2}(a+b)(a^2-4ab+b^2)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}(a-b)(a^2+4}{d}}{}$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

$\Big\downarrow$ 1103

$$\frac{\frac{14e^3\left(\frac{1}{2}(a-b)(a^2+4ab+b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)(a^2-4ab+b^2)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}\right.}{2\sqrt{2}\sqrt{e}}\right.}{d}}{7}$$

$$\frac{2b^2(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}{7de}$$

input
```
Int[(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3,x]
```

output
```
(-2*b^2*(e*Cot[c + d*x])^(5/2)*(a + b*Cot[c + d*x]))/(7*d*e) + ((-14*a*(a^
2 - 3*b^2)*e^2*Sqrt[e*Cot[c + d*x]])/d - (14*b*(3*a^2 - b^2)*e*(e*Cot[c +
d*x])^(3/2))/(3*d) - (32*a*b^2*(e*Cot[c + d*x])^(5/2))/(5*d) + (14*e^3*(((
a - b)*(a^2 + 4*a*b + b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sq
rt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt
[e]]/(Sqrt[2]*Sqrt[e])))/2 + ((a + b)*(a^2 - 4*a*b + b^2)*(-1/2*Log[e + e*
Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + L
og[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*S
qrt[e])))/2))/d)/(7*e)
```

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

3.62. $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx$

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.62.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{2\left(\frac{b^3(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{3aeb^2(e\cot(dx+c))^{\frac{5}{2}}}{5}+a^2e^2b(e\cot(dx+c))^{\frac{3}{2}}-\frac{b^3e^2(e\cot(dx+c))^{\frac{3}{2}}}{3}+a^3e^3\sqrt{e\cot(dx+c)}-3\sqrt{e\cot(}\right.}{}$
default	$-\frac{2\left(\frac{b^3(e\cot(dx+c))^{\frac{7}{2}}}{7}+\frac{3aeb^2(e\cot(dx+c))^{\frac{5}{2}}}{5}+a^2e^2b(e\cot(dx+c))^{\frac{3}{2}}-\frac{b^3e^2(e\cot(dx+c))^{\frac{3}{2}}}{3}+a^3e^3\sqrt{e\cot(dx+c)}-3\sqrt{e\cot(}\right.}{}$
parts	$-\frac{2a^3e\left(\sqrt{e\cot(dx+c)}-\frac{(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}+1\right)-2\,\mathrm{arcta}\right.}{8}\right.}{d}$

input
```
int((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d/e^2*(1/7*b^3*(e*cot(d*x+c))^(7/2)+3/5*a*e*b^2*(e*cot(d*x+c))^(5/2)+a^
2*e^2*b*(e*cot(d*x+c))^(3/2)-1/3*b^3*e^2*(e*cot(d*x+c))^(3/2)+a^3*e^3*(e*c
ot(d*x+c))^(1/2)-3*(e*cot(d*x+c))^(1/2)*a*b^2*e^3-e^4*(1/8*(a^3*e-3*a*b^2*
e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^
(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2
*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(3*a^2*b-b^3)/(e
^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/
2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^
2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-
2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1795 vs. $2(311) = 622$.

Time = 0.34 (sec) , antiderivative size = 1795, normalized size of antiderivative = 4.83

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/210*(105*(d*cos(2*d*x + 2*c) - d)*sqrt(-(2*(3*a^5*b - 10*a^3*b^3 + 3*a*
b^5)*e^3 + sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4
*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^2)/d^2)*log(-(a^12 - 12*a^10*b^2 - 2
7*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*e^4*sqrt((e*cos(2*d*x + 2*c)
+ e)/sin(2*d*x + 2*c)) + ((a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*
a*b^8)*d*e^3 + (3*a^2*b - b^3)*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 4
52*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^3)*sqrt(-(2*(3*a
^5*b - 10*a^3*b^3 + 3*a*b^5)*e^3 + sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4
 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^2)/d^2))*sin
(2*d*x + 2*c) - 105*(d*cos(2*d*x + 2*c) - d)*sqrt(-(2*(3*a^5*b - 10*a^3*b^
3 + 3*a*b^5)*e^3 + sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 +
 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^2)/d^2)*log(-(a^12 - 12*a^10
*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*e^4*sqrt((e*cos(2*d*x
 + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*
b^6 + 3*a*b^8)*d*e^3 + (3*a^2*b - b^3)*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8
*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^3)*sqrt(
-(2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*e^3 + sqrt(-(a^12 - 30*a^10*b^2 + 255
*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^6/d^4)*d^2)/d
^2))*sin(2*d*x + 2*c) + 105*(d*cos(2*d*x + 2*c) - d)*sqrt(-(2*(3*a^5*b - 1
0*a^3*b^3 + 3*a*b^5)*e^3 - sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 45...
```

3.62.6 Sympy [F]

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx = \int (e\cot{(c+dx)})^{\frac{3}{2}}\,(a+b\cot{(c+dx)})^3\,dx$$

input
```
integrate((e*cot(d*x+c))**(3/2)*(a+b*cot(d*x+c))**3,x)
```

output
```
Integral((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**3, x)
```

3.62.7 Maxima [F(-2)]

Exception generated.

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.62.8 Giac [F]

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx = \int (b\cot(dx+c)+a)^3(e\cot(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)*(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2), x)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 18.06 (sec) , antiderivative size = 2317, normalized size of antiderivative = 6.23

$$\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))^3,x)
```

output
```
(e*cot(c + d*x))^(3/2)*((2*b^3)/(3*d) - (2*a^2*b)/d) - (e*cot(c + d*x))^(1
/2)*((2*a^3*e)/d - (6*a*b^2*e)/d) - atan((((16*(e*cot(c + d*x))^(1/2)*(a^6
*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 - (8*(4*a^3*d^2*e^5
 - 12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^
3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2))/d^
3)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15
i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*1i + ((16*(e*cot(c +
d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b^2*e^6))/d^2 + (
8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*
e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4
*d^2))^(1/2))/d^3)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3
- a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d^2))^(1/2)*1i)/(
((16*(e*cot(c + d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e^6 - 15*a^4*b
^2*e^6))/d^2 + (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(b^6*e^3*1i - a^6*e
^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4
*b^2*e^3*15i)/(4*d^2))^(1/2))/d^3)*(-(b^6*e^3*1i - a^6*e^3*1i + 6*a*b^5*e^
3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a^3*b^3*e^3 + a^4*b^2*e^3*15i)/(4*d
^2))^(1/2) - ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^6 - b^6*e^6 + 15*a^2*b^4*e
^6 - 15*a^4*b^2*e^6))/d^2 - (8*(4*a^3*d^2*e^5 - 12*a*b^2*d^2*e^5)*(-(b^6*e
^3*1i - a^6*e^3*1i + 6*a*b^5*e^3 + 6*a^5*b*e^3 - a^2*b^4*e^3*15i - 20*a...
```

3.62. $\int (e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3\,dx$

3.63 $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx$

3.63.1 Optimal result

Integrand size = 25, antiderivative size = 342

$$\begin{aligned}
&\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx\\
&=\frac{(a+b)\left(a^2-4ab+b^2\right)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}\\
&\quad-\frac{(a+b)\left(a^2-4ab+b^2\right)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d}-\frac{2b(3a^2-b^2)\sqrt{e\cot(c+dx)}}{d}\\
&\quad-\frac{8ab^2(e\cot(c+dx))^{3/2}}{5de}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}\\
&\quad-\frac{(a-b)\left(a^2+4ab+b^2\right)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}\\
&\quad+\frac{(a-b)\left(a^2+4ab+b^2\right)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d}
\end{aligned}$$

output
```
-8/5*a*b^2*(e*cot(d*x+c))^(3/2)/d/e-2/5*b^2*(e*cot(d*x+c))^(3/2)*(a+b*cot(
d*x+c))/d/e+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2
)/e^(1/2))*e^(1/2)/d*2^(1/2)-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*(e
*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*ln
(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2
)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d
*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-2*b*(3*a^2-b^2)*(e*cot(d*x+c))^(1/2)/d
```

3.63.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.64 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.72

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx =$$
$$-\frac{\sqrt{e\cot(c+dx)}\Big(2ab^2\cot^{\frac{3}{2}}(c+dx)+\frac{2}{5}b^3\cot^{\frac{5}{2}}(c+dx)+\frac{2}{3}a(a^2-3b^2)\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^3,x]
```

output
```
-((Sqrt[e*Cot[c + d*x]]*(2*a*b^2*Cot[c + d*x]^(3/2) + (2*b^3*Cot[c + d*x]^
(5/2))/5 + (2*a*(a^2 - 3*b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1,
 7/4, -Cot[c + d*x]^2])/3 - (b*(-3*a^2 + b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2
]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] +
 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
 + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/4)
)/(d*Sqrt[Cot[c + d*x]]))
```

3.63.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.93, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3042, 4049, 27, 3042, 4113, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4049}$$

$$-\frac{2\int -\frac{1}{2}\sqrt{e\cot(c+dx)}\left(12ab^2e\cot^2(c+dx)+5b\left(3a^2-b^2\right)e\cot(c+dx)+a\left(5a^2-3b^2\right)e\right)dx}{5e}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$\big\downarrow$ 27

$$\frac{\int \sqrt{e\cot(c+dx)}\left(12ab^2e\cot^2(c+dx)+5b\left(3a^2-b^2\right)e\cot(c+dx)+a\left(5a^2-3b^2\right)e\right)dx}{5e}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$\big\downarrow$ 3042

$$\frac{\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2e\tan\left(c+dx+\frac{\pi}{2}\right)^2-5b\left(3a^2-b^2\right)e\tan\left(c+dx+\frac{\pi}{2}\right)+a\left(5a^2-3b^2\right)e\right)dx}{5e}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$\big\downarrow$ 4113

$$\frac{\int \sqrt{e\cot(c+dx)}\left(5a\left(a^2-3b^2\right)e+5b\left(3a^2-b^2\right)\cot(c+dx)e\right)dx-\frac{8ab^2(e\cot(c+dx))^{3/2}}{d}}{5e}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$\big\downarrow$ 3042

$$\frac{\int \sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(5a\left(a^2-3b^2\right)e-5b\left(3a^2-b^2\right)e\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{8ab^2(e\cot(c+dx))^{3/2}}{d}}{5e}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$\big\downarrow$ 4011

$$\frac{\int \frac{5a\left(a^2-3b^2\right)e^2\cot(c+dx)-5b\left(3a^2-b^2\right)e^2}{\sqrt{e\cot(c+dx)}}dx-\frac{10be\left(3a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{8ab^2(e\cot(c+dx))^{3/2}}{d}}{5e}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{-5b\left(3a^2-b^2\right)e^2-5a\left(a^2-3b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{10be\left(3a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{8ab^2(e\cot(c+dx))^{3/2}}{d}}{5e}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$\big\downarrow$ 4017

3.63. $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx$

$$\frac{\frac{2\int \frac{5e^2\left(b\left(3a^2-b^2\right)e-a\left(a^2-3b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{10be\left(3a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{8ab^2(e\cot(c+dx))^{3/2}}{d}}{5e}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{10e^2\int \frac{b\left(3a^2-b^2\right)e-a\left(a^2-3b^2\right)e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{10be\left(3a^2-b^2\right)\sqrt{e\cot(c+dx)}}{d}-\frac{8ab^2(e\cot(c+dx))^{3/2}}{d}}{5e}-\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$$\Big\downarrow \; 1482$$

$$\frac{\frac{10e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-\frac{10be\left(3a^2-b^2\right)\sqrt{e\cot}}{d}}{5e}$$

$$\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{10e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int\right.}{d}}{5e}$$

$$\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{10e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}}{}\right.\right.}{d}}{5e}$$

$$\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{10e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right.}{d}}{5e}$$

$$\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$$\Big\downarrow \; 1479$$

3.63. $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx$

$$\frac{10e^2\left(\frac{1}{2}(a-b)(a^2+4ab+b^2)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)(\cdots}{d}}{5e}$$

$$\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$\downarrow$ 25

$$\frac{10e^2\left(\frac{1}{2}(a-b)(a^2+4ab+b^2)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)(a^2-4\cdots}{d}}{5e}$$

$$\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$\downarrow$ 27

$$\frac{10e^2\left(\frac{1}{2}(a-b)(a^2+4ab+b^2)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)-\frac{1}{2}(a+b)(a^2-4\cdots}{d}}{5e}$$

$$\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

$\downarrow$ 1103

$$\frac{10e^2\left(\frac{1}{2}(a-b)(a^2+4ab+b^2)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(e\cot(c+dx)-\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)(a^2-4ab+b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right.}{\cdots}\right.}{d}}{5e}$$

$$\frac{2b^2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}{5de}$$

input
```
Int[Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^3,x]
```

3.63. $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx$

output
```
(-2*b^2*(e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x]))/(5*d*e) + ((-10*b*(3*
a^2 - b^2)*e*Sqrt[e*Cot[c + d*x]])/d - (8*a*b^2*(e*Cot[c + d*x])^(3/2))/d
+ (10*e^2*(-1/2*((a + b)*(a^2 - 4*a*b + b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e
*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*C
ot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e]))) + ((a - b)*(a^2 + 4*a*b + b^2)*
(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt
[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x
]]]/(2*Sqrt[2]*Sqrt[e])))/2))/d)/(5*e)
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

3.63. $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx$

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.63.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{2\left(\frac{b^3(e\cot(dx+c))^{\frac{5}{2}}}{5}+ae\,b^2(e\cot(dx+c))^{\frac{3}{2}}+3a^2b\,e^2\sqrt{e\cot(dx+c)}-\sqrt{e\cot(dx+c)}\,b^3e^2+e^3\left(\frac{\left(-3a^2be+b^3e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\right.}{}\right.\right.}{}$
default	$-\frac{2\left(\frac{b^3(e\cot(dx+c))^{\frac{5}{2}}}{5}+ae\,b^2(e\cot(dx+c))^{\frac{3}{2}}+3a^2b\,e^2\sqrt{e\cot(dx+c)}-\sqrt{e\cot(dx+c)}\,b^3e^2+e^3\left(\frac{\left(-3a^2be+b^3e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\right.}{}\right.\right.}{}$
parts	$-\frac{a^3e\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{4d\left(e^2\right)^{\frac{1}{4}}}$

input
```
int((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d/e^2*(1/5*b^3*(e*cot(d*x+c))^(5/2)+a*e*b^2*(e*cot(d*x+c))^(3/2)+3*a^2*
b*e^2*(e*cot(d*x+c))^(1/2)-(e*cot(d*x+c))^(1/2)*b^3*e^2+e^3*(1/8*(-3*a^2*b
*e+b^3*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^3-3*a*
b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1643 vs. $2(285) = 570$.

Time = 0.34 (sec) , antiderivative size = 1643, normalized size of antiderivative = 4.80

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/10*(5*(d*cos(2*d*x + 2*c) - d)*sqrt((d^2*sqrt(-(a^12 - 30*a^10*b^2 + 255
*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) + 2*(3
*a^5*b - 10*a^3*b^3 + 3*a*b^5)*e)/d^2)*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b
^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(
2*d*x + 2*c)) + ((a^3 - 3*a*b^2)*d^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b
^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) - (3*a^8*b -
 46*a^6*b^3 + 60*a^4*b^5 - 18*a^2*b^7 + b^9)*d*e)*sqrt((d^2*sqrt(-(a^12 -
30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12
)*e^2/d^4) + 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*e)/d^2)) - 5*(d*cos(2*d*x
+ 2*c) - d)*sqrt((d^2*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^
6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) + 2*(3*a^5*b - 10*a^3*b^3 +
 3*a*b^5)*e)/d^2)*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*
a^2*b^10 - b^12)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^3
 - 3*a*b^2)*d^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 25
5*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) - (3*a^8*b - 46*a^6*b^3 + 60*a^4*
b^5 - 18*a^2*b^7 + b^9)*d*e)*sqrt((d^2*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8
*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)*e^2/d^4) + 2*(3*a^5
*b - 10*a^3*b^3 + 3*a*b^5)*e)/d^2)) - 5*(d*cos(2*d*x + 2*c) - d)*sqrt(-(d^
2*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30
*a^2*b^10 + b^12)*e^2/d^4) - 2*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*e)/d^2)...
```

3.63.6 Sympy [F]

$$\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx = \int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx$$

input
```
integrate((e*cot(d*x+c))**(1/2)*(a+b*cot(d*x+c))**3,x)
```

output
```
Integral(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))**3, x)
```

3.63.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{e \cot(c+dx)}(a+b\cot(c+dx))^3\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.63.8 Giac [F]

$$\int \sqrt{e \cot(c+dx)}(a+b\cot(c+dx))^3\,dx = \int (b\cot(dx+c)+a)^3\sqrt{e\cot(dx+c)}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)*(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^3*sqrt(e*cot(d*x + c)), x)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 15.06 (sec) , antiderivative size = 2071, normalized size of antiderivative = 6.06

$$\int \sqrt{e \cot(c+dx)}(a+b\cot(c+dx))^3\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))^3,x)
```

output
```
(e*cot(c + d*x))^(1/2)*((2*b^3)/d - (6*a^2*b)/d) + atan((((16*(e*cot(c + d
*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 - (8
*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i
- 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^
3)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i +
6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2)*1i + ((16*(e*cot(c + d*x))^(1/2)*(a^
6*e^4 - b^6*e^4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 + (8*(4*b^3*d^2*e^
4 - 12*a^2*b*d^2*e^4)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e
 + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((b^6*e*1i
- a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*
a^5*b*e)/(4*d^2))^(1/2)*1i)/(((16*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^
4 + 15*a^2*b^4*e^4 - 15*a^4*b^2*e^4))/d^2 - (8*(4*b^3*d^2*e^4 - 12*a^2*b*d
^2*e^4)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*1
5i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((b^6*e*1i - a^6*e*1i - a
^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^
2))^(1/2) - ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^4 - b^6*e^4 + 15*a^2*b^4*e^
4 - 15*a^4*b^2*e^4))/d^2 + (8*(4*b^3*d^2*e^4 - 12*a^2*b*d^2*e^4)*((b^6*e*1
i - a^6*e*1i - a^2*b^4*e*15i - 20*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e +
6*a^5*b*e)/(4*d^2))^(1/2))/d^3)*((b^6*e*1i - a^6*e*1i - a^2*b^4*e*15i - 20
*a^3*b^3*e + a^4*b^2*e*15i + 6*a*b^5*e + 6*a^5*b*e)/(4*d^2))^(1/2) + (1...
```

3.63. $\int \sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3\,dx$

3.64 $\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx$

3.64.1 Optimal result

Integrand size = 25, antiderivative size = 313

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx &= \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} \\ &\quad - \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}d\sqrt{e}} \\ &\quad - \frac{16ab^2\sqrt{e\cot(c+dx)}}{3de} - \frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de} \\ &\quad + \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}} \\ &\quad - \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}d\sqrt{e}}\end{aligned}$$

output
```
1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d
*2^(1/2)/e^(1/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))
^(1/2)/e^(1/2))/d*2^(1/2)/e^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot
(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d*2^(1/2)/e^(1/2)-1/4*(a+b)*
(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2)
)/d*2^(1/2)/e^(1/2)-16/3*a*b^2*(e*cot(d*x+c))^(1/2)/d/e-2/3*b^2*(a+b*cot(d
*x+c))*(e*cot(d*x+c))^(1/2)/d/e
```

3.64.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.07 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = -\frac{\sqrt{\cot(c+dx)}\Big(72ab^2\sqrt{\cot(c+dx)}+8b^3\cot^{\frac{3}{2}}(c+dx)-8b(-3a^2+b^2)\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2}$$

input
```
Integrate[(a + b*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]],x]
```

output
```
-1/12*(Sqrt[Cot[c + d*x]]*(72*a*b^2*Sqrt[Cot[c + d*x]] + 8*b^3*Cot[c + d*x
]^(3/2) - 8*b*(-3*a^2 + b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1,
7/4, -Cot[c + d*x]^2] - 3*Sqrt[2]*a*(a^2 - 3*b^2)*(2*ArcTan[1 - Sqrt[2]*Sq
rt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqr
t[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]] + Cot[c + d*x]])))/(d*Sqrt[e*Cot[c + d*x]])
```

3.64.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.91, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4049, 27, 3042, 4113, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4049}$$

$$-\frac{2\int -\frac{8ab^2e\cot^2(c+dx)+3b(3a^2-b^2)e\cot(c+dx)+a(3a^2-b^2)e}{2\sqrt{e\cot(c+dx)}}dx}{3e}-\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{8ab^2e\cot^2(c+dx)+3b(3a^2-b^2)e\cot(c+dx)+a(3a^2-b^2)e}{\sqrt{e\cot(c+dx)}}dx}{3e}-\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{8ab^2e\tan\left(c+dx+\frac{\pi}{2}\right)^2-3b(3a^2-b^2)e\tan\left(c+dx+\frac{\pi}{2}\right)+a(3a^2-b^2)e}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e}-\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \; 4113$$

$$\frac{\int \frac{3a(a^2-3b^2)e+3b(3a^2-b^2)\cot(c+dx)e}{\sqrt{e\cot(c+dx)}}dx-\frac{16ab^2\sqrt{e\cot(c+dx)}}{d}}{3e}-\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{3a(a^2-3b^2)e-3b(3a^2-b^2)e\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{16ab^2\sqrt{e\cot(c+dx)}}{d}}{3e}-\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \; 4017$$

$$\frac{\frac{2\int -\frac{3e\left(a\left(a^2-3b^2\right)e+b\left(3a^2-b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{16ab^2\sqrt{e\cot(c+dx)}}{d}}{3e}-\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{6e\int \frac{a\left(a^2-3b^2\right)e+b\left(3a^2-b^2\right)\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{16ab^2\sqrt{e\cot(c+dx)}}{d}}{3e}-\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \; 1482$$

$$\frac{-\frac{6e\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-\frac{16ab^2\sqrt{e\cot(c+dx)}}{d}}{3e}-\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \; 1476$$

3.64. $\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx$

$$-\frac{\frac{6e\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \ldots\right.\right.}{d}}{3e}$$

$$\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \text{ 1082}$$

$$-\frac{\frac{6e\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}\ldots}{\ldots}\right.\right.}{d}}{3e}$$

$$\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{\frac{6e\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\ldots\right.}{d}}{3e}$$

$$\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \text{ 1479}$$

$$-\frac{\frac{6e\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-b)(\ldots\right.}{d}}{3e}$$

$$\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{6e\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-b)\left(a^2+4\ldots\right.\right.}{d}}{3e}$$

$$\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \text{ 27}$$

3.64. $\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx$

$$-\frac{\frac{6e\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}(a-b)\left(a^2+4\right.}{d}}{3e}$$

$$\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{\frac{6e\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+d\right.}{2\sqrt{2}\sqrt{e}}\right.}{d}}{3e}$$

$$\frac{2b^2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}{3de}$$

input
```
Int[(a + b*Cot[c + d*x])^3/Sqrt[e*Cot[c + d*x]],x]
```

output
```
(-2*b^2*Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x]))/(3*d*e) + ((-16*a*b^2*S
qrt[e*Cot[c + d*x]])/d - (6*e*(((a - b)*(a^2 + 4*a*b + b^2)*(-(ArcTan[1 -
(Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (S
qrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 + ((a + b)*(a^
2 - 4*a*b + b^2)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot
[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*S
qrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/d)/(3*e)
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

3.64. $\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx$

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.64.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{2\left(\frac{b^3(e\cot(dx+c))^{\frac{3}{2}}}{3}+3\sqrt{e\cot(dx+c)}\,a\,b^2e+e^2\left(\frac{\left(a^3e-3ae\,b^2\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)\right.}{8e^2}\right.\right.}{}$
default	$-\frac{2\left(\frac{b^3(e\cot(dx+c))^{\frac{3}{2}}}{3}+3\sqrt{e\cot(dx+c)}\,a\,b^2e+e^2\left(\frac{\left(a^3e-3ae\,b^2\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)\right.}{8e^2}\right.\right.}{}$
parts	$-\frac{a^3\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot}}{\left(e^2\right)}\right.\right.}{4de}$

input
```
int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/e^2*(1/3*b^3*(e*cot(d*x+c))^(3/2)+3*(e*cot(d*x+c))^(1/2)*a*b^2*e+e^2*
(1/8*(a^3*e-3*a*b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+
1/8*(3*a^2*b-b^3)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1633 vs. $2(258) = 516$.

Time = 0.41 (sec) , antiderivative size = 1633, normalized size of antiderivative = 5.22

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*d*e*sqrt(-(6*a^5*b - 20*a^3*b^3 + 6*a*b^5 + d^2*e*sqrt(-(a^12 - 30*
a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(
d^4*e^2)))/(d^2*e))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 1
2*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((3*a
^2*b - b^3)*d^3*e^2*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6
+ 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^2)) + (a^9 - 18*a^7*b^2 + 60*a^
5*b^4 - 46*a^3*b^6 + 3*a*b^8)*d*e)*sqrt(-(6*a^5*b - 20*a^3*b^3 + 6*a*b^5 +
 d^2*e*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8
 - 30*a^2*b^10 + b^12)/(d^4*e^2)))/(d^2*e)))*sin(2*d*x + 2*c) - 3*d*e*sqrt
(-(6*a^5*b - 20*a^3*b^3 + 6*a*b^5 + d^2*e*sqrt(-(a^12 - 30*a^10*b^2 + 255*
a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^2)))/(d^2
*e))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^
12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((3*a^2*b - b^3)*d^3
*e^2*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 -
 30*a^2*b^10 + b^12)/(d^4*e^2)) + (a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*
b^6 + 3*a*b^8)*d*e)*sqrt(-(6*a^5*b - 20*a^3*b^3 + 6*a*b^5 + d^2*e*sqrt(-(a
^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10
+ b^12)/(d^4*e^2)))/(d^2*e)))*sin(2*d*x + 2*c) - 3*d*e*sqrt(-(6*a^5*b - 20
*a^3*b^3 + 6*a*b^5 - d^2*e*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a
^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^2)))/(d^2*e))*log(-(a...
```

3.64.6 Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx=\int \frac{(a+b\cot{(c+dx)})^3}{\sqrt{e\cot{(c+dx)}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(1/2),x)
```

output
```
Integral((a + b*cot(c + d*x))**3/sqrt(e*cot(c + d*x)), x)
```

3.64.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.64.8 Giac [F]

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx=\int \frac{(b\cot{(dx+c)}+a)^3}{\sqrt{e\cot{(dx+c)}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^3/sqrt(e*cot(d*x + c)), x)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 1896, normalized size of antiderivative = 6.06

$$\int \frac{(a+b\cot(c+dx))^3}{\sqrt{e\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(1/2),x)
```

output
```
atan((((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15
*a^4*b^2*e^2))/d^2 - (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a
^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e)
)^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20
i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*1i + ((16*(e*cot(c + d*x))^(1/2)*(a^6
*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 + (8*(4*a^3*d^2*e^3
 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3
*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i -
 a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2)*1
i)/(((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2 - b^6*e^2 + 15*a^2*b^4*e^2 - 15*a
^4*b^2*e^2))/d^2 + (8*(4*a^3*d^2*e^3 - 12*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5
*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^
(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i
+ 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2) - ((16*(e*cot(c + d*x))^(1/2)*(a^6*e^2
- b^6*e^2 + 15*a^2*b^4*e^2 - 15*a^4*b^2*e^2))/d^2 - (8*(4*a^3*d^2*e^3 - 12
*a*b^2*d^2*e^3)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*
20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2))/d^3)*(((a*b^5*6i + a^5*b*6i - a^6
+ b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e))^(1/2) + (16*
(3*a^8*b*e^2 - b^9*e^2 + 6*a^4*b^5*e^2 + 8*a^6*b^3*e^2))/d^3))*(((a*b^5*6i
 + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4...
```

3.65 $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx$

3.65.1 Optimal result

Integrand size = 25, antiderivative size = 313

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx &= -\frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\ &\quad+\frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{3/2}} \\ &\quad-\frac{2b(a^2+b^2)\sqrt{e\cot(c+dx)}}{de^2}+\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}} \\ &\quad+\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}} \\ &\quad-\frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{3/2}}\end{aligned}$$

output
```
-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/
d/e^(3/2)*2^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c)
)^(1/2)/e^(1/2))/d/e^(3/2)*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+co
t(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(3/2)*2^(1/2)-1/4*(a-b)
*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2
))/d/e^(3/2)*2^(1/2)+2*a^2*(a+b*cot(d*x+c))/d/e/(e*cot(d*x+c))^(1/2)-2*b*(
a^2+b^2)*(e*cot(d*x+c))^(1/2)/d/e^2
```

3.65.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.55 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.62

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx =$$
$$-\frac{-24ab^2+8b^3\cot(c+dx)-8a(a^2-3b^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\cot^2(c+dx)\right)+\sqrt{2}b(-3a^2+b^2\ldots}{}$$

input
```
Integrate[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2),x]
```

output
```
-1/4*(-24*a*b^2 + 8*b^3*Cot[c + d*x] - 8*a*(a^2 - 3*b^2)*Hypergeometric2F1
[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*b*(-3*a^2 + b^2)*Sqrt[Cot[c + d*
x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[
Cot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[
1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(d*e*Sqrt[e*Cot[c + d*x]]
)
```

3.65.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.92, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3042, 4048, 27, 3042, 4113, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \; 4048$$

$$\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}-\frac{2\int -\frac{b(a^2+b^2)\cot^2(c+dx)e^2+4a^2be^2-a(a^2-3b^2)\cot(c+dx)e^2}{2\sqrt{e\cot(c+dx)}}dx}{e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b(a^2+b^2)\cot^2(c+dx)e^2+4a^2be^2-a(a^2-3b^2)\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx}{e^3}+\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(a^2+b^2)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+4a^2be^2+a(a^2-3b^2)\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^3}+\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{4113}$$

$$\frac{\int \frac{b(3a^2-b^2)e^2-a(a^2-3b^2)e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx-\frac{2be(a^2+b^2)\sqrt{e\cot(c+dx)}}{d}}{e^3}+\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(3a^2-b^2)e^2+a(a^2-3b^2)\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2be(a^2+b^2)\sqrt{e\cot(c+dx)}}{d}}{e^3}+\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\frac{2\int -\frac{e^2\left(b\left(3a^2-b^2\right)e-a\left(a^2-3b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2be(a^2+b^2)\sqrt{e\cot(c+dx)}}{d}}{e^3}+\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{2\int \frac{e^2\left(b\left(3a^2-b^2\right)e-a\left(a^2-3b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2be(a^2+b^2)\sqrt{e\cot(c+dx)}}{d}}{e^3}+\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{2e^2\int \frac{b\left(3a^2-b^2\right)e-a\left(a^2-3b^2\right)e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}-\frac{2be(a^2+b^2)\sqrt{e\cot(c+dx)}}{d}}{e^3}+\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \text{1482}$$

3.65. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx$

$$-\frac{\frac{2e^2\left(\frac{1}{2}(a-b)(a^2+4ab+b^2)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)(a^2-4ab+b^2)\int\frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}-\frac{2be(a^2+b^2)\sqrt{e\cot(}}{d}}{e^3}$$
$$\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 1476

$$-\frac{\frac{2e^2\left(\frac{1}{2}(a-b)(a^2+4ab+b^2)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)(a^2-4ab+b^2)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int\right.\right.}{d}}{e^3}$$
$$\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 1082

$$-\frac{\frac{2e^2\left(\frac{1}{2}(a-b)(a^2+4ab+b^2)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)(a^2-4ab+b^2)\left(\frac{\int\frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e\cot(c+dx)-}}{}\right.\right.}{d}}{e^3}$$
$$\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 217

$$-\frac{\frac{2e^2\left(\frac{1}{2}(a-b)(a^2+4ab+b^2)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)(a^2-4ab+b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right.\right.}{d}}{e^3}$$
$$\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 1479

$$-\frac{\frac{2e^2\left(\frac{1}{2}(a-b)(a^2+4ab+b^2)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\right.}{d}}{e^3}$$
$$\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$\downarrow$ 25

3.65. $\int\frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx$

$$-\frac{\frac{2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-\right.}{d}}{e^3}$$

$$\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-\right.}{d}}{e^3}$$

$$\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{\frac{2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(e\cot(c+dx)-\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.}{d}}{e^3}$$

$$\frac{2a^2(a+b\cot(c+dx))}{de\sqrt{e\cot(c+dx)}}$$

input
```
Int[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(3/2),x]
```

output
```
(2*a^2*(a + b*Cot[c + d*x]))/(d*e*Sqrt[e*Cot[c + d*x]]) + ((-2*b*(a^2 + b^
2)*e*Sqrt[e*Cot[c + d*x]])/d - (2*e^2*(-1/2*((a + b)*(a^2 - 4*a*b + b^2)*(
-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) +
ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e]))) + (
(a - b)*(a^2 + 4*a*b + b^2)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]
*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2
]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/d)/e^3
```

3.65. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx$

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.65.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{2\left(\sqrt{e\cot(dx+c)}\,b^3-e\left(\frac{\left(-3a^2be+b^3e\right)\left(e^2\right)^{\frac14}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac14}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac14}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(d\cdots}{\left(e^2\right)^{\frac14}}\cdots\right.\right.}{8e^2}\cdots\right.\right.}{\cdots}$
default	$-\frac{2\left(\sqrt{e\cot(dx+c)}\,b^3-e\left(\frac{\left(-3a^2be+b^3e\right)\left(e^2\right)^{\frac14}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac14}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac14}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(d\cdots}{\left(e^2\right)^{\frac14}}\cdots\right.\right.}{8e^2}\cdots\right.\right.}{\cdots}$
parts	$-\frac{2a^3e\left(-\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac14}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac14}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac14}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac14}}\cdots\right.\right.}{8e^2\left(e^2\right)^{\frac14}}\cdots\right.}{d}\cdots$

input
```
int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/e^2*((e*cot(d*x+c))^(1/2)*b^3-e*(1/8*(-3*a^2*b*e+b^3*e)*(e^2)^(1/4)/e
^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2
)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2
)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)
/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^3-3*a*b^2)/(e^2)^(1/4)*2^(1/2
)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/
(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arc
tan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)+1)))-a^3*e/(e*cot(d*x+c))^(1/2))
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. $2(262) = 524$.

Time = 0.38 (sec) , antiderivative size = 1679, normalized size of antiderivative = 5.36

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt((6*a^5*b - 20*a^3*b^3 + 6*a*b^
5 + d^2*e^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^
4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)))/(d^2*e^3))*log(-(a^12 - 12*a^10*b^
2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c)
 + e)/sin(2*d*x + 2*c)) + ((a^3 - 3*a*b^2)*d^3*e^5*sqrt(-(a^12 - 30*a^10*b
^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^
6)) - (3*a^8*b - 46*a^6*b^3 + 60*a^4*b^5 - 18*a^2*b^7 + b^9)*d*e^2)*sqrt((
6*a^5*b - 20*a^3*b^3 + 6*a*b^5 + d^2*e^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a
^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)))/(d^2*
e^3))) - (d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt((6*a^5*b - 20*a^3*b^3 + 6*a
*b^5 + d^2*e^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255
*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)))/(d^2*e^3))*log(-(a^12 - 12*a^10
*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2
*c) + e)/sin(2*d*x + 2*c)) - ((a^3 - 3*a*b^2)*d^3*e^5*sqrt(-(a^12 - 30*a^1
0*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4
*e^6)) - (3*a^8*b - 46*a^6*b^3 + 60*a^4*b^5 - 18*a^2*b^7 + b^9)*d*e^2)*sqr
t((6*a^5*b - 20*a^3*b^3 + 6*a*b^5 + d^2*e^3*sqrt(-(a^12 - 30*a^10*b^2 + 25
5*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^6)))/(d
^2*e^3))) - (d*e^2*cos(2*d*x + 2*c) + d*e^2)*sqrt((6*a^5*b - 20*a^3*b^3 +
6*a*b^5 - d^2*e^3*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6...
```

3.65.6 Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx=\int \frac{(a+b\cot{(c+dx)})^3}{(e\cot{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*cot(c + d*x))**3/(e*cot(c + d*x))**(3/2), x)
```

3.65.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.65.8 Giac [F]

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \int \frac{(b\cot(dx+c)+a)^3}{(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^3/(e*cot(d*x + c))^(3/2), x)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 1951, normalized size of antiderivative = 6.23

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(3/2),x)
```

output
```
(2*a^3)/(d*e*(e*cot(c + d*x))^(1/2)) - atan((((e*cot(c + d*x))^(1/2)*(16*a
^6*d^3*e^5 - 16*b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) +
 (32*b^3*d^4*e^7 - 96*a^2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 +
 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*
6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*
d^2*e^3))^(1/2)*1i + ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^5 - 16*b^6*d^3*
e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) - (32*b^3*d^4*e^7 - 96*a^
2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20
i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*6i + a^6 - b^
6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2)*1i)/(((e
*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^5 - 16*b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^
5 - 240*a^4*b^2*d^3*e^5) - (32*b^3*d^4*e^7 - 96*a^2*b*d^4*e^7)*(-((a*b^5*6
i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d
^2*e^3))^(1/2))*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3
*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^
6*d^3*e^5 - 16*b^6*d^3*e^5 + 240*a^2*b^4*d^3*e^5 - 240*a^4*b^2*d^3*e^5) +
(32*b^3*d^4*e^7 - 96*a^2*b*d^4*e^7)*(-((a*b^5*6i + a^5*b*6i + a^6 - b^6 +
15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^3))^(1/2))*(-((a*b^5*6
i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d
^2*e^3))^(1/2) - 16*a^9*d^2*e^4 + 48*a*b^8*d^2*e^4 + 128*a^3*b^6*d^2*e^...
```

3.65. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{3/2}}\,dx$

3.66 $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx$

3.66.1 Optimal result

Integrand size = 25, antiderivative size = 313

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx &= -\frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} \\ &+ \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{5/2}} \\ &+ \frac{16a^2b}{3de^2\sqrt{e\cot(c+dx)}}+\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}} \\ &- \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}} \\ &+ \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{5/2}}\end{aligned}$$

output
```
2/3*a^2*(a+b*cot(d*x+c))/d/e/(e*cot(d*x+c))^(3/2)-1/2*(a-b)*(a^2+4*a*b+b^2
)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)*2^(1/2)+1/2*(a-
b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2
)*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*
(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/
2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(5/2)*2^(1/2)+16/3
*a^2*b/d/e^2/(e*cot(d*x+c))^(1/2)
```

3.66.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.33

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \frac{-6b(-3a^2+b^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\cot^2(c+dx)\right)+2a(a^2-3b^2)}{3de^2\sqrt{e}}$$

input
```
Integrate[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(5/2),x]
```

output
```
(-6*b*(-3*a^2 + b^2)*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + 2*
a*(a^2 - 3*b^2)*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2]*Tan[c + d
*x] + 6*b^2*(b + a*Tan[c + d*x]))/(3*d*e^2*Sqrt[e*Cot[c + d*x]])
```

3.66.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.91, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4048, 27, 3042, 4111, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4048}$$

$$\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}-\frac{2\int -\frac{-b(a^2-3b^2)\cot^2(c+dx)e^2+8a^2be^2-3a(a^2-3b^2)\cot(c+dx)e^2}{2(e\cot(c+dx))^{3/2}}dx}{3e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-b(a^2-3b^2)\cot^2(c+dx)e^2+8a^2be^2-3a(a^2-3b^2)\cot(c+dx)e^2}{(e\cot(c+dx))^{3/2}}dx}{3e^3}+\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-b(a^2-3b^2)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+8a^2be^2+3a(a^2-3b^2)\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3e^3}+\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4111}$$

$$\frac{\frac{\int -\frac{3\left(a\left(a^2-3b^2\right)e^3+b\left(3a^2-b^2\right)\cot(c+dx)e^3\right)}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{16a^2be}{d\sqrt{e\cot(c+dx)}}}{3e^3}+\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{16a^2be}{d\sqrt{e\cot(c+dx)}}-\frac{3\int \frac{a\left(a^2-3b^2\right)e^3+b\left(3a^2-b^2\right)\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}}dx}{e^2}}{3e^3}+\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{16a^2be}{d\sqrt{e\cot(c+dx)}}-\frac{3\int \frac{a\left(a^2-3b^2\right)e^3-b\left(3a^2-b^2\right)e^3\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}}{3e^3}+\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\frac{16a^2be}{d\sqrt{e\cot(c+dx)}}-\frac{6\int -\frac{e^3\left(a\left(a^2-3b^2\right)e+b\left(3a^2-b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}}{3e^3}+\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{6\int \frac{e^3\left(a\left(a^2-3b^2\right)e+b\left(3a^2-b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}+\frac{16a^2be}{d\sqrt{e\cot(c+dx)}}}{3e^3}+\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{6e\int \frac{a\left(a^2-3b^2\right)e+b\left(3a^2-b^2\right)\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}+\frac{16a^2be}{d\sqrt{e\cot(c+dx)}}}{3e^3}+\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{1482}$$

3.66. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx$

$$\frac{\frac{6e\left(\frac{1}{2}(a+b)(a^2-4ab+b^2)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)(a^2+4ab+b^2)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}+\frac{16a^2be}{d\sqrt{e\cot(c+dx)}}}{3e^3}+$$

$$\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$\downarrow$ 1476

$$\frac{6e\left(\frac{1}{2}(a+b)(a^2-4ab+b^2)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)(a^2+4ab+b^2)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{}{\cot}\right.}{d}$$
$$3e^3$$

$$\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$\downarrow$ 1082

$$\frac{6e\left(\frac{1}{2}(a+b)(a^2-4ab+b^2)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)(a^2+4ab+b^2)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d(}{\sqrt{}}\right.\right.}{d}$$
$$3e^3$$

$$\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$\downarrow$ 217

$$\frac{6e\left(\frac{1}{2}(a+b)(a^2-4ab+b^2)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)(a^2+4ab+b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d}$$
$$3e^3$$

$$\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$\downarrow$ 1479

$$\frac{6e\left(\frac{1}{2}(a+b)(a^2-4ab+b^2)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-b)(a^2\right.}{d}$$
$$3e^3$$

$$\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$\downarrow$ 25

3.66. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx$

$$\frac{6e\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-b)\left(a^2+4ab\right.}{d}}{3e^3}$$

$$\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{6e\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}(a-b)\left(a^2+4ab\right.}{d}}{3e^3}$$

$$\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow 1103$$

$$\frac{6e\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+\right.}{2\sqrt{2}\sqrt{e}}\right.}{d}}{3e^3}$$

$$\frac{2a^2(a+b\cot(c+dx))}{3de(e\cot(c+dx))^{3/2}}$$

input

```
Int[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(5/2),x]
```

output

```
(2*a^2*(a + b*Cot[c + d*x]))/(3*d*e*(e*Cot[c + d*x])^(3/2)) + ((16*a^2*b*e
)/(d*Sqrt[e*Cot[c + d*x]]) + (6*e*(((a - b)*(a^2 + 4*a*b + b^2)*(-(ArcTan[
1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 + ((a + b)
*(a^2 - 4*a*b + b^2)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e
*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[
e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/d)/(3*e^3)
```

3.66. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx$

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

3.66. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.66.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{2\left(\frac{\left(-a^{3}e+3aeb^{2}\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8e^{2}}\right.}{}$
default	$-\frac{2\left(\frac{\left(-a^{3}e+3aeb^{2}\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8e^{2}}\right.}{}$
parts	$-\frac{2a^{3}e\left(-\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{e\cot}}{\left(e^{2}\right)}\right.\right.}{8e^{4}}\right.}{d}$

input
```
int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/e^2*(1/8*(-a^3*e+3*a*b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)
+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1))+1/8*(-3*a^2*b+b^3)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)
)-1/3*a^3*e/(e*cot(d*x+c))^(3/2)-3*a^2*b/(e*cot(d*x+c))^(1/2))
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1692 vs. $2(258) = 516$.

Time = 0.37 (sec) , antiderivative size = 1692, normalized size of antiderivative = 5.41

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^12 - 30*a
^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d
^4*e^10)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^5))*log(-(a^12 - 12*a^1
0*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x +
2*c) + e)/sin(2*d*x + 2*c)) + ((3*a^2*b - b^3)*d^3*e^8*sqrt(-(a^12 - 30*a^
10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^
4*e^10)) + (a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)*d*e^3)*s
qrt(-(d^2*e^5*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*
a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^10)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5
)/(d^2*e^5))) - 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a
^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10
+ b^12)/(d^4*e^10)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^5))*log(-(a^1
2 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*co
s(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((3*a^2*b - b^3)*d^3*e^8*sqrt(-(a^
12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 +
 b^12)/(d^4*e^10)) + (a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8
)*d*e^3)*sqrt(-(d^2*e^5*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*
b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^10)) + 6*a^5*b - 20*a^3*b^3
 + 6*a*b^5)/(d^2*e^5))) - 3*(d*e^3*cos(2*d*x + 2*c) + d*e^3)*sqrt((d^2*e^5
*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - ...
```

3.66.6 Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*cot(c + d*x))**3/(e*cot(c + d*x))**(5/2), x)
```

3.66.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.66.8 Giac [F]

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \int \frac{(b\cot(dx+c)+a)^3}{(e\cot(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^3/(e*cot(d*x + c))^(5/2), x)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 1946, normalized size of antiderivative = 6.22

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(5/2),x)
```

output
```
((2*a^3*e)/3 + 6*a^2*b*e*cot(c + d*x))/(d*e^2*(e*cot(c + d*x))^(3/2)) - at
an((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4
*d^3*e^8 - 240*a^4*b^2*d^3*e^8) + (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*((
(a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*
1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 -
a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2)*1i + ((e*cot(c + d*x))^(1
/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d
^3*e^8) - (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i + a
^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*
(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2
)*1i)/(4*d^2*e^5))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16
*b^6*d^3*e^8 + 240*a^2*b^4*d^3*e^8 - 240*a^4*b^2*d^3*e^8) + (32*a^3*d^4*e^
11 - 96*a*b^2*d^4*e^11)*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 -
a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i +
 a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2)
 - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^8 - 16*b^6*d^3*e^8 + 240*a^2*b^4*
d^3*e^8 - 240*a^4*b^2*d^3*e^8) - (32*a^3*d^4*e^11 - 96*a*b^2*d^4*e^11)*(((
a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a^3*b^3*20i - 15*a^4*b^2)*1
i)/(4*d^2*e^5))^(1/2))*(((a*b^5*6i + a^5*b*6i + a^6 - b^6 + 15*a^2*b^4 - a
^3*b^3*20i - 15*a^4*b^2)*1i)/(4*d^2*e^5))^(1/2) - 16*b^9*d^2*e^6 + 48*a...
```

3.66. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{5/2}}\,dx$

3.67 $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

3.67.1 Optimal result

Integrand size = 25, antiderivative size = 343

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} - \frac{(a+b)\left(a^2-4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{7/2}} + \frac{8a^2b}{5de^2(e\cot(c+dx))^{3/2}} - \frac{2a(a^2-3b^2)}{de^3\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}} - \frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}} + \frac{(a-b)\left(a^2+4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{7/2}}$$

output
```
8/5*a^2*b/d/e^2/(e*cot(d*x+c))^(3/2)+2/5*a^2*(a+b*cot(d*x+c))/d/e/(e*cot(d
*x+c))^(5/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/
2)/e^(1/2))/d/e^(7/2)*2^(1/2)-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*(
e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(7/2)*2^(1/2)-1/4*(a-b)*(a^2+4*a*b+b^2)*l
n(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(7/2)*2^(1/
2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(
d*x+c))^(1/2))/d/e^(7/2)*2^(1/2)-2*a*(a^2-3*b^2)/d/e^3/(e*cot(d*x+c))^(1/2
)
```

3.67.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.31

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \frac{2\left(3a(a^2-3b^2)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\cot^2(c+dx)\right)+b\left(b(9a+5b\right.\right.}{15de(e}$$

input
```
Integrate[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(7/2),x]
```

output
```
(2*(3*a*(a^2 - 3*b^2)*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2] +
b*(b*(9*a + 5*b*Cot[c + d*x]) + 5*(3*a^2 - b^2)*Cot[c + d*x]*Hypergeometri
c2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2])))/(15*d*e*(e*Cot[c + d*x])^(5/2))
```

3.67.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.94, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 4048, 27, 3042, 4111, 27, 3042, 4012, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4048}$$

$$\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}-\frac{2\int -\frac{-b(3a^2-5b^2)\cot^2(c+dx)e^2+12a^2be^2-5a(a^2-3b^2)\cot(c+dx)e^2}{2(e\cot(c+dx))^{5/2}}dx}{5e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-b(3a^2-5b^2)\cot^2(c+dx)e^2+12a^2be^2-5a(a^2-3b^2)\cot(c+dx)e^2}{(e\cot(c+dx))^{5/2}}dx}{5e^3}+\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-b(3a^2-5b^2)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+12a^2be^2+5a(a^2-3b^2)\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5e^3}+\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4111}$$

$$\frac{\frac{\int -\frac{5\left(a\left(a^2-3b^2\right)e^3+b\left(3a^2-b^2\right)\cot(c+dx)e^3\right)}{(e\cot(c+dx))^{3/2}}dx}{e^2}+\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}}{5e^3}+\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\int \frac{a\left(a^2-3b^2\right)e^3+b\left(3a^2-b^2\right)\cot(c+dx)e^3}{(e\cot(c+dx))^{3/2}}dx}{e^2}}{5e^3}+\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\int \frac{a\left(a^2-3b^2\right)e^3-b\left(3a^2-b^2\right)e^3\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{e^2}}{5e^3}+\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{\int \frac{b\left(3a^2-b^2\right)e^4-a\left(a^2-3b^2\right)e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}\right)}{e^2}}{5e^3}+\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{\int \frac{b\left(3a^2-b^2\right)e^4+a\left(a^2-3b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)e^4}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}+\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}\right)}{e^2}}{5e^3}+\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{2\int -\frac{e^4\left(b\left(3a^2-b^2\right)e-a\left(a^2-3b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}+\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}\right)}{e^2}}{5e^3}+\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

3.67. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

$\downarrow$ 25

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2\int\frac{e^4\left(b\left(3a^2-b^2\right)e-a\left(a^2-3b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}\right)}{e^2}}{5e^3}+\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e^2\int\frac{b\left(3a^2-b^2\right)e-a\left(a^2-3b^2\right)e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}\right)}{e^2}}{5e^3}+\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$\downarrow$ 1482

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int\frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(\right.}{d}}{e^2}}{5e^3}\qquad \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$\downarrow$ 1476

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\,c}\right.}{d}}{e^2}}{5e^3}\qquad \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$\downarrow$ 1082

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int\frac{1}{-e\cot(c+dx)-1}d\left(1-\sqrt{\right.}}{\sqrt{2}\sqrt{e}}\right.}{d}}{e^2}}{5e^3}\qquad \frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

3.67. $\int\frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

$\big\downarrow$ 217

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+\cdots\right)}{\sqrt{2}\sqrt{e}}\cdots\right)\right)}{d}\cdots\right)}{e^2}\cdots}{5e^3}$$

$$\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 1479

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(\cdots}\right.}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cdots}}\cdots}{2\sqrt{2}\cdots}\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{e^2}\cdots}{5e^3}$$

$$\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 25

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx}\cdots\right.}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+\cdots}}\cdots}{2\sqrt{2}\sqrt{e}}\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{e^2}\cdots}{5e^3}$$

$$\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+\cdots}}\cdots}{2\sqrt{e}}\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{e^2}\cdots}{5e^3}$$

$$\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

3.67. $\int\frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{8a^2be}{d(e\cot(c+dx))^{3/2}}-\frac{5\left(\frac{2ae^2\left(a^2-3b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(e\cot(c+dx)-\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right)\cdots\right.}{d}\right.}{e^2}}{5e^3}$$

$$\frac{2a^2(a+b\cot(c+dx))}{5de(e\cot(c+dx))^{5/2}}$$

input
```
Int[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(7/2),x]
```

output
```
(2*a^2*(a + b*Cot[c + d*x]))/(5*d*e*(e*Cot[c + d*x])^(5/2)) + ((8*a^2*b*e)
/(d*(e*Cot[c + d*x])^(3/2)) - (5*((2*a*(a^2 - 3*b^2)*e^2)/(d*Sqrt[e*Cot[c
+ d*x]]) - (2*e^2*(-1/2*((a + b)*(a^2 - 4*a*b + b^2)*(-(ArcTan[1 - (Sqrt[2
]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*
Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e]))) + ((a - b)*(a^2 + 4*a*b
 + b^2)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]
]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot
[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/d))/e^2)/(5*e^3)
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

3.67. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.67.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{2\left(\frac{\left(-3a^2be+b^3e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8e^2}\right.}{}$
default	$-\frac{2\left(\frac{\left(-3a^2be+b^3e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(dx+c)}}{\left(e^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{8e^2}\right.}{}$
parts	$-\frac{2a^3e\left(-\frac{1}{5e^2(e\cot(dx+c))^{\frac{5}{2}}}+\frac{1}{e^4\sqrt{e\cot(dx+c)}}+\frac{\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{e\cot(d}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e^4\left(e^2\right)^{\frac{1}{4}}}\right.}{d}$

input

```
int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output

```
-2/d/e^2*(1/e*(1/8*(-3*a^2*b*e+b^3*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d
*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1))+1/8*(a^3-3*a*b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2
)+1)))-1/5*a^3*e/(e*cot(d*x+c))^(5/2)-a^2*b/(e*cot(d*x+c))^(3/2)+a/e*(a^2-
3*b^2)/(e*cot(d*x+c))^(1/2))
```

3.67. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. $2(286) = 572$.

Time = 0.36 (sec) , antiderivative size = 1804, normalized size of antiderivative = 5.26

$$\int \frac{(a + b\cot(c + dx))^3}{(e\cot(c + dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/10*(5*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)*sqrt
((d^2*e^7*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*
b^8 - 30*a^2*b^10 + b^12)/(d^4*e^14)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d
^2*e^7))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10
- b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^3 - 3*a*b^2)
*d^3*e^11*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*
b^8 - 30*a^2*b^10 + b^12)/(d^4*e^14)) - (3*a^8*b - 46*a^6*b^3 + 60*a^4*b^5
 - 18*a^2*b^7 + b^9)*d*e^4)*sqrt((d^2*e^7*sqrt(-(a^12 - 30*a^10*b^2 + 255*
a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^14)) + 6*
a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^7))) - 5*(d*e^4*cos(2*d*x + 2*c)^2 +
2*d*e^4*cos(2*d*x + 2*c) + d*e^4)*sqrt((d^2*e^7*sqrt(-(a^12 - 30*a^10*b^2
+ 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^14)
) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^7))*log(-(a^12 - 12*a^10*b^2 -
27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e
)/sin(2*d*x + 2*c)) - ((a^3 - 3*a*b^2)*d^3*e^11*sqrt(-(a^12 - 30*a^10*b^2
+ 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^14)
) - (3*a^8*b - 46*a^6*b^3 + 60*a^4*b^5 - 18*a^2*b^7 + b^9)*d*e^4)*sqrt((d^
2*e^7*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8
- 30*a^2*b^10 + b^12)/(d^4*e^14)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e
^7))) - 5*(d*e^4*cos(2*d*x + 2*c)^2 + 2*d*e^4*cos(2*d*x + 2*c) + d*e^4)...
```

3.67.6 Sympy [F]

$$\int \frac{(a + b\cot(c + dx))^3}{(e\cot(c + dx))^{7/2}}\,dx = \int \frac{(a + b\cot{(c + dx)})^3}{(e\cot{(c + dx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(7/2),x)
```

output
```
Integral((a + b*cot(c + d*x))**3/(e*cot(c + d*x))**(7/2), x)
```

3.67.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.67.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.67.9 Mupad [B] (verification not implemented)

Time = 15.67 (sec) , antiderivative size = 1969, normalized size of antiderivative = 5.74

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(7/2),x)
```

output
```
atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2
*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^
15)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^
4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^
2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2)*1i + ((e*cot(c +
d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 24
0*a^4*b^2*d^3*e^11) - (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-((a*b^5*6i +
 a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*
e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20
i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^
6*d^3*e^11 - 16*b^6*d^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11
) - (32*b^3*d^4*e^15 - 96*a^2*b*d^4*e^15)*(-((a*b^5*6i + a^5*b*6i - a^6 +
b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a
*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i
)/(4*d^2*e^7))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^11 - 16*b^6*d
^3*e^11 + 240*a^2*b^4*d^3*e^11 - 240*a^4*b^2*d^3*e^11) + (32*b^3*d^4*e^15
- 96*a^2*b*d^4*e^15)*(-((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^
3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2))*(-((a*b^5*6i + a^5*b*6i -
a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^7))^(1/2)
- 16*a^9*d^2*e^8 + 48*a*b^8*d^2*e^8 + 128*a^3*b^6*d^2*e^8 + 96*a^5*b^4*...
```

3.67. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{7/2}}\,dx$

3.68 $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

3.68.1 Optimal result

Integrand size = 25, antiderivative size = 377

$$\begin{aligned}\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx &= \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{9/2}} \\ &- \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}de^{9/2}} + \frac{32a^2b}{35de^2(e\cot(c+dx))^{5/2}} \\ &- \frac{2a(a^2-3b^2)}{3de^3(e\cot(c+dx))^{3/2}} - \frac{2b(3a^2-b^2)}{de^4\sqrt{e\cot(c+dx)}} + \frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}} \\ &+ \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{9/2}} \\ &- \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}de^{9/2}}\end{aligned}$$

output
```
32/35*a^2*b/d/e^2/(e*cot(d*x+c))^(5/2)-2/3*a*(a^2-3*b^2)/d/e^3/(e*cot(d*x+
c))^(3/2)+2/7*a^2*(a+b*cot(d*x+c))/d/e/(e*cot(d*x+c))^(7/2)+1/2*(a-b)*(a^2
+4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/d/e^(9/2)*2^(1/
2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2)
)/d/e^(9/2)*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2
)-2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(9/2)*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2
)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/d/e^(9/2)*2^
(1/2)-2*b*(3*a^2-b^2)/d/e^4/(e*cot(d*x+c))^(1/2)
```

3.68.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.80 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.31

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \frac{2\sqrt{e\cot(c+dx)}\left(5a(a^2-3b^2)\,\text{Hypergeometric2F1}\left(-\frac{7}{4},1,-\frac{3}{4},-\cot^2(c+dx)\right)\right.}{}$$

input
```
Integrate[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(9/2),x]
```

output
```
(2*Sqrt[e*Cot[c + d*x]]*(5*a*(a^2 - 3*b^2)*Hypergeometric2F1[-7/4, 1, -3/4
, -Cot[c + d*x]^2] + b*(b*(15*a + 7*b*Cot[c + d*x]) + 7*(3*a^2 - b^2)*Cot[
c + d*x]*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[c + d*x]^2]))*Tan[c + d*x]^
4)/(35*d*e^5)
```

3.68.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.97, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4048, 27, 3042, 4111, 27, 3042, 4012, 3042, 4012, 25, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 4048}$$

$$\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}-\frac{2\int -\frac{-b(5a^2-7b^2)\cot^2(c+dx)e^2+16a^2be^2-7a(a^2-3b^2)\cot(c+dx)e^2}{2(e\cot(c+dx))^{7/2}}dx}{7e^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-b(5a^2-7b^2)\cot^2(c+dx)e^2+16a^2be^2-7a(a^2-3b^2)\cot(c+dx)e^2}{(e\cot(c+dx))^{7/2}}dx}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-b(5a^2-7b^2)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+16a^2be^2+7a(a^2-3b^2)\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4111}$$

$$\frac{\frac{\int -\frac{7\left(a\left(a^2-3b^2\right)e^3+b\left(3a^2-b^2\right)\cot(c+dx)e^3\right)}{(e\cot(c+dx))^{5/2}}dx}{e^2}+\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\int \frac{a\left(a^2-3b^2\right)e^3+b\left(3a^2-b^2\right)\cot(c+dx)e^3}{(e\cot(c+dx))^{5/2}}dx}{e^2}}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\int \frac{a\left(a^2-3b^2\right)e^3-b\left(3a^2-b^2\right)e^3\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{e^2}}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\int \frac{b\left(3a^2-b^2\right)e^4-a\left(a^2-3b^2\right)e^4\cot(c+dx)}{(e\cot(c+dx))^{3/2}}dx}{e^2}+\frac{2ae^2\left(a^2-3b^2\right)}{3d(e\cot(c+dx))^{3/2}}\right)}{e^2}}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\int \frac{b\left(3a^2-b^2\right)e^4+a\left(a^2-3b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)e^4}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{e^2}+\frac{2ae^2\left(a^2-3b^2\right)}{3d(e\cot(c+dx))^{3/2}}\right)}{e^2}}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{\int -\frac{a\left(a^2-3b^2\right)e^5+b\left(3a^2-b^2\right)\cot(c+dx)e^5}{\sqrt{e\cot(c+dx)}}dx}{e^2}+\frac{2be^3\left(3a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2ae^2\left(a^2-3b^2\right)}{3d(e\cot(c+dx))^{3/2}}\right)}{e^2}}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

3.68. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

$\Big\downarrow$ 25

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2be^3\left(3a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{a\left(a^2-3b^2\right)e^5+b\left(3a^2-b^2\right)\cot(c+dx)e^5}{\sqrt{e\cot(c+dx)}}dx}{e^2}}{e^2}+\frac{2ae^2\left(a^2-3b^2\right)}{3d(e\cot(c+dx))^{3/2}}\right)}{e^2}}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2be^3\left(3a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{\int \frac{a\left(a^2-3b^2\right)e^5-b\left(3a^2-b^2\right)e^5\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{e^2}}{e^2}+\frac{2ae^2\left(a^2-3b^2\right)}{3d(e\cot(c+dx))^{3/2}}\right)}{e^2}}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$\Big\downarrow$ 4017

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2be^3\left(3a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}-\frac{2\int -\frac{e^5\left(a\left(a^2-3b^2\right)e+b\left(3a^2-b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}}{e^2}+\frac{2ae^2\left(a^2-3b^2\right)}{3d(e\cot(c+dx))^{3/2}}\right)}{e^2}}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2\int \frac{e^5\left(a\left(a^2-3b^2\right)e+b\left(3a^2-b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{de^2}+\frac{2be^3\left(3a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2ae^2\left(a^2-3b^2\right)}{3d(e\cot(c+dx))^{3/2}}\right)}{e^2}}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$\Big\downarrow$ 27

3.68. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2e^3\int\frac{a\left(a^2-3b^2\right)e+b\left(3a^2-b^2\right)\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d}+\frac{2be^3\left(3a^2-b^2\right)}{d\sqrt{e\cot(c+dx)}}}{e^2}+\frac{2ae^2\left(a^2-3b^2\right)}{3d(e\cot(c+dx))^{3/2}}\right)}{e^2}}{7e^3}+\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$\downarrow$ 1482

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2e^3\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int\frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d}+\frac{2be^3(\ldots}{d\sqrt{e\,c\ldots}}}{e^2}\ldots}{e^2}}{7e^3}\ldots$$

$$\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$\downarrow$ 1476

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2e^3\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{\ldots}\right.\right.}{d}\ldots}{e^2}\ldots}{e^2}}{7e^3}\ldots$$

$$\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$\downarrow$ 1082

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2e^3\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int\frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\ldots\right.\right.}{d}\ldots}{e^2}\ldots}{e^2}}{7e^3}\ldots$$

$$\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$\downarrow$ 217

3.68. $\int\frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2e^3\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\cdots\right.}{\cdots}\cdots\right)\cdots\right)}{d}}{e^2}\cdots\right)}{e^2}}{7e^3}-\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$\downarrow$ 1479

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2e^3\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{\cdots}}{2\sqrt{2}\sqrt{e}}\cdots\right)\cdots\right)}{d}}{e^2}\cdots\right)}{\cdots}}{7e^3}-\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$\downarrow$ 25

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2e^3\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(\cdots}}{2\sqrt{2}\sqrt{e}}\cdots\right)\cdots\right)}{d}}{e^2}\cdots\right)}{\cdots}}{7e^3}-\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$\downarrow$ 27

3.68. $\int\frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2e^3\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c\cdots}{2\sqrt{e}}\cdots\right.\right.}{d}\cdots}{e^2}\cdots\right.}{\cdots}}{7e^3}\cdots$$

$$\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{32a^2be}{5d(e\cot(c+dx))^{5/2}}-\frac{7\left(\frac{\frac{2e^3\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\log\left(e\cot(c+\cdots\right.\right.\right.}{\cdots}}{d}\cdots}{e^2}\cdots\right.}{e^2\cdots}}{7e^3}\cdots$$

$$\frac{2a^2(a+b\cot(c+dx))}{7de(e\cot(c+dx))^{7/2}}$$

input
```
Int[(a + b*Cot[c + d*x])^3/(e*Cot[c + d*x])^(9/2),x]
```

output
```
(2*a^2*(a + b*Cot[c + d*x]))/(7*d*e*(e*Cot[c + d*x])^(7/2)) + ((32*a^2*b*e
)/(5*d*(e*Cot[c + d*x])^(5/2)) - (7*((2*a*(a^2 - 3*b^2)*e^2)/(3*d*(e*Cot[c
 + d*x])^(3/2)) + ((2*b*(3*a^2 - b^2)*e^3)/(d*Sqrt[e*Cot[c + d*x]]) + (2*e
^3*(((a - b)*(a^2 + 4*a*b + b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x
]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]]
)/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 + ((a + b)*(a^2 - 4*a*b + b^2)*(-1/2*Log[
e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e
]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqr
t[2]*Sqrt[e])))/2))/d)/e^2))/e^2)/(7*e^3)
```

3.68. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

3.68.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

3.68. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.68. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

3.68.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.03

method	result
derivativedivides	$-\frac{2\left(\frac{\left(a^{3}e-3ae\,b^{2}\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{\ }}{}\right.\right.}{8e^{2}}}{}$
default	$-\frac{2\left(\frac{\left(a^{3}e-3ae\,b^{2}\right)\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{\left(e^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{\ }}{}\right.\right.}{8e^{2}}}{}$
parts	$-\frac{2a^{3}e\left(-\frac{1}{7e^{2}(e\cot(dx+c))^{\frac{7}{2}}}+\frac{1}{3e^{4}(e\cot(dx+c))^{\frac{3}{2}}}+\frac{\left(e^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}{e\cot(dx+c)-\left(e^{2}\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}}{}\right.\right.}{8e^{6}}\right.}{d}$

input
```
int((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/e^2*(1/e^2*(1/8*(a^3*e-3*a*b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(
d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)
-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1))+1/8*(3*a^2*b-b^3)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)+1)))-1/7*a^3*e/(e*cot(d*x+c))^(7/2)-3/5*a^2*b/(e*cot(d*x+c))^(5/2)+1/3*
a/e*(a^2-3*b^2)/(e*cot(d*x+c))^(3/2)+b*(3*a^2-b^2)/e^2/(e*cot(d*x+c))^(1/2
))
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1839 vs. $2(316) = 632$.

Time = 0.34 (sec) , antiderivative size = 1839, normalized size of antiderivative = 4.88

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
1/210*(105*(d*e^5*cos(2*d*x + 2*c)^2 + 2*d*e^5*cos(2*d*x + 2*c) + d*e^5)*s
qrt(-(d^2*e^9*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*
a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^18)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5
)/(d^2*e^9))*log(-(a^12 - 12*a^10*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b
^10 - b^12)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((3*a^2*b -
b^3)*d^3*e^14*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*
a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^18)) + (a^9 - 18*a^7*b^2 + 60*a^5*b^4
 - 46*a^3*b^6 + 3*a*b^8)*d*e^5)*sqrt(-(d^2*e^9*sqrt(-(a^12 - 30*a^10*b^2 +
 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^18))
 + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^9))) - 105*(d*e^5*cos(2*d*x + 2*
c)^2 + 2*d*e^5*cos(2*d*x + 2*c) + d*e^5)*sqrt(-(d^2*e^9*sqrt(-(a^12 - 30*a
^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d
^4*e^18)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^5)/(d^2*e^9))*log(-(a^12 - 12*a^1
0*b^2 - 27*a^8*b^4 + 27*a^4*b^8 + 12*a^2*b^10 - b^12)*sqrt((e*cos(2*d*x +
2*c) + e)/sin(2*d*x + 2*c)) - ((3*a^2*b - b^3)*d^3*e^14*sqrt(-(a^12 - 30*a
^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255*a^4*b^8 - 30*a^2*b^10 + b^12)/(d
^4*e^18)) + (a^9 - 18*a^7*b^2 + 60*a^5*b^4 - 46*a^3*b^6 + 3*a*b^8)*d*e^5)*
sqrt(-(d^2*e^9*sqrt(-(a^12 - 30*a^10*b^2 + 255*a^8*b^4 - 452*a^6*b^6 + 255
*a^4*b^8 - 30*a^2*b^10 + b^12)/(d^4*e^18)) + 6*a^5*b - 20*a^3*b^3 + 6*a*b^
5)/(d^2*e^9))) - 105*(d*e^5*cos(2*d*x + 2*c)^2 + 2*d*e^5*cos(2*d*x + 2*...
```

3.68.6 Sympy [F]

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))**3/(e*cot(d*x+c))**(9/2),x)
```

output
```
Integral((a + b*cot(c + d*x))**3/(e*cot(c + d*x))**(9/2), x)
```

3.68.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.68.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cot(d*x+c))^3/(e*cot(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

3.68.9 Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 1992, normalized size of antiderivative = 5.28

$$\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx = \text{Too large to display}$$

input
```
int((a + b*cot(c + d*x))^3/(e*cot(c + d*x))^(9/2),x)
```

output
```
atan((((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2
*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) + (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^
19)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4
*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*
b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2)*1i + ((e*cot(c + d*
x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14 - 240*
a^4*b^2*d^3*e^14) - (32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i + a^
5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9
))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i +
15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2)*1i)/(((e*cot(c + d*x))^(1/2)*(16*a^6*d^
3*e^14 - 16*b^6*d^3*e^14 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) +
(32*a^3*d^4*e^19 - 96*a*b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 -
 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6
i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d
^2*e^9))^(1/2) - ((e*cot(c + d*x))^(1/2)*(16*a^6*d^3*e^14 - 16*b^6*d^3*e^1
4 + 240*a^2*b^4*d^3*e^14 - 240*a^4*b^2*d^3*e^14) - (32*a^3*d^4*e^19 - 96*a
*b^2*d^4*e^19)*(((a*b^5*6i + a^5*b*6i - a^6 + b^6 - 15*a^2*b^4 - a^3*b^3*2
0i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2))*(((a*b^5*6i + a^5*b*6i - a^6 + b^
6 - 15*a^2*b^4 - a^3*b^3*20i + 15*a^4*b^2)*1i)/(4*d^2*e^9))^(1/2) - 16*b^9
*d^2*e^10 + 48*a^8*b*d^2*e^10 + 96*a^4*b^5*d^2*e^10 + 128*a^6*b^3*d^2*e...
```

3.68. $\int \frac{(a+b\cot(c+dx))^3}{(e\cot(c+dx))^{9/2}}\,dx$

3.69 $\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx$

3.69.1 Optimal result

Integrand size = 25, antiderivative size = 325

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = \frac{2a^{5/2}e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}(a^2+b^2)\,d} - \frac{(a+b)e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)\,d} + \frac{(a+b)e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)\,d} - \frac{2e^2\sqrt{e\cot(c+dx)}}{bd} + \frac{(a-b)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)\,d} - \frac{(a-b)e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)\,d}$$

output
```
2*a^(5/2)*e^(5/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/b^(
3/2)/(a^2+b^2)/d-1/2*(a+b)*e^(5/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e
^(1/2))/(a^2+b^2)/d*2^(1/2)+1/2*(a+b)*e^(5/2)*arctan(1+2^(1/2)*(e*cot(d*x+
c))^(1/2)/e^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*e^(5/2)*ln(e^(1/2)+cot(d*
x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*e
^(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^
2)/d*2^(1/2)-2*e^2*(e*cot(d*x+c))^(1/2)/b/d
```

3.69.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.88

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = \frac{(e\cot(c+dx))^{5/2}\left(8ab^{3/2}\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx\right.\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x]),x]
```

output
```
((e*Cot[c + d*x])^(5/2)*(8*a*b^(3/2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[
3/4, 1, 7/4, -Cot[c + d*x]^2] - 3*(2*Sqrt[2]*b^(5/2)*ArcTan[1 - Sqrt[2]*Sq
rt[Cot[c + d*x]]] - 2*Sqrt[2]*b^(5/2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
]] - 8*a^(5/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]] + 8*a^2*Sqrt[b
]*Sqrt[Cot[c + d*x]] + 8*b^(5/2)*Sqrt[Cot[c + d*x]] + Sqrt[2]*b^(5/2)*Log[
1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*b^(5/2)*Log[1 + S
qrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(12*b^(3/2)*(a^2 + b^2)*d*Cot
[c + d*x]^(5/2))
```

3.69.3 Rubi [A] (warning: unable to verify)

Time = 1.25 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.90, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4049, 27, 3042, 4136, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4049}$$

$$-\frac{2\int \frac{a\cot^2(c+dx)e^3+ae^3+b\cot(c+dx)e^3}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$\downarrow$ 27

$$-\frac{\int \frac{a\cot^2(c+dx)e^3+ae^3+b\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$\downarrow$ 3042

$$-\frac{\int \frac{a\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3+ae^3-b\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$\downarrow$ 4136

$$-\frac{\frac{\int \frac{b^2e^3+ab\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}+\frac{a^3e^3\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$\downarrow$ 3042

$$-\frac{\frac{\int \frac{b^2e^3-abe^3\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}+\frac{a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$\downarrow$ 4017

$$-\frac{\frac{2\int -\frac{be^3(be+a\cot(c+dx)e)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}+\frac{a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$\downarrow$ 25

$$-\frac{\frac{a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int \frac{be^3(be+a\cot(c+dx)e)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$\downarrow$ 27

$$-\frac{\frac{a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2be^3\int \frac{be+a\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$\downarrow$ 1482

$$-\frac{\frac{a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2be^3\left(\frac{1}{2}(a+b)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d(a^2+b^2)}}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

3.69. $\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx$

$$\Big\downarrow \ 1476$$

$$-\frac{\frac{a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2be^3\left(\frac{1}{2}(a+b)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e+\sqrt{2}\cdots}\right.\right.}{d(a^2+b^2)}}{b}$$

$$-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$$\Big\downarrow \ 1082$$

$$-\frac{\frac{a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2be^3\left(\frac{1}{2}(a+b)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{e\cot(c+d\cdots}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}}{b}$$

$$-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$$\Big\downarrow \ 217$$

$$-\frac{\frac{a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2be^3\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\int \frac{e-\cdots}{\cot^2\cdots}\right.}{d(a^2+b^2)}}{b}$$

$$-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$$\Big\downarrow \ 1479$$

$$-\frac{\frac{a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2be^3\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(-\frac{\int \cdots}{\cdots}\right.\right.}{\cdots}}{b}$$

$$-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2be^3\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\int \frac{\cdots}{\cot\cdots}}{\cdots}\right.\right.}{d(\cdots}}{b}$$

$$-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$$\Big\downarrow \ 27$$

3.69. $\displaystyle\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx$

$$-\frac{\frac{a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2be^3\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\int \frac{}{\cot}\right.\right.}{d(}}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$\downarrow$ 1103

$$-\frac{\frac{a^3e^3\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2be^3\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(e\right.}{}\right.\right.}{d(a^2+b^2)}}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$\downarrow$ 4117

$$-\frac{\frac{a^3e^3\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}-\frac{2be^3\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log}{}\right.\right.}{d(a^2+b^2)}}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$\downarrow$ 73

$$-\frac{-\frac{2a^3e^2\int \frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}-\frac{2be^3\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(e\cot(c+\right.}{}\right.\right.}{d(a^2+b^2)}}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

$\downarrow$ 218

$$-\frac{\frac{2a^{5/2}e^{5/2}\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}d(a^2+b^2)}-\frac{2be^3\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{\right.}{2\sqrt{2}\sqrt{}}\right.\right.}{d(a^2+b^2)}}{b}-\frac{2e^2\sqrt{e\cot(c+dx)}}{bd}$$

input
```
Int[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x]),x]
```

3.69. $\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx$

output
```
(-2*e^2*Sqrt[e*Cot[c + d*x]])/(b*d) - ((2*a^(5/2)*e^(5/2)*ArcTan[(Sqrt[b]*
Cot[c + d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt[b]*(a^2 + b^2)*d) - (2*b*e^3*(((a
+ b)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e
])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])
))/2 - ((a - b)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[
c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sq
rt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/((a^2 + b^2)*d))/b
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

3.69. $\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx$

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.69.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{2e^2\left(\frac{\sqrt{e\cot(dx+c)}}{b}-\frac{a^3 e\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{b\left(a^2+b^2\right)\sqrt{aeb}}-\frac{e\left(\frac{b\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\,\mathrm{arcta}\cdots\right.}{8e}\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2e^2\left(\frac{\sqrt{e\cot(dx+c)}}{b}-\frac{a^3 e\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{b\left(a^2+b^2\right)\sqrt{aeb}}-\frac{e\left(\frac{b\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\,\mathrm{arcta}\cdots\right.}{8e}\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^2*((e*cot(d*x+c))^(1/2)/b-1/b*a^3*e/(a^2+b^2)/(a*e*b)^(1/2)*arctan(
(e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2))-e/(a^2+b^2)*(1/8*b/e*(e^2)^(1/4)*2^(
1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2
))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*
arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*a/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c
)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)+1))))
```

3.69. $\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx$

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1602 vs. $2(262) = 524$.

Time = 0.37 (sec) , antiderivative size = 3267, normalized size of antiderivative = 10.05

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(2*a^2*sqrt(-a*e/b)*e^2*log((b*e*cos(2*d*x + 2*c) - a*e*sin(2*d*x + 2
*c) + 2*b*sqrt(-a*e/b)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))*sin
(2*d*x + 2*c) + b*e)/(b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)) - 4*(a
^2 + b^2)*e^2*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - (a^2*b + b
^3)*d*sqrt(-(2*a*b*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*
b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^
4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^7*sqrt((e*cos(2*d*x + 2*c) +
 e)/sin(2*d*x + 2*c)) + ((a^2*b - b^3)*d*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^
4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^5 + 2*a^
3*b^2 + a*b^4)*d^3)*sqrt(-(2*a*b*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/
((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 +
b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) + (a^2*b + b^3)*d*sqrt(-(2*a*b*e
^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*a^2*b^2 + b^4)
*d^2))*log(-(a^2 - b^2)*e^7*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)
) - ((a^2*b - b^3)*d*e^5 - sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^5 + 2*a^3*b^2 + a*b^4)*d^3)*
sqrt(-(2*a*b*e^5 + sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^10/((a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*(a^4 + 2*a^2*b^2 + b^4)*d^2)/((a^4 + 2*
a^2*b^2 + b^4)*d^2))) - (a^2*b + b^3)*d*sqrt(-(2*a*b*e^5 - sqrt(-(a^4 -...
```

3.69.6 Sympy [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = \int \frac{(e\cot{(c+dx)})^{\frac{5}{2}}}{a+b\cot{(c+dx)}}\,dx$$

input
```
integrate((e*cot(d*x+c))**(5/2)/(a+b*cot(d*x+c)),x)
```

output
```
Integral((e*cot(c + d*x))**(5/2)/(a + b*cot(c + d*x)), x)
```

3.69.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.69.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = \int \frac{(e\cot(dx+c))^{\frac{5}{2}}}{b\cot(dx+c)+a}\,dx$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(5/2)/(b*cot(d*x + c) + a), x)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 14.68 (sec) , antiderivative size = 5579, normalized size of antiderivative = 17.17

$$\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(5/2)/(a + b*cot(c + d*x)),x)
```

output
```
(atan(((((32*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^4) + (((
32*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/(b*d^5) +
 (((32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e^15 + 4*a
^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4) - (((32*(4*a*b^8*d^4*e^13 +
 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) + (32*(e*cot(c + d*x))^
(1/2)*(-a^5*b^3*e^5)^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^10 - 16*a^
4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b^4*d^5*(a^2 + b^2)))*(-a^5*b^3*e^
5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*
(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2)*1i)/(b^3*d
*(a^2 + b^2)) + (((32*(e*cot(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d
^4) - (((32*(12*a^6*b*d^2*e^18 + a^2*b^5*d^2*e^18 - 15*a^4*b^3*d^2*e^18))/
(b*d^5) - (((32*(e*cot(c + d*x))^(1/2)*(16*a^7*b*d^2*e^15 - 14*a*b^7*d^2*e
^15 + 4*a^3*b^5*d^2*e^15 + 2*a^5*b^3*d^2*e^15))/(b*d^4) + (((32*(4*a*b^8*d
^4*e^13 + 8*a^3*b^6*d^4*e^13 + 4*a^5*b^4*d^4*e^13))/(b*d^5) - (32*(e*cot(c
 + d*x))^(1/2)*(-a^5*b^3*e^5)^(1/2)*(16*b^10*d^4*e^10 + 16*a^2*b^8*d^4*e^1
0 - 16*a^4*b^6*d^4*e^10 - 16*a^6*b^4*d^4*e^10))/(b^4*d^5*(a^2 + b^2)))*(-a
^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2
+ b^2)))*(-a^5*b^3*e^5)^(1/2))/(b^3*d*(a^2 + b^2)))*(-a^5*b^3*e^5)^(1/2)*1
i)/(b^3*d*(a^2 + b^2)))/((64*(a^5*e^23 - a^3*b^2*e^23))/(b*d^5) + (((32*(e
*cot(c + d*x))^(1/2)*(2*a^6*e^20 - b^6*e^20))/(b*d^4) + (((32*(12*a^6*b...
```

3.69. $\int \frac{(e\cot(c+dx))^{5/2}}{a+b\cot(c+dx)}\,dx$

3.70 $\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx$

3.70.1 Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = -\frac{2a^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)\,d} - \frac{(a-b)e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a-b)e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a+b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a+b)e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}$$

output
```
-1/2*(a-b)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2
)/d*2^(1/2)+1/2*(a-b)*e^(3/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2
))/(a^2+b^2)/d*2^(1/2)-1/4*(a+b)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(
1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)+1/4*(a+b)*e^(3/2)*ln(e^(1/2
)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)-2*a
^(3/2)*e^(3/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/(a^2+b
^2)/d/b^(1/2)
```

3.70.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.82

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx =$$
$$-\frac{(e\cot(c+dx))^{3/2}\left(8b^{3/2}\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx)\right)+3a\left(2\sqrt{2}\sqrt{b}\arctan\left(\right.\right.\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x]),x]
```

output
```
-1/12*((e*Cot[c + d*x])^(3/2)*(8*b^(3/2)*Cot[c + d*x]^(3/2)*Hypergeometric
2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + 3*a*(2*Sqrt[2]*Sqrt[b]*ArcTan[1 - Sqrt
[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*Sqrt[b]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c
+ d*x]]] + 8*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]] + Sqrt[2
]*Sqrt[b]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Sqr
t[b]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(Sqrt[b]*(a^2 +
 b^2)*d*Cot[c + d*x]^(3/2))
```

3.70.3 Rubi [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.87, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4056, 25, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4056}$$

$$\frac{a^2e^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}+\frac{\int -\frac{ae^2-be^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}$$

$\downarrow$ 25

$$\frac{a^2e^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}-\frac{\int \frac{ae^2-be^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}$$

$\downarrow$ 3042

$$\frac{a^2e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{\int \frac{ae^2+b\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}$$

$\downarrow$ 4017

$$\frac{a^2e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int -\frac{e^2(ae-be\cot(c+dx))}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}$$

$\downarrow$ 25

$$\frac{a^2e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{2\int \frac{e^2(ae-be\cot(c+dx))}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}$$

$\downarrow$ 27

$$\frac{a^2e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{2e^2\int \frac{ae-be\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}$$

$\downarrow$ 1482

$$\frac{a^2e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{2e^2\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d\left(a^2+b^2\right)}$$

$\downarrow$ 1476

$$\frac{a^2e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{2e^2\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\right.\right.}{d\left(a^2+b^2\right)}$$

$\downarrow$ 1082

3.70. $\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx$

$$\frac{a^2e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+$$

$$\frac{2e^2\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\cdots}{}\right)}{\sqrt{2}\sqrt{e\cdots}}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{2e^2\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d\left(a^2+b^2\right)}$$

$$\frac{a^2e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2e^2\left(\frac{1}{2}(a+b)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-\cdots\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a^2e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2e^2\left(\frac{1}{2}(a+b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-b)\left(\cdots\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a^2e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{27}$$

3.70. $\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx$

$$\frac{2e^2\left(\frac{1}{2}(a+b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}(a-b)\left(\cdots\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a^2e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 1103

$$\frac{a^2e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+$$

$$\frac{2e^2\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\cdots\right.}{d\left(a^2+b^2\right)}$$

$\downarrow$ 4117

$$\frac{a^2e^2\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}+$$

$$\frac{2e^2\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\cdots\right.}{d\left(a^2+b^2\right)}$$

$\downarrow$ 73

$$\frac{2e^2\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\cdots\right.}{d\left(a^2+b^2\right)}$$

$$\frac{2a^2e\int \frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}$$

$\downarrow$ 218

$$\frac{2e^2\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\cdots\right.}{d\left(a^2+b^2\right)}$$

$$\frac{2a^{3/2}e^{3/2}\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}d\left(a^2+b^2\right)}$$

3.70. $\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx$

input
```
Int[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x]),x]
```

output
```
(2*a^(3/2)*e^(3/2)*ArcTan[(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt
[b]*(a^2 + b^2)*d) + (2*e^2*(((a - b)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c
+ d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c +
d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 + ((a + b)*(-1/2*Log[e + e*Cot[c + d
*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*
Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))
/2))/((a^2 + b^2)*d)
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4056
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(c^2 + d^2)   Int[Simp[a^2*c - b^2*c +
2*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x
]], x], x] + Simp[(b*c - a*d)^2/(c^2 + d^2)   Int[(1 + Tan[e + f*x]^2)/(Sqr
t[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.70. $\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx$

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.70.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{2e^2\left(\frac{a^2\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{\left(a^2+b^2\right)\sqrt{aeb}}+\frac{-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$
default	$-\frac{2e^2\left(\frac{a^2\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{\left(a^2+b^2\right)\sqrt{aeb}}+\frac{-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$

input
```
int((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^2*(a^2/(a^2+b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b
)^(1/2))+1/(a^2+b^2)*(-1/8*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)
^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)
*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+
1))+1/8*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1537 vs. 2(241) = 482.

Time = 0.38 (sec) , antiderivative size = 3137, normalized size of antiderivative = 10.39

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*((a^2 + b^2)*d*sqrt((2*a*b*e^3 + (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4
- 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^
4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e^4*sqrt((e*cos(2
*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^3 - a*b^2)*d*e^3 + (a^4*b + 2*a^2
*b^3 + b^5)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8)*d^4))*d^3)*sqrt((2*a*b*e^3 + (a^4 + 2*a^2*b^2 + b^4)*
sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) - (a^2 + b^2)*d*sqrt((
2*a*b*e^3 + (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2
+ b^4)*d^2))*log(-(a^2 - b^2)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x
+ 2*c)) - ((a^3 - a*b^2)*d*e^3 + (a^4*b + 2*a^2*b^3 + b^5)*sqrt(-(a^4 - 2*
a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*
d^3)*sqrt((2*a*b*e^3 + (a^4 + 2*a^2*b^2 + b^4)*sqrt(-(a^4 - 2*a^2*b^2 + b^
4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4))*d^2)/((a^4 +
 2*a^2*b^2 + b^4)*d^2))) + (a^2 + b^2)*d*sqrt((2*a*b*e^3 - (a^4 + 2*a^2*b^
2 + b^4)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8)*d^4))*d^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^
2)*e^4*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^3 - a*b^2)*d*
e^3 - (a^4*b + 2*a^2*b^3 + b^5)*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^6/((a^8...
```

3.70.6 Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = \int \frac{(e\cot(c+dx))^{\frac{3}{2}}}{a+b\cot(c+dx)}\,dx$$

input
```
integrate((e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c)),x)
```

output
```
Integral((e*cot(c + d*x))**(3/2)/(a + b*cot(c + d*x)), x)
```

3.70.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.70.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{b\cot(dx+c)+a}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(3/2)/(b*cot(d*x + c) + a), x)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 5129, normalized size of antiderivative = 16.98

$$\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(3/2)/(a + b*cot(c + d*x)),x)
```

output

```
atan((((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^1
2))/d^5 - (32*(e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b
*d^2*2i)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e
^10 - 16*a^6*b^3*d^4*e^10))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2
*2i)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d
^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b
*d^2*2i)))^(1/2) + (32*(a*b^5*d^2*e^15 + 4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2
*e^15))/d^5)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(
e*cot(c + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b*e^16))/d^4)*((e^3*1i)/(4*(b^2*d^
2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i - (((((32*(4*a^2*b^6*d^4*e^12 + 8*a^4
*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d^5 + (32*(e*cot(c + d*x))^(1/2)*((e^
3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*(16*b^9*d^4*e^10 + 16*a^
2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)*((e^3*1i
)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) - (32*(e*cot(c + d*x))^(1/2)
*(14*a*b^6*d^2*e^13 - 4*a^3*b^4*d^2*e^13 + 14*a^5*b^2*d^2*e^13))/d^4)*((e^
3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2) + (32*(a*b^5*d^2*e^15 +
4*a^5*b*d^2*e^15 - 15*a^3*b^3*d^2*e^15))/d^5)*((e^3*1i)/(4*(b^2*d^2 - a^2*
d^2 + a*b*d^2*2i)))^(1/2) + (32*(e*cot(c + d*x))^(1/2)*(b^5*e^16 + 2*a^4*b
*e^16))/d^4)*((e^3*1i)/(4*(b^2*d^2 - a^2*d^2 + a*b*d^2*2i)))^(1/2)*1i)/(((
(((32*(4*a^2*b^6*d^4*e^12 + 8*a^4*b^4*d^4*e^12 + 4*a^6*b^2*d^4*e^12))/d...
```

3.70. $\int \frac{(e\cot(c+dx))^{3/2}}{a+b\cot(c+dx)}\,dx$

3.71 $\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx$

3.71.1 Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \frac{2\sqrt{a}\sqrt{b}\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{(a^2+b^2)\,d} + \frac{(a+b)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a+b)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a-b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a-b)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d}$$

output
```
1/2*(a+b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/(a^2+b^2)
/d*2^(1/2)-1/2*(a+b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2
)/(a^2+b^2)/d*2^(1/2)-1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*c
ot(d*x+c))^(1/2))*e^(1/2)/(a^2+b^2)/d*2^(1/2)+1/4*(a-b)*ln(e^(1/2)+cot(d*x
+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/(a^2+b^2)/d*2^(1/2)+2*ar
ctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*a^(1/2)*b^(1/2)*e^(1/2)
/(a^2+b^2)/d
```

3.71.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx$$
$$= \frac{\sqrt{e\cot(c+dx)}\left(6\sqrt{2}b\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-6\sqrt{2}b\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+24\sqrt{a}\sqrt{b}\right.}{}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x]),x]
```

output
```
(Sqrt[e*Cot[c + d*x]]*(6*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]
- 6*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*Sqrt[a]*Sqrt[b]*
ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]] - 8*a*Cot[c + d*x]^(3/2)*Hype
rgeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + 3*Sqrt[2]*b*Log[1 - Sqrt[2]*
Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 3*Sqrt[2]*b*Log[1 + Sqrt[2]*Sqrt[Cot[
c + d*x]] + Cot[c + d*x]]))/(12*(a^2 + b^2)*d*Sqrt[Cot[c + d*x]])
```

3.71.3 Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.86, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4055, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{4055}$$

$$\frac{\int \frac{be+a\cot(c+dx)e}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}-\frac{abe\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{be-ae\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}-\frac{abe\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\Big\downarrow$ 4017

$$\frac{2\int -\frac{e(be+a\cot(c+dx)e)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{abe\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\Big\downarrow$ 25

$$-\frac{2\int \frac{e(be+a\cot(c+dx)e)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{abe\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\Big\downarrow$ 27

$$-\frac{2e\int \frac{be+a\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{abe\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\Big\downarrow$ 1482

$$-\frac{2e\left(\frac{1}{2}(a+b)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d\left(a^2+b^2\right)}-\frac{abe\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\Big\downarrow$ 1476

$$-\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{abe\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\Big\downarrow$ 1082

3.71. $\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx$

$$-\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}\right.}{d\left(a^2+b^2\right)}$$

$$\frac{abe\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$$\Big\downarrow \; 217$$

$$-\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d\left(a^2+b^2\right)}$$

$$\frac{abe\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$$\Big\downarrow \; 1479$$

$$-\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\surd}{2\sqrt{2}\sqrt{e}}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{abe\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\,\mathrm{co}}}{2\sqrt{2}\sqrt{e}}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{abe\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$$\Big\downarrow \; 27$$

3.71. $\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx$

$$-\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot}}{2\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}$$

$$\frac{abe\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 1103

$$-\frac{abe\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-$$

$$\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}$$

$\downarrow$ 4117

$$-\frac{abe\int\frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}-$$

$$\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}$$

$\downarrow$ 73

$$\frac{2ab\int\frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}-$$

$$\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}$$

$\downarrow$ 218

$$-\frac{2\sqrt{a}\sqrt{b}\sqrt{e}\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}\sqrt{e}}\right)}{d(a^2+b^2)}-$$

$$\frac{2e\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}$$

3.71. $\int\frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}dx$

input
```
Int[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x]),x]
```

output
```
(-2*Sqrt[a]*Sqrt[b]*Sqrt[e]*ArcTan[(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sqrt[e]
)])/((a^2 + b^2)*d) - (2*e*(((a + b)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c +
 d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d
*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 - ((a - b)*(-1/2*Log[e + e*Cot[c + d*
x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*C
ot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/
2))/((a^2 + b^2)*d)
```

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4055
```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[1/(c^2 + d^2)   Int[Simp[a*c + b*d + (b*c -
 a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Simp[d*((b*c - a*
d)/(c^2 + d^2))   Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d
*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.71. $\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx$

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.71.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{ab\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{e\left(a^2+b^2\right)\sqrt{aeb}}+\frac{\frac{b\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right)}{}$
default	$-\frac{2e^2\left(-\frac{ab\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{e\left(a^2+b^2\right)\sqrt{aeb}}+\frac{\frac{b\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right)}{}$

input
```
int((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^2*(-a/e*b/(a^2+b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*
e*b)^(1/2))+1/e/(a^2+b^2)*(1/8*b/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)+(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(
1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1
/2)+1))+1/8*a/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1523 vs. $2(241) = 482$.

Time = 0.36 (sec) , antiderivative size = 3088, normalized size of antiderivative = 10.23

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*((a^2 + b^2)*d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2
*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2
*a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e*sqrt((e*cos(2*d*
x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*sqrt(-(a^
4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*
d^4)) - (a^2*b - b^3)*d*e)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 -
 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4
)) + 2*a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))) - (a^2 + b^2)*d*sqrt(-((a^4
+ 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*
d^2))*log(-(a^2 - b^2)*e*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) -
 ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 +
4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) - (a^2*b - b^3)*d*e)*sqrt(-
((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4)) + 2*a*b*e)/((a^4 + 2*a^2*b^2 +
 b^4)*d^2))) - (a^2 + b^2)*d*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(-(a^4
- 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^
4)) - 2*a*b*e)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-(a^2 - b^2)*e*sqrt((e*c
os(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*sq
rt(-(a^4 - 2*a^2*b^2 + b^4)*e^2/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b...
```

3.71.6 Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \int \frac{\sqrt{e\cot{(c+dx)}}}{a+b\cot{(c+dx)}}\,dx$$

input
```
integrate((e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c)),x)
```

output
```
Integral(sqrt(e*cot(c + d*x))/(a + b*cot(c + d*x)), x)
```

3.71.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.71.8 Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{b\cot(dx+c)+a}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*cot(d*x + c))/(b*cot(d*x + c) + a), x)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 4808, normalized size of antiderivative = 15.92

$$\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(1/2)/(a + b*cot(c + d*x)),x)
```

output
```
(atan(((((32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4 - (((
32*(13*a^2*b^4*d^2*e^12 + a^4*b^2*d^2*e^12))/d^5 + (((32*(e*cot(c + d*x))^
(1/2)*(20*a^3*b^4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4
+ (((32*(12*a*b^7*d^4*e^11 + 24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d
^5 - (32*(e*cot(c + d*x))^(1/2)*(-a*b*e)^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b
^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/(d^5*(a^2 + b^2)
))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b
*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2)*1i)/(d*(a^2 + b^2)) + (((32*(e*
cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4 + (((32*(13*a^2*b^4*d
^2*e^12 + a^4*b^2*d^2*e^12))/d^5 - (((32*(e*cot(c + d*x))^(1/2)*(20*a^3*b^
4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4 - (((32*(12*a*b^
7*d^4*e^11 + 24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 + (32*(e*cot(
c + d*x))^(1/2)*(-a*b*e)^(1/2)*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16
*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/(d^5*(a^2 + b^2)))*(-a*b*e)^(1/2
))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a^2 + b^2)))*(-a*b*e)^(1/2))/(d*(a
^2 + b^2)))*(-a*b*e)^(1/2)*1i)/(d*(a^2 + b^2)))/((64*a*b^3*e^13)/d^5 - (((
32*(e*cot(c + d*x))^(1/2)*(b^5*e^12 - 2*a^2*b^3*e^12))/d^4 - (((32*(13*a^2
*b^4*d^2*e^12 + a^4*b^2*d^2*e^12))/d^5 + (((32*(e*cot(c + d*x))^(1/2)*(20*
a^3*b^4*d^2*e^11 - 14*a*b^6*d^2*e^11 + 2*a^5*b^2*d^2*e^11))/d^4 + (((32*(1
2*a*b^7*d^4*e^11 + 24*a^3*b^5*d^4*e^11 + 12*a^5*b^3*d^4*e^11))/d^5 - (3...
```

3.71. $\int \frac{\sqrt{e\cot(c+dx)}}{a+b\cot(c+dx)}\,dx$

3.72 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx$

3.72.1 Optimal result

Integrand size = 25, antiderivative size = 302

$$\begin{aligned}\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx &= -\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)\,d\sqrt{e}} + \frac{(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}} \\ &\quad - \frac{(a-b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}} \\ &\quad + \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}} \\ &\quad - \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,d\sqrt{e}}\end{aligned}$$

output
```
1/2*(a-b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)/d*2^(1/
2)/e^(1/2)-1/2*(a-b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b
^2)/d*2^(1/2)/e^(1/2)+1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*c
ot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)/e^(1/2)-1/4*(a+b)*ln(e^(1/2)+cot(d*x
+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d*2^(1/2)/e^(1/2)-2*b^
(3/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/(a^2+b^2)/d/a^(
1/2)/e^(1/2)
```

3.72.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx = -\frac{\sqrt{\cot(c+dx)}\left(\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}(a^2+b^2)}-\frac{2b\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx)\right)}{3(a^2+b^2)}-\frac{a\left(2\sqrt{2}\arctan\left(1-\sqrt{2}\right.\right.}{}\right.}{d\sqrt{e\cot(c}}$$

input
```
Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])),x]
```

output
```
-((Sqrt[Cot[c + d*x]]*((2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt
[a]])/(Sqrt[a]*(a^2 + b^2)) - (2*b*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/
4, 1, 7/4, -Cot[c + d*x]^2])/(3*(a^2 + b^2)) - (a*(2*Sqrt[2]*ArcTan[1 - Sq
rt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*
Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(4*(a^2 + b^2))))/(d*
Sqrt[e*Cot[c + d*x]]))
```

3.72.3 Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.86, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4057, 3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 4057}$$

$$\frac{b^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{a-b\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}$$

$\downarrow$ 3042

$$\frac{b^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{\int \frac{a+b\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}$$

$\downarrow$ 4017

$$\frac{2\int -\frac{ae-be\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}+\frac{b^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 25

$$\frac{b^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int \frac{ae-be\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}$$

$\downarrow$ 1482

$$\frac{2\left(-\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d(a^2+b^2)}+\frac{b^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 1476

$$\frac{2\left(-\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int\right.\right.}{d(a^2+b^2)}$$
$$\frac{b^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 1082

$$\frac{2\left(-\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}}{}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}$$
$$\frac{b^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 217

3.72. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx$

$$\frac{2\left(-\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d(a^2+b^2)}$$

$$\frac{b^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 1479

$$\frac{2\left(-\frac{1}{2}(a+b)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-\right.}{d(a^2+b^2)}$$

$$\frac{b^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 25

$$\frac{2\left(-\frac{1}{2}(a+b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\right.\right.}{d(a^2+b^2)}$$

$$\frac{b^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 27

$$\frac{2\left(-\frac{1}{2}(a+b)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\right.\right.}{d(a^2+b^2)}$$

$$\frac{b^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 1103

3.72. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx$

$$\frac{b^2 \int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+$$
$$\frac{2\left(-\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right.\right.}{d\left(a^2+b^2\right)}$$

$\downarrow$ 4117

$$\frac{b^2 \int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}+$$
$$\frac{2\left(-\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right.\right.}{d\left(a^2+b^2\right)}$$

$\downarrow$ 73

$$\frac{2\left(-\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right.\right.}{d\left(a^2+b^2\right)}$$
$$\frac{2b^2 \int \frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{de\left(a^2+b^2\right)}$$

$\downarrow$ 218

$$\frac{2\left(-\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}\right.\right.}{d\left(a^2+b^2\right)}$$
$$\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}d\sqrt{e}\left(a^2+b^2\right)}$$

input
```
Int[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])),x]
```

3.72. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx$

output
```
(2*b^(3/2)*ArcTan[(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*(a^2
 + b^2)*d*Sqrt[e]) + (2*(-1/2*((a - b)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c
 + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c +
 d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e]))) - ((a + b)*(-1/2*Log[e + e*Cot[c + d*
x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*C
ot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/
2))/((a^2 + b^2)*d)
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4057
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[1/(c^2 + d^2)   Int[(a + b*Tan[e + f*x])^m
*(c - d*Tan[e + f*x]), x], x] + Simp[d^2/(c^2 + d^2)   Int[(a + b*Tan[e + f
*x])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] &&  !IntegerQ[m]
```

3.72. $\int \frac{1}{\sqrt{e \cot(c+dx)}(a+b\cot(c+dx))} \, dx$

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.72.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{2e^2\left(\frac{b^2\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{e^2(a^2+b^2)\sqrt{aeb}}+\frac{\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{2e^2\left(\frac{b^2\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{e^2(a^2+b^2)\sqrt{aeb}}+\frac{\frac{a(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+c)}}{(e^2)^{\frac{1}{4}}}\cdots\right.\right.}{8e}\cdots}{\cdots}\cdots\right.}{\cdots}$

input
```
int(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^2*(b^2/e^2/(a^2+b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a
*e*b)^(1/2))+1/e^2/(a^2+b^2)*(1/8*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c
)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)+1))-1/8*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c)
)^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^
(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1558 vs. $2(241) = 482$.

Time = 0.38 (sec) , antiderivative size = 3160, normalized size of antiderivative = 10.46

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*((a^2 + b^2)*d*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^2
*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) + 2
*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x
+ 2*c) + e)/sin(2*d*x + 2*c)) + ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^2*sqrt(-(
a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^
4*e^2)) + (a^3 - a*b^2)*d*e)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^
4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*
e^2)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e))) - (a^2 + b^2)*d*sqrt(((a^
4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4
)*d^2*e))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c))
 - ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) + (a^3 - a*b^2)*d*e)*
sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 +
4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^2)) + 2*a*b)/((a^4 + 2*a^2*
b^2 + b^4)*d^2*e))) - (a^2 + b^2)*d*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e*s
qrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8)*d^4*e^2)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e))*log(-(a^2 - b^2)*s
qrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) + ((a^4*b + 2*a^2*b^3 + b^5
)*d^3*e^2*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +...
```

3.72.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx = \int \frac{1}{\sqrt{e\cot{(c+dx)}}\,(a+b\cot{(c+dx)})}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c)),x)
```

output
```
Integral(1/(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))), x)
```

3.72.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.72.8 Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx = \int \frac{1}{(b\cot(dx+c)+a)\sqrt{e\cot(dx+c)}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*cot(d*x + c) + a)*sqrt(e*cot(d*x + c))), x)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 4871, normalized size of antiderivative = 16.13

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx = \text{Too large to display}$$

input
```
int(1/((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))),x)
```

output
```
atan(((((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - ((((1/(b^2*d^2*e*1i - a^2
*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*((32*(16*b^8*d^2*e^10 + 28*a^2*b^6*d^2*e^1
0 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 - (16*(e*cot(c + d*x))^(
1/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*(16*b^9*d^4*e^1
0 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3*d^4*e^10))/d^4)
)/2 - (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a*b^6*d^2*e^9 + 2
*a^5*b^2*d^2*e^9))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1
/2))/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2))/2 + (96*b^5
*e^8*(e*cot(c + d*x))^(1/2))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*
d^2*e))^(1/2)*1i)/2 - (((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))/d^3 - ((((1/(b^
2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*((32*(16*b^8*d^2*e^10 + 28
*a^2*b^6*d^2*e^10 + 8*a^4*b^4*d^2*e^10 - 4*a^6*b^2*d^2*e^10))/d^3 + (16*(e
*cot(c + d*x))^(1/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)
*(16*b^9*d^4*e^10 + 16*a^2*b^7*d^4*e^10 - 16*a^4*b^5*d^4*e^10 - 16*a^6*b^3
*d^4*e^10))/d^4))/2 + (32*(e*cot(c + d*x))^(1/2)*(4*a^3*b^4*d^2*e^9 - 30*a
*b^6*d^2*e^9 + 2*a^5*b^2*d^2*e^9))/d^4)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i +
2*a*b*d^2*e))^(1/2))/2)*(1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1
/2))/2 - (96*b^5*e^8*(e*cot(c + d*x))^(1/2))/d^4)*(1/(b^2*d^2*e*1i - a^2*d
^2*e*1i + 2*a*b*d^2*e))^(1/2)*1i)/2)/((((((32*(5*a*b^5*e^9 + a^3*b^3*e^9))
/d^3 - ((((1/(b^2*d^2*e*1i - a^2*d^2*e*1i + 2*a*b*d^2*e))^(1/2)*((32*(1...
```

3.72. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}\,dx$

3.73 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx$

3.73.1 Optimal result

Integrand size = 25, antiderivative size = 325

$$\begin{aligned}\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx &= \frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}(a^2+b^2)\,de^{3/2}} \\ &- \frac{(a+b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,de^{3/2}} + \frac{(a+b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)\,de^{3/2}} \\ &+ \frac{2}{ade\sqrt{e\cot(c+dx)}} + \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,de^{3/2}} \\ &- \frac{(a-b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)\,de^{3/2}}\end{aligned}$$

output
```
2*b^(5/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/a^(3/2)/(a^
2+b^2)/d/e^(3/2)-1/2*(a+b)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/
(a^2+b^2)/d/e^(3/2)*2^(1/2)+1/2*(a+b)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2
)/e^(1/2))/(a^2+b^2)/d/e^(3/2)*2^(1/2)+1/4*(a-b)*ln(e^(1/2)+cot(d*x+c)*e^(
1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d/e^(3/2)*2^(1/2)-1/4*(a-b)*l
n(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)/d/e^(
3/2)*2^(1/2)+2/a/d/e/(e*cot(d*x+c))^(1/2)
```

3.73.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \frac{8b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{b\cot(c+dx)}{a}\right)+a\Big(8a\,\text{Hyperg}}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])),x]
```

output
```
(8*b^2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*Cot[c + d*x])/a)] + a*(8*a*Hyp
ergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*b*Sqrt[Cot[c + d*x
]]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[
Cot[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] + Log[
1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(4*a*(a^2 + b^2)*d*e*Sqr
t[e*Cot[c + d*x]])
```

3.73.3 Rubi [A] (warning: unable to verify)

Time = 1.30 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.91, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4052, 27, 3042, 4136, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 4052$$

$$\frac{2\int -\frac{b\cot^2(c+dx)e^2+be^2+a\cot(c+dx)e^2}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae^3}+\frac{2}{ade\sqrt{e\cot(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\int \frac{b\cot^2(c+dx)e^2+be^2+a\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\int \frac{b\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+be^2-a\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}$$

$$\Big\downarrow \textbf{4136}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{\int \frac{abe^2+a^2\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}+\frac{b^3e^2\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{ae^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{\int \frac{abe^2-a^2e^2\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}+\frac{b^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{ae^3}$$

$$\Big\downarrow \textbf{4017}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{2\int -\frac{ae^2(be+a\cot(c+dx)e)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}+\frac{b^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{ae^3}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int \frac{ae^2(be+a\cot(c+dx)e)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{ae^3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2ae^2\int \frac{be+a\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{ae^3}$$

$$\Big\downarrow \textbf{1482}$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2ae^2\left(\frac{1}{2}(a+b)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a-b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d(a^2+b^2)}}{ae^3}$$

3.73. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx$

$$\Big\downarrow \; 1476$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2ae^2\left(\frac{1}{2}(a+b)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}}{ae^3}$$

$$\Big\downarrow \; 1082$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2ae^2\left(\frac{1}{2}(a+b)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}}{ae^3}$$

$$\Big\downarrow \; 217$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2ae^2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\int \frac{e-e\,c}{\cot^2(c+}\right.}{d(a^2+b^2)}}{ae^3}$$

$$\Big\downarrow \; 1479$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2ae^2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(-\frac{\int -\frac{}{c}}{}\right.\right.}{d(}}{ae^3}$$

$$\Big\downarrow \; 25$$

$$\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^2\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2ae^2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\int}{\cot(c}\right.\right.}{d(a^2}}{ae^3}$$

$$\Big\downarrow \; 27$$

3.73. $\displaystyle\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx$

$$\frac{\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^2\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2ae^2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\int\frac{}{\cot(c}}{}\right.\right.}{d(a^2}}{ae^3}$$

$\downarrow$ 1103

$$\frac{\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^2\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2ae^2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(e\cot\right.}{}\right.\right.}{d(a^2+b^2)}}{ae^3}$$

$\downarrow$ 4117

$$\frac{\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^2\int\frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}-\frac{2ae^2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(e\right.}{}\right.\right.}{d(a^2+b^2)}}{ae^3}$$

$\downarrow$ 73

$$\frac{\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{-\frac{2b^3e\int\frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}-\frac{2ae^2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(e\cot(c+dx)+\right.}{}\right.\right.}{d(a^2+b^2)}}{ae^3}$$

$\downarrow$ 218

$$\frac{\frac{2}{ade\sqrt{e\cot(c+dx)}}-\frac{\frac{2b^{5/2}e^{3/2}\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}d(a^2+b^2)}-\frac{2ae^2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{}\right.}{2\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}}{ae^3}$$

input
```
Int[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])),x]
```

3.73. $\int\frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx$

output
```
2/(a*d*e*Sqrt[e*Cot[c + d*x]]) - ((2*b^(5/2)*e^(3/2)*ArcTan[(Sqrt[b]*Cot[c
 + d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*(a^2 + b^2)*d) - (2*a*e^2*(((a + b)*
(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) +
 ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2
- ((a - b)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d
*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*
Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/((a^2 + b^2)*d))/(a*e^3)
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

3.73. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx$

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.73.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{2e^2\left(-\frac{b^3\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{a\,e^3(a^2+b^2)\sqrt{aeb}}+\frac{-\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$
default	$-\frac{2e^2\left(-\frac{b^3\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{a\,e^3(a^2+b^2)\sqrt{aeb}}+\frac{-\frac{b(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\,\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx}{(e^2)^{\frac{1}{4}}}\right.\right.}{8e}}{}\right.}{}$

input
```
int(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^2*(-1/a/e^3*b^3/(a^2+b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)
*b/(a*e*b)^(1/2))+1/(a^2+b^2)/e^3*(-1/8*b/e*(e^2)^(1/4)*2^(1/2)*(ln((e*cot
(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c
)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)+1))-1/8*a/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(
e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/a
/e^3/(e*cot(d*x+c))^(1/2))
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(262) = 524$.

Time = 0.41 (sec) , antiderivative size = 3637, normalized size of antiderivative = 11.19

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(((a^3 + a*b^2)*d*e^2*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*e^2)*sqrt(-(
(a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) + 2*a*b)/((a^4 + 2*a^2*b^2
+ b^4)*d^2*e^3))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x
+ 2*c)) + ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)
/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) - (a^2*b - b^3
)*d*e^2)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b
^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) + 2*a*b)/((
a^4 + 2*a^2*b^2 + b^4)*d^2*e^3))) - ((a^3 + a*b^2)*d*e^2*cos(2*d*x + 2*c)
+ (a^3 + a*b^2)*d*e^2)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3*sqrt(-(a^4 -
 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^6
)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3))*log(-(a^2 - b^2)*sqrt((e*co
s(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^5 + 2*a^3*b^2 + a*b^4)*d^3*e^5
*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)*d^4*e^6)) - (a^2*b - b^3)*d*e^2)*sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*
e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*d^4*e^6)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^3))) - ((a^3 +
a*b^2)*d*e^2*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*e^2)*sqrt(((a^4 + 2*a^2*b^
2 + b^4)*d^2*e^3*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b
^4 + 4*a^2*b^6 + b^8)*d^4*e^6)) - 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e...
```

3.73.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \int \frac{1}{\left(e\cot\left(c+dx\right)\right)^{\frac{3}{2}}\left(a+b\cot\left(c+dx\right)\right)}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c)),x)
```

output
```
Integral(1/((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))), x)
```

3.73.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.73.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \int \frac{1}{(b\cot(dx+c)+a)(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*cot(d*x + c) + a)*(e*cot(d*x + c))^(3/2)), x)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 13.93 (sec) , antiderivative size = 4899, normalized size of antiderivative = 15.07

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx = \text{Too large to display}$$

input
```
int(1/((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))),x)
```

output
```
(log((e*cot(c + d*x))^(1/2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) -
((-1/(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2)*(((((-1/(b^2
*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2)*(((e*cot(c + d*x))^(1
/2)*(-1/(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2)*(512*a^9*
b^9*d^9*e^19 + 512*a^11*b^7*d^9*e^19 - 512*a^13*b^5*d^9*e^19 - 512*a^15*b^
3*d^9*e^19))/2 - 512*a^8*b^9*d^8*e^18 - 640*a^10*b^7*d^8*e^18 + 256*a^12*b
^5*d^8*e^18 + 384*a^14*b^3*d^8*e^18))/2 - (e*cot(c + d*x))^(1/2)*(512*a^8*
b^8*d^7*e^16 - 448*a^10*b^6*d^7*e^16 + 128*a^12*b^4*d^7*e^16 + 64*a^14*b^2
*d^7*e^16))*(-1/(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2))/
2 - 128*a^7*b^8*d^6*e^15 + 32*a^11*b^4*d^6*e^15 + 32*a^13*b^2*d^6*e^15))/2
)*(-1/(b^2*d^2*e^3*1i - a^2*d^2*e^3*1i + 2*a*b*d^2*e^3))^(1/2))/2 - atan((
(-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((e*cot(c + d
*x))^(1/2)*(64*a^7*b^7*d^5*e^13 - 32*a^9*b^5*d^5*e^13) - (-1i/(4*(b^2*d^2*
e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^
2*e^3 + a*b*d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*
d^2*e^3*2i)))^(1/2)*((-1i/(4*(b^2*d^2*e^3 - a^2*d^2*e^3 + a*b*d^2*e^3*2i))
)^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^9*b^9*d^9*e^19 + 512*a^11*b^7*d^9*e^
19 - 512*a^13*b^5*d^9*e^19 - 512*a^15*b^3*d^9*e^19) - 512*a^8*b^9*d^8*e^18
 - 640*a^10*b^7*d^8*e^18 + 256*a^12*b^5*d^8*e^18 + 384*a^14*b^3*d^8*e^18)
- (e*cot(c + d*x))^(1/2)*(512*a^8*b^8*d^7*e^16 - 448*a^10*b^6*d^7*e^16 ...
```

3.73. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}\,dx$

3.74 $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx$

3.74.1 Optimal result

Integrand size = 25, antiderivative size = 351

$$\begin{aligned}\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx &= -\frac{2b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}(a^2+b^2)\,de^{5/2}} \\ &- \frac{(a-b)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)\,de^{5/2}} + \frac{(a-b)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)\,de^{5/2}} \\ &+ \frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{2b}{a^2de^2\sqrt{e\cot(c+dx)}} \\ &- \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)\,de^{5/2}} \\ &+ \frac{(a+b)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)\,de^{5/2}}\end{aligned}$$

output
```
-2*b^(7/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/a^(5/2)/(a
^2+b^2)/d/e^(5/2)+2/3/a/d/e/(e*cot(d*x+c))^(3/2)-1/2*(a-b)*arctan(1-2^(1/2
)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)/d/e^(5/2)*2^(1/2)+1/2*(a-b)*arct
an(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)/d/e^(5/2)*2^(1/2)-1/4
*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^
2)/d/e^(5/2)*2^(1/2)+1/4*(a+b)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*co
t(d*x+c))^(1/2))/(a^2+b^2)/d/e^(5/2)*2^(1/2)-2*b/a^2/d/e^2/(e*cot(d*x+c))^
(1/2)
```

3.74.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.31

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \frac{2\left(b^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{b\cot(c+dx)}{a}\right)+a\left(a\,\text{Hype}\right.\right.}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(5/2)*(a + b*Cot[c + d*x])),x]
```

output
```
(2*(b^2*Hypergeometric2F1[-3/2, 1, -1/2, -((b*Cot[c + d*x])/a)] + a*(a*Hyp
ergeometric2F1[-3/4, 1, 1/4, -Cot[c + d*x]^2] - 3*b*Cot[c + d*x]*Hypergeom
etric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2])))/(3*a*(a^2 + b^2)*d*e*(e*Cot[c +
 d*x])^(3/2))
```

3.74.3 Rubi [A] (warning: unable to verify)

Time = 1.69 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.94, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4052, 27, 3042, 4132, 27, 3042, 4137, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4052}$$

$$\frac{2\int -\frac{3\left(b\cot^2(c+dx)e^2+be^2+a\cot(c+dx)e^2\right)}{2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}dx}{3ae^3}+\frac{2}{3ade(e\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\int \frac{b\cot^2(c+dx)e^2+be^2+a\cot(c+dx)e^2}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}dx}{ae^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\int \frac{b\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+be^2-a\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}$$

$$\Big\downarrow \text{4132}$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{2\int \frac{\left(a^2-b^2\right)e^4-b^2e^4\cot^2(c+dx)}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae^3}+\frac{2be}{ad\sqrt{e\cot(c+dx)}}}{ae^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{\int \frac{\left(a^2-b^2\right)e^4-b^2e^4\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae^3}+\frac{2be}{ad\sqrt{e\cot(c+dx)}}}{ae^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{\int \frac{\left(a^2-b^2\right)e^4-b^2e^4\tan\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}+\frac{2be}{ad\sqrt{e\cot(c+dx)}}}{ae^3}$$

$$\Big\downarrow \text{4137}$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{\frac{\int \frac{a^3e^4-a^2be^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}-\frac{b^4e^4\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{ae^3}+\frac{2be}{ad\sqrt{e\cot(c+dx)}}}{ae^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{\frac{\int \frac{a^3e^4+a^2b\tan\left(c+dx+\frac{\pi}{2}\right)e^4}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}-\frac{b^4e^4\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{ae^3}+\frac{2be}{ad\sqrt{e\cot(c+dx)}}}{ae^3}$$

$$\Big\downarrow \text{4017}$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{\frac{2\int -\frac{a^2e^4(ae-be\cot(c+dx))}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{b^4e^4\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{ae^3}+\frac{2be}{ad\sqrt{e\cot(c+dx)}}}{ae^3}$$

3.74. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}dx$

$\big\downarrow$ 25

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{-\frac{2\int \frac{a^2e^4(ae-be\cot(c+dx))}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{b^4e^4\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{ae^3}+\frac{2be}{ad\sqrt{e\cot(c+dx)}}}{ae^3}$$

$\big\downarrow$ 27

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{-\frac{2a^2e^4\int \frac{ae-be\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{b^4e^4\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{ae^3}+\frac{2be}{ad\sqrt{e\cot(c+dx)}}}{ae^3}$$

$\big\downarrow$ 1482

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{-\frac{2a^2e^4\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d\left(a^2+b^2\right)}-\frac{b^4e^4\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{ae^3}+\frac{\ }{ad\ldots}}{ae^3}$$

$\big\downarrow$ 1476

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{-\frac{2a^2e^4\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\ldots\right.\right.}{d\left(a^2+b^2\right)}\ldots}{ae^3}\ldots}{ae^3}$$

$\big\downarrow$ 1082

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{\frac{-\frac{2a^2e^4\left(\frac{1}{2}(a+b)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d\left(a^2+b^2\right)}-\frac{b\ldots}{\ldots}}{ae^3}\ldots}{ae^3}$$

$\big\downarrow$ 217

3.74. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{-\frac{2a^2e^4\left(\frac{1}{2}(a+b)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d(a^2+b^2)}-\frac{b^4e^4\int\frac{\tan(c+dx\ldots}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}(\ldots}}{a^2+b^{\ldots}}}{ae^3}}{ae^3}$$

$\downarrow$ 1479

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{-\frac{2a^2e^4\left(\frac{1}{2}(a+b)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\ldots}}{\ldots}\right.}{\sqrt{2}\ldots}\ldots\right.\right.}{d(a^2+b^2)}\ldots}{ae^3}}{ae^3}$$

$\downarrow$ 25

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{-\frac{2a^2e^4\left(\frac{1}{2}(a+b)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c\ldots}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\ldots\right.\right.}{d(a^2+b^2)}\ldots}{ae^3}}{ae^3}$$

$\downarrow$ 27

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{-\frac{2a^2e^4\left(\frac{1}{2}(a+b)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c\ldots}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\ldots\right.\right.}{d(a^2+b^2)}\ldots}{ae^3}}{ae^3}$$

$\downarrow$ 1103

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}}-\frac{-\frac{b^4e^4\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a^2e^4\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c\ldots\right.}{\ldots}\ldots\right.\right.}{d(a^2+b^2)}}{ae^3}}{ae^3}$$

$\downarrow$ 4117

3.74. $\int\frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{-\frac{b^4e^4\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)} - \frac{2a^2e^4\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\frac{\log(e\cot(c\cdots}{}\right.\right.}{d(a^2+b^2)}}{ae^3}}{ae^3}$$

$$\Big\downarrow 73$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{\frac{2b^4e^3\int \frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)} - \frac{2a^2e^4\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}}{ae^3}}{ae^3}$$

$$\Big\downarrow 218$$

$$\frac{2}{3ade(e\cot(c+dx))^{3/2}} - \frac{-\frac{2a^2e^4\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}} - \frac{\log\left(e\cot(c+dx)-\sqrt{2}\sqrt{\cdots}\right.}{2\sqrt{2}\cdots}\right.\right.}{d(a^2+b^2)}}{ae^3}}{ae^3}$$

input
```
Int[1/((e*Cot[c + d*x])^(5/2)*(a + b*Cot[c + d*x])),x]
```

output
```
2/(3*a*d*e*(e*Cot[c + d*x])^(3/2)) - ((2*b*e)/(a*d*Sqrt[e*Cot[c + d*x]]) +
 ((-2*b^(7/2)*e^(7/2)*ArcTan[(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sqrt[e])])/(S
qrt[a]*(a^2 + b^2)*d) - (2*a^2*e^4*(((a - b)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e
*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*C
ot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 + ((a + b)*(-1/2*Log[e + e*Co
t[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log
[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqr
t[e])))/2))/((a^2 + b^2)*d))/(a*e^3))/(a*e^3)
```

3.74. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx$

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.74. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.74. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx$

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4137
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Sim
p[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*Simp[a*(A - C) - (A*b - b*C)*T
an[e + f*x], x], x], x] + Simp[(A*b^2 + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan
[e + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{
a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.74.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{2e^2\left(\frac{b^4\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{a^2e^4\left(a^2+b^2\right)\sqrt{aeb}}+\frac{-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}}{\cdots}\right.}{\cdots}$
default	$-\frac{2e^2\left(\frac{b^4\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{a^2e^4\left(a^2+b^2\right)\sqrt{aeb}}+\frac{-\frac{a\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx+c)}\sqrt{2}+\sqrt{e^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(dx+}}{\left(e^2\right)^{\frac{1}{4}}}\right.\right.}{8e}}{\cdots}\right.}{\cdots}$

input
```
int(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^2*(1/a^2/e^4*b^4/(a^2+b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2
)*b/(a*e*b)^(1/2))+1/(a^2+b^2)/e^4*(-1/8*a/e*(e^2)^(1/4)*2^(1/2)*(ln((e*co
t(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+
c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)
/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(
d*x+c))^(1/2)+1))+1/8*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot
(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/
3/a/e^3/(e*cot(d*x+c))^(3/2)+1/a^2/e^4*b/(e*cot(d*x+c))^(1/2))
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1849 vs. 2(284) = 568.

Time = 0.47 (sec) , antiderivative size = 3742, normalized size of antiderivative = 10.66

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/6*(3*((a^4 + a^2*b^2)*d*e^3*cos(2*d*x + 2*c) + (a^4 + a^2*b^2)*d*e^3)*
sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) + 2*a*b)/((a^4 + 2*a
^2*b^2 + b^4)*d^2*e^5))*log(-(a^2 - b^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin
(2*d*x + 2*c)) + ((a^4*b + 2*a^2*b^3 + b^5)*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2
 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) + (a^3
 - a*b^2)*d*e^3)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*
b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) + 2
*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5))) - 3*((a^4 + a^2*b^2)*d*e^3*cos(2
*d*x + 2*c) + (a^4 + a^2*b^2)*d*e^3)*sqrt(((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5
*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
 b^8)*d^4*e^10)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5))*log(-(a^2 - b
^2)*sqrt((e*cos(2*d*x + 2*c) + e)/sin(2*d*x + 2*c)) - ((a^4*b + 2*a^2*b^3
+ b^5)*d^3*e^8*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8)*d^4*e^10)) + (a^3 - a*b^2)*d*e^3)*sqrt(((a^4 + 2*a^2*b
^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) + 2*a*b)/((a^4 + 2*a^2*b^2 + b^4)*d^2*e^
5))) - 3*((a^4 + a^2*b^2)*d*e^3*cos(2*d*x + 2*c) + (a^4 + a^2*b^2)*d*e^3)*
sqrt(-((a^4 + 2*a^2*b^2 + b^4)*d^2*e^5*sqrt(-(a^4 - 2*a^2*b^2 + b^4)/((a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d^4*e^10)) - 2*a*b)/((a^4 +...
```

3.74.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \int \frac{1}{(e\cot(c+dx))^{\frac{5}{2}}\,(a+b\cot(c+dx))}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))**(5/2)/(a+b*cot(d*x+c)),x)
```

output
```
Integral(1/((e*cot(c + d*x))**(5/2)*(a + b*cot(c + d*x))), x)
```

3.74.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.74.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \int \frac{1}{(b\cot(dx+c)+a)\,(e\cot(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*cot(d*x + c) + a)*(e*cot(d*x + c))^(5/2)), x)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 14.96 (sec) , antiderivative size = 6042, normalized size of antiderivative = 17.21

$$\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx = \text{Too large to display}$$

input
```
int(1/((e*cot(c + d*x))^(5/2)*(a + b*cot(c + d*x))),x)
```

output
```
(2/(3*a*e) - (2*b*cot(c + d*x))/(a^2*e))/(d*(e*cot(c + d*x))^(3/2)) - atan
(((((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 + 32*a^18*b^5*d^5*e^18))/
2 + ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((((1/(b
^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(((e*cot(c + d*x))^
(1/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(1/2)*(512*a^1
8*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*b^5*d^9*e^28 - 512*a^24*
b^3*d^9*e^28))/4 - 256*a^16*b^10*d^8*e^26 - 256*a^18*b^8*d^8*e^26 + 192*a^
20*b^6*d^8*e^26 + 128*a^22*b^4*d^8*e^26 - 64*a^24*b^2*d^8*e^26))/2 - ((e*c
ot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 448*a^19*b^6*d^7*e^23 - 128*a
^21*b^4*d^7*e^23 - 64*a^23*b^2*d^7*e^23))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*
e^5*1i + 2*a*b*d^2*e^5))^(1/2))/2 + 192*a^15*b^9*d^6*e^21 - 16*a^19*b^5*d^
6*e^21 - 16*a^21*b^3*d^6*e^21))/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2
*a*b*d^2*e^5))^(1/2)*1i + (((e*cot(c + d*x))^(1/2)*(64*a^14*b^9*d^5*e^18 +
 32*a^18*b^5*d^5*e^18))/2 + ((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d
^2*e^5))^(1/2)*(((((1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*d^2*e^5))^(
1/2)*(((e*cot(c + d*x))^(1/2)*(1/(b^2*d^2*e^5*1i - a^2*d^2*e^5*1i + 2*a*b*
d^2*e^5))^(1/2)*(512*a^18*b^9*d^9*e^28 + 512*a^20*b^7*d^9*e^28 - 512*a^22*
b^5*d^9*e^28 - 512*a^24*b^3*d^9*e^28))/4 + 256*a^16*b^10*d^8*e^26 + 256*a^
18*b^8*d^8*e^26 - 192*a^20*b^6*d^8*e^26 - 128*a^22*b^4*d^8*e^26 + 64*a^24*
b^2*d^8*e^26))/2 - ((e*cot(c + d*x))^(1/2)*(512*a^15*b^10*d^7*e^23 + 44...
```

3.74. $\int \frac{1}{(e\cot(c+dx))^{5/2}(a+b\cot(c+dx))}\,dx$

3.75 $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx$

3.75.1 Optimal result

Integrand size = 25, antiderivative size = 437

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \frac{a^{5/2}(3a^2+7b^2)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{5/2}\left(a^2+b^2\right)^2 d} + \frac{(a^2-2ab-b^2)\,e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d} - \frac{(a^2-2ab-b^2)\,e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d} - \frac{(3a^2+2b^2)\,e^3\sqrt{e\cot(c+dx)}}{b^2\left(a^2+b^2\right)d} + \frac{a^2e^2(e\cot(c+dx))^{3/2}}{b\left(a^2+b^2\right)d(a+b\cot(c+dx))} + \frac{(a^2+2ab-b^2)\,e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d} - \frac{(a^2+2ab-b^2)\,e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d}$$

output
```
a^(5/2)*(3*a^2+7*b^2)*e^(7/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/
e^(1/2))/b^(5/2)/(a^2+b^2)^2/d+a^2*e^2*(e*cot(d*x+c))^(3/2)/b/(a^2+b^2)/d/
(a+b*cot(d*x+c))+1/2*(a^2-2*a*b-b^2)*e^(7/2)*arctan(1-2^(1/2)*(e*cot(d*x+c
))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/2*(a^2-2*a*b-b^2)*e^(7/2)*arctan
(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)+1/4*(a^2+2*
a*b-b^2)*e^(7/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2
))/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*e^(7/2)*ln(e^(1/2)+cot(d*x+c)
*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)-(3*a^2+2*b^2)
*e^3*(e*cot(d*x+c))^(1/2)/b^2/(a^2+b^2)/d
```

3.75.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.19 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.05

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx =$$

$$-\frac{(e\cot(c+dx))^{7/2}\left(\frac{4a^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}(a^2+b^2)^2}-\frac{4a^4\sqrt{\cot(c+dx)}}{b^2(a^2+b^2)^2}+\frac{4a^3\cot^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)^2}-\frac{4a^2\cot^{\frac{5}{2}}(c+dx)}{5(a^2+b^2)^2}+\frac{4ab\cot^{\frac{7}{2}}(c+dx)}{7(a^2+b^2)^2}+\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(7/2)/(a + b*Cot[c + d*x])^2,x]
```

output
```
-(((e*Cot[c + d*x])^(7/2)*((4*a^(9/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/
Sqrt[a]])/(b^(5/2)*(a^2 + b^2)^2) - (4*a^4*Sqrt[Cot[c + d*x]])/(b^2*(a^2 +
 b^2)^2) + (4*a^3*Cot[c + d*x]^(3/2))/(3*b*(a^2 + b^2)^2) - (4*a^2*Cot[c +
 d*x]^(5/2))/(5*(a^2 + b^2)^2) + (4*a*b*Cot[c + d*x]^(7/2))/(7*(a^2 + b^2)
^2) + (4*a*b*(7*Cot[c + d*x]^(3/2) - 3*Cot[c + d*x]^(7/2) - 7*Cot[c + d*x]
^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(21*(a^2 + b^2)^2
) + (2*b^2*Cot[c + d*x]^(9/2)*Hypergeometric2F1[2, 9/2, 11/2, -((b*Cot[c +
 d*x])/a)])/(9*a^2*(a^2 + b^2)) - ((a - b)*(a + b)*(10*Sqrt[2]*ArcTan[1 -
Sqrt[2]*Sqrt[Cot[c + d*x]]] - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d
*x]]] + 40*Sqrt[Cot[c + d*x]] - 8*Cot[c + d*x]^(5/2) + 5*Sqrt[2]*Log[1 - S
qrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt
[Cot[c + d*x]] + Cot[c + d*x]]))/(20*(a^2 + b^2)^2)))/(d*Cot[c + d*x]^(7/2
)))
```

3.75.3 Rubi [A] (warning: unable to verify)

Time = 2.03 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.91, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.960, Rules used = {3042, 4048, 27, 3042, 4130, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4048}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\left(a^2+b^2\right)(a+b\cot(c+dx))}-\frac{\int -\frac{\sqrt{e\cot(c+dx)}\left(3a^2e^3+\left(3a^2+2b^2\right)\cot^2(c+dx)e^3-2ab\cot(c+dx)e^3\right)}{2(a+b\cot(c+dx))}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{e\cot(c+dx)}\left(3a^2e^3+\left(3a^2+2b^2\right)\cot^2(c+dx)e^3-2ab\cot(c+dx)e^3\right)}{a+b\cot(c+dx)}dx}{2b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(3a^2e^3+\left(3a^2+2b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3+2ab\tan\left(c+dx+\frac{\pi}{2}\right)e^3\right)}{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{4130}$$

$$\frac{-\frac{2\int \frac{a\left(3a^2+4b^2\right)\cot^2(c+dx)e^4+a\left(3a^2+2b^2\right)e^4+2b^3\cot(c+dx)e^4}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{b}-\frac{2e^3\left(3a^2+2b^2\right)\sqrt{e\cot(c+dx)}}{bd}}{2b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{a(3a^2+4b^2)\cot^2(c+dx)e^4+a(3a^2+2b^2)e^4+2b^3\cot(c+dx)e^4}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{b}-\frac{2e^3(3a^2+2b^2)\sqrt{e\cot(c+dx)}}{bd}}{2b(a^2+b^2)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{a(3a^2+4b^2)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^4+a(3a^2+2b^2)e^4-2b^3\tan\left(c+dx+\frac{\pi}{2}\right)e^4}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2e^3(3a^2+2b^2)\sqrt{e\cot(c+dx)}}{bd}}{2b(a^2+b^2)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{4136}$$

$$\frac{-\frac{\frac{\int -\frac{2\left(b^2(a^2-b^2)e^4-2ab^3e^4\cot(c+dx)\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}+\frac{a^3e^4(3a^2+7b^2)\int\frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{b}-\frac{2e^3(3a^2+2b^2)\sqrt{e\cot(c+dx)}}{bd}}{2b(a^2+b^2)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\frac{a^3e^4(3a^2+7b^2)\int\frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}-\frac{2\int\frac{b^2(a^2-b^2)e^4-2ab^3e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{b}-\frac{2e^3(3a^2+2b^2)\sqrt{e\cot(c+dx)}}{bd}}{2b(a^2+b^2)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\frac{a^3e^4(3a^2+7b^2)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int\frac{b^2(a^2-b^2)e^4+2ab^3\tan\left(c+dx+\frac{\pi}{2}\right)e^4}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{b}-\frac{2e^3(3a^2+2b^2)\sqrt{e\cot(c+dx)}}{bd}}{2b(a^2+b^2)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{4017}$$

3.75. $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx$

$$-\frac{\frac{\frac{a^3e^4(3a^2+7b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4\int -\frac{b^2e^4\left(\left(a^2-b^2\right)e-2abe\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}}{b}-\frac{2e^3(3a^2+2b^2)\sqrt{e\cot(c+dx)}}{bd}}{2b(a^2+b^2)}$$
$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$\big\downarrow$ 25

$$-\frac{\frac{\frac{4\int \frac{b^2e^4\left(\left(a^2-b^2\right)e-2abe\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a^3e^4(3a^2+7b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{b}-\frac{2e^3(3a^2+2b^2)\sqrt{e\cot(c+dx)}}{bd}}{2b(a^2+b^2)}+$$
$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$\big\downarrow$ 27

$$-\frac{\frac{\frac{4b^2e^4\int \frac{\left(a^2-b^2\right)e-2abe\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a^3e^4(3a^2+7b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{b}-\frac{2e^3(3a^2+2b^2)\sqrt{e\cot(c+dx)}}{bd}}{2b(a^2+b^2)}+$$
$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$\big\downarrow$ 1482

$$-\frac{\frac{\frac{4b^2e^4\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d\left(a^2+b^2\right)}+\frac{a^3e^4(3a^2+7b^2)\int \frac{\tan\left(c+dx+\right.}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-\right.}}{a^2+b^2}}{b}}{2b(a^2+b^2)}$$
$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$\big\downarrow$ 1476

$$-\frac{\frac{4b^2e^4\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e+\sqrt{2}\right.\right.}{d\left(a^2+b^2\right)}}{b}}{2b(a^2+b^2)}$$
$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

3.75. $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx$

$$\Big\downarrow \; 1082$$

$$-\frac{\frac{4b^2e^4\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\int\frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{e\cot(c\cdots}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\cdots\right.\right.}{d\left(a^2+b^2\right)}\cdots}{b}\cdots}{2b\left(a^2+b^2\right)}\cdots$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{4b^2e^4\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d\left(a^2+b^2\right)}+\frac{a^3e^4\left(3a^2+\cdots\right.}{\cdots}}{b}\cdots}{2b\left(a^2+b^2\right)}\cdots$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 1479$$

$$-\frac{\frac{4b^2e^4\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(a^2-2ab-b^2\right)\cdots\right.}{d\left(a^2+b^2\right)}\cdots}{b}\cdots}{2b\cdots}\cdots$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{4b^2e^4\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\mathrm{arc}\cdots}{\cdots}\right.\right.}{d\left(a^2+b^2\right)}\cdots}{b}\cdots}{2b\left(a\cdots\right.}\cdots$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 27$$

3.75. $\int\frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx$

$$-\frac{\frac{4b^2e^4\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\text{arc}\cdots}{\cdots}\right.}{d\left(a^2+b^2\right)}}{b}}{2b\,(a\cdots}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\,(a^2+b^2)\,(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 1103$$

$$-\frac{-\frac{\frac{a^3e^4\left(3a^2+7b^2\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{4b^2e^4\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\cdots}{\cdots}}{b}}{2b\,(a^2+b^2)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\,(a^2+b^2)\,(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 4117$$

$$-\frac{-\frac{\frac{a^3e^4\left(3a^2+7b^2\right)\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}+\frac{4b^2e^4\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\cdots}{\cdots}}{b}}{2b\,(a^2+b^2)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\,(a^2+b^2)\,(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 73$$

$$-\frac{-\frac{\frac{4b^2e^4\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log\left(e\cdots\right.}{\cdots}\right.}{d\left(a^2+b^2\right)}}{b}}{2b\,(a^2+b^2)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\,(a^2+b^2)\,(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 218$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{bd\,(a^2+b^2)\,(a+b\cot(c+dx))}+$$

$$\frac{-\frac{2e^3\left(3a^2+2b^2\right)\sqrt{e\cot(c+dx)}}{bd}-\frac{4b^2e^4\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\log\left(e\cot(c+\cdots\right.}{\cdots}\right.}{d\left(a^2+b^2\right)}}{2b\,(a^2+b^2)}$$

3.75. $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx$

input
```
Int[(e*Cot[c + d*x])^(7/2)/(a + b*Cot[c + d*x])^2,x]
```

output
```
(a^2*e^2*(e*Cot[c + d*x])^(3/2))/(b*(a^2 + b^2)*d*(a + b*Cot[c + d*x])) +
((-2*(3*a^2 + 2*b^2)*e^3*Sqrt[e*Cot[c + d*x]])/(b*d) - ((2*a^(5/2)*(3*a^2
+ 7*b^2)*e^(7/2)*ArcTan[(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt[b
]*(a^2 + b^2)*d) + (4*b^2*e^4*(((a^2 - 2*a*b - b^2)*(-(ArcTan[1 - (Sqrt[2]
*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*S
qrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 + ((a^2 + 2*a*b - b^2)
*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqr
t[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*
x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/((a^2 + b^2)*d))/b)/(2*b*(a^2 + b^2))
```

3.75.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

3.75. $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx$

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.75. $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx$

3.75.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{2e^3\left(\frac{\sqrt{e\cot(dx+c)}}{b^2}-\frac{a^3 e\left(\frac{\left(-\frac{a^2}{2}-\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{\left(3a^2+7b^2\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{b^2\left(a^2+b^2\right)^2}+\frac{e\left(\frac{\left(a^2e-b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$
default	$-\frac{2e^3\left(\frac{\sqrt{e\cot(dx+c)}}{b^2}-\frac{a^3 e\left(\frac{\left(-\frac{a^2}{2}-\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{\left(3a^2+7b^2\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{b^2\left(a^2+b^2\right)^2}+\frac{e\left(\frac{\left(a^2e-b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$

input
```
int((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^3*((e*cot(d*x+c))^(1/2)/b^2-a^3*e/b^2/(a^2+b^2)^2*((-1/2*a^2-1/2*b^
2)*(e*cot(d*x+c))^(1/2)/(e*cot(d*x+c)*b+a*e)+1/2*(3*a^2+7*b^2)/(a*e*b)^(1/
2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2)))+e/(a^2+b^2)^2*(1/8*(a^2*e
-b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4*a*b/(e^2)^(
1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1
/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3170 vs. $2(372) = 744$.

Time = 0.71 (sec) , antiderivative size = 6403, normalized size of antiderivative = 14.65

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))**(7/2)/(a+b*cot(d*x+c))**2,x)
```

output
```
Timed out
```

3.75.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.75.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \int \frac{(e\cot(dx+c))^{\frac{7}{2}}}{(b\cot(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(7/2)/(b*cot(d*x + c) + a)^2, x)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 16.41 (sec) , antiderivative size = 13244, normalized size of antiderivative = 30.31

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(7/2)/(a + b*cot(c + d*x))^2,x)
```

output
```
(atan(((((16*(e*cot(c + d*x))^(1/2)*(9*a^12*e^24 + 2*b^12*e^24 + 4*a^2*b^1
0*e^24 + 2*a^4*b^8*e^24 - 49*a^6*b^6*e^24 + 7*a^8*b^4*e^24 + 33*a^10*b^2*e
^24))/(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*
d^4) + (((16*(30*a^6*b^8*d^2*e^21 - 224*a^4*b^10*d^2*e^21 - 18*a^14*d^2*e^
21 + 600*a^8*b^6*d^2*e^21 + 388*a^10*b^4*d^2*e^21 + 24*a^12*b^2*d^2*e^21))
/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 + a^8*b^3*d^5)
- (((16*(e*cot(c + d*x))^(1/2)*(72*a^15*b*d^2*e^17 - 60*a*b^15*d^2*e^17 -
52*a^3*b^13*d^2*e^17 + 72*a^5*b^11*d^2*e^17 + 448*a^7*b^9*d^2*e^17 + 1108*
a^9*b^7*d^2*e^17 + 1132*a^11*b^5*d^2*e^17 + 480*a^13*b^3*d^2*e^17))/(b^11*
d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*d^4) + (((16
*(8*a*b^17*d^4*e^14 + 96*a^3*b^15*d^4*e^14 + 360*a^5*b^13*d^4*e^14 + 640*a
^7*b^11*d^4*e^14 + 600*a^9*b^9*d^4*e^14 + 288*a^11*b^7*d^4*e^14 + 56*a^13*
b^5*d^4*e^14))/(b^11*d^5 + 4*a^2*b^9*d^5 + 6*a^4*b^7*d^5 + 4*a^6*b^5*d^5 +
 a^8*b^3*d^5) - (8*(e*cot(c + d*x))^(1/2)*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(
1/2)*(32*b^20*d^4*e^10 + 160*a^2*b^18*d^4*e^10 + 288*a^4*b^16*d^4*e^10 + 1
60*a^6*b^14*d^4*e^10 - 160*a^8*b^12*d^4*e^10 - 288*a^10*b^10*d^4*e^10 - 16
0*a^12*b^8*d^4*e^10 - 32*a^14*b^6*d^4*e^10))/((b^9*d + 2*a^2*b^7*d + a^4*b
^5*d)*(b^11*d^4 + 4*a^2*b^9*d^4 + 6*a^4*b^7*d^4 + 4*a^6*b^5*d^4 + a^8*b^3*
d^4)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d + a^4
*b^5*d)))*(3*a^2 + 7*b^2)*(-a^5*b^5*e^7)^(1/2))/(2*(b^9*d + 2*a^2*b^7*d...
```

3.76 $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx$

3.76.1 Optimal result

Integrand size = 25, antiderivative size = 393

$$\begin{aligned}\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx &= -\frac{a^{3/2}(a^2+5b^2)\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{b^{3/2}\,(a^2+b^2)^2\,d} \\ &- \frac{(a^2+2ab-b^2)\,e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\ &+ \frac{(a^2+2ab-b^2)\,e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{a^2e^2\sqrt{e\cot(c+dx)}}{b\,(a^2+b^2)\,d(a+b\cot(c+dx))} \\ &+ \frac{(a^2-2ab-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d} \\ &- \frac{(a^2-2ab-b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}\end{aligned}$$

output
```
-a^(3/2)*(a^2+5*b^2)*e^(5/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e
^(1/2))/b^(3/2)/(a^2+b^2)^2/d-1/2*(a^2+2*a*b-b^2)*e^(5/2)*arctan(1-2^(1/2)
*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)+1/2*(a^2+2*a*b-b^2)*e
^(5/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2
)+1/4*(a^2-2*a*b-b^2)*e^(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot
(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*e^(5/2)*ln(e^(1/
2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)+
a^2*e^2*(e*cot(d*x+c))^(1/2)/b/(a^2+b^2)/d/(a+b*cot(d*x+c))
```

3.76.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.05 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.99

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx =$$
$$-\frac{(e\cot(c+dx))^{5/2}\left(-28a^2b^{3/2}(a^2-b^2)\cot^{\frac{3}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\cot^2(c+dx)\right)+12b^{7/2}\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^2,x]
```

output
```
-1/42*((e*Cot[c + d*x])^(5/2)*(-28*a^2*b^(3/2)*(a^2 - b^2)*Cot[c + d*x]^(3
/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2] + 12*b^(7/2)*(a^2 + b^
2)*Cot[c + d*x]^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*Cot[c + d*x])/a)
] - 7*a^2*(-6*Sqrt[2]*a*b^(5/2)*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 6
*Sqrt[2]*a*b^(5/2)*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 24*a^(7/2)*Arc
Tan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]] - 24*a^3*Sqrt[b]*Sqrt[Cot[c + d*
x]] - 24*a*b^(5/2)*Sqrt[Cot[c + d*x]] + 4*a^2*b^(3/2)*Cot[c + d*x]^(3/2) +
 4*b^(7/2)*Cot[c + d*x]^(3/2) - 3*Sqrt[2]*a*b^(5/2)*Log[1 - Sqrt[2]*Sqrt[C
ot[c + d*x]] + Cot[c + d*x]] + 3*Sqrt[2]*a*b^(5/2)*Log[1 + Sqrt[2]*Sqrt[Co
t[c + d*x]] + Cot[c + d*x]])))/(a^2*b^(3/2)*(a^2 + b^2)^2*d*Cot[c + d*x]^(
5/2))
```

3.76.3 Rubi [A] (warning: unable to verify)

Time = 1.48 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.90, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4048, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4048}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}-\frac{\int -\frac{a^2e^3+\left(a^2+2b^2\right)\cot^2(c+dx)e^3-2ab\cot(c+dx)e^3}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2e^3+\left(a^2+2b^2\right)\cot^2(c+dx)e^3-2ab\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{2b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2e^3+\left(a^2+2b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3+2ab\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$$\Big\downarrow \text{4136}$$

$$\frac{\frac{a^2e^3\left(a^2+5b^2\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}+\frac{\int -\frac{2\left(2ab^2e^3+b\left(a^2-b^2\right)\cot(c+dx)e^3\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a^2e^3\left(a^2+5b^2\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}-\frac{2\int \frac{2ab^2e^3+b\left(a^2-b^2\right)\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a^2e^3\left(a^2+5b^2\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int \frac{2ab^2e^3-b\left(a^2-b^2\right)e^3\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$$\Big\downarrow \text{4017}$$

3.76. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx$

$$\frac{\frac{a^2e^3(a^2+5b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4\int -\frac{be^3\left(2abe+\left(a^2-b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{2b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 25

$$\frac{\frac{a^2e^3(a^2+5b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{4\int \frac{be^3\left(2abe+\left(a^2-b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{2b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 27

$$\frac{\frac{a^2e^3(a^2+5b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{4be^3\int \frac{2abe+\left(a^2-b^2\right)\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{2b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 1482

$$\frac{\frac{a^2e^3(a^2+5b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{4be^3\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{e-e\cot}{\cot^2(c+}\right.}{d(a^2+b^2)}}{2b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 1476

$$\frac{\frac{a^2e^3(a^2+5b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{4be^3\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{}{\cot}\right.\right.}{}}{2b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 1082

3.76. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx$

$$\frac{\frac{a^2e^3(a^2+5b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{4be^3\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d}{\sqrt{}}\right.\right.}{d(a^2+b^2}}{2b(a^2+b^2)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{4be^3\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a^2-2ab-b^2)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d(a^2+b^2)}+\frac{a^2e^3}{}}{2b(a^2+b^2)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{4be^3\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a^2-2ab-b^2)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot}}{2\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}}{2b(a^2+b^2)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{4be^3\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+}}{2\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}}{2b(a^2+b^2)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{4be^3\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+}}{2\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}}{2b(a^2+b^2)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

3.76. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx$

$\Big\downarrow$ 1103

$$\frac{\frac{a^2e^3(a^2+5b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{4be^3\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-}{}}{2b(a^2+b^2)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$\Big\downarrow$ 4117

$$\frac{\frac{a^2e^3(a^2+5b^2)\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}+\frac{4be^3\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)}{}}{2b(a^2+b^2)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$\Big\downarrow$ 73

$$\frac{\frac{4be^3\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log(e}{}\right.\right.}{d(a^2+b^2)}}{2b(a^2+b^2)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}$$

$\Big\downarrow$ 218

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}+$$

$$\frac{\frac{4be^3\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+e\right)}{2\sqrt{2}\sqrt{e}}-\frac{\log(e}{}\right.\right.}{d(a^2+b^2)}}{2b(a^2+b^2)}$$

input
```
Int[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^2,x]
```

3.76. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx$

output
```
(a^2*e^2*Sqrt[e*Cot[c + d*x]])/(b*(a^2 + b^2)*d*(a + b*Cot[c + d*x])) + ((
2*a^(3/2)*(a^2 + 5*b^2)*e^(5/2)*ArcTan[(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sqr
t[e])])/(Sqrt[b]*(a^2 + b^2)*d) + (4*b*e^3*(((a^2 + 2*a*b - b^2)*(-(ArcTan
[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1
 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 - ((a^2 -
 2*a*b - b^2)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c
+ d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt
[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/((a^2 + b^2)*d))/(2*b*(a^2 + b
^2))
```

3.76.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

3.76. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx$

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.76.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{2e^3\left(\frac{a^2\left(-\frac{(a^2+b^2)\sqrt{e\cot(dx+c)}}{2b(e\cot(dx+c)b+ae)}+\frac{(a^2+5b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2b\sqrt{aeb}}\right)}{(a^2+b^2)^2}+\frac{-\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx}}\right.\right.}{}}{}\right.}{}$
default	$-\frac{2e^3\left(\frac{a^2\left(-\frac{(a^2+b^2)\sqrt{e\cot(dx+c)}}{2b(e\cot(dx+c)b+ae)}+\frac{(a^2+5b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2b\sqrt{aeb}}\right)}{(a^2+b^2)^2}+\frac{-\frac{ab(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cot(dx}}\right.\right.}{}}{}\right.}{}$

input
```
int((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^3*(a^2/(a^2+b^2)^2*(-1/2*(a^2+b^2)/b*(e*cot(d*x+c))^(1/2)/(e*cot(d*
x+c)*b+a*e)+1/2*(a^2+5*b^2)/b/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/
(a*e*b)^(1/2)))+1/(a^2+b^2)^2*(-1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d
*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x
+c))^(1/2)+1))+1/8*(-a^2+b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
))))
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3098 vs. $2(330) = 660$.

Time = 0.59 (sec) , antiderivative size = 6258, normalized size of antiderivative = 15.92

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))**(5/2)/(a+b*cot(d*x+c))**2,x)
```

output
```
Timed out
```

3.76.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.76.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx = \int \frac{(e\cot(dx+c))^{\frac{5}{2}}}{(b\cot(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(5/2)/(b*cot(d*x + c) + a)^2, x)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 15.91 (sec) , antiderivative size = 12617, normalized size of antiderivative = 32.10

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(5/2)/(a + b*cot(c + d*x))^2,x)
```

output
```
atan(((((((8*(96*a^2*b^14*d^4*e^13 + 480*a^4*b^12*d^4*e^13 + 960*a^6*b^10*
d^4*e^13 + 960*a^8*b^8*d^4*e^13 + 480*a^10*b^6*d^4*e^13 + 96*a^12*b^4*d^4*
e^13))/(b^9*d^5 + a^8*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^
5) - (16*(e*cot(c + d*x))^(1/2)*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^
2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2)*(32*b^18*d^4*e^10 + 160*a^2*b
^16*d^4*e^10 + 288*a^4*b^14*d^4*e^10 + 160*a^6*b^12*d^4*e^10 - 160*a^8*b^1
0*d^4*e^10 - 288*a^10*b^8*d^4*e^10 - 160*a^12*b^6*d^4*e^10 - 32*a^14*b^4*d
^4*e^10))/(b^9*d^4 + a^8*b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3
*d^4))*((e^5*1i)/(4*(a^4*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a
^2*b^2*d^2)))^(1/2) + (16*(e*cot(c + d*x))^(1/2)*(60*a*b^13*d^2*e^15 + 8*a
^13*b*d^2*e^15 + 52*a^3*b^11*d^2*e^15 + 128*a^5*b^9*d^2*e^15 + 424*a^7*b^7
*d^2*e^15 + 380*a^9*b^5*d^2*e^15 + 100*a^11*b^3*d^2*e^15))/(b^9*d^4 + a^8*
b*d^4 + 4*a^2*b^7*d^4 + 6*a^4*b^5*d^4 + 4*a^6*b^3*d^4))*((e^5*1i)/(4*(a^4*
d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (8*
(4*a*b^11*d^2*e^18 + 16*a^11*b*d^2*e^18 - 304*a^3*b^9*d^2*e^18 - 120*a^5*b
^7*d^2*e^18 + 320*a^7*b^5*d^2*e^18 + 148*a^9*b^3*d^2*e^18))/(b^9*d^5 + a^8
*b*d^5 + 4*a^2*b^7*d^5 + 6*a^4*b^5*d^5 + 4*a^6*b^3*d^5))*((e^5*1i)/(4*(a^4
*d^2 + b^4*d^2 + a*b^3*d^2*4i - a^3*b*d^2*4i - 6*a^2*b^2*d^2)))^(1/2) + (1
6*(e*cot(c + d*x))^(1/2)*(a^10*e^20 - 2*b^10*e^20 - 4*a^2*b^8*e^20 - 27*a^
4*b^6*e^20 + 15*a^6*b^4*e^20 + 9*a^8*b^2*e^20))/(b^9*d^4 + a^8*b*d^4 + ...
```

3.77 $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx$

3.77.1 Optimal result

Integrand size = 25, antiderivative size = 387

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = -\frac{\sqrt{a}(a^2-3b^2)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,d} - \frac{(a^2-2ab-b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{(a^2-2ab-b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} - \frac{ae\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} - \frac{(a^2+2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{(a^2+2ab-b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}$$

output
```
-1/2*(a^2-2*a*b-b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2)
)/(a^2+b^2)^2/d*2^(1/2)+1/2*(a^2-2*a*b-b^2)*e^(3/2)*arctan(1+2^(1/2)*(e*co
t(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2+2*a*b-b^2)*e^(3/2)
*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d
*2^(1/2)+1/4*(a^2+2*a*b-b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)
*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)-(a^2-3*b^2)*e^(3/2)*arctan(b^
(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*a^(1/2)/(a^2+b^2)^2/d/b^(1/2)-
a*e*(e*cot(d*x+c))^(1/2)/(a^2+b^2)/d/(a+b*cot(d*x+c))
```

3.77.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.83

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx =$$

$$-\frac{(e\cot(c+dx))^{3/2}\left(\frac{240a^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}}-240a^2\sqrt{\cot(c+dx)}+80ab\cot^{\frac{3}{2}}(c+dx)+80ab\cot^{\frac{3}{2}}(c+d\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x])^2,x]
```

output
```
-1/60*((e*Cot[c + d*x])^(3/2)*((240*a^(5/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d
*x]])/Sqrt[a]])/Sqrt[b] - 240*a^2*Sqrt[Cot[c + d*x]] + 80*a*b*Cot[c + d*x]
^(3/2) + 80*a*b*Cot[c + d*x]^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -C
ot[c + d*x]^2]) + (24*b^2*(a^2 + b^2)*Cot[c + d*x]^(5/2)*Hypergeometric2F1
[2, 5/2, 7/2, -((b*Cot[c + d*x])/a)])/a^2 + 15*(a - b)*(a + b)*(2*Sqrt[2]*
ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt
[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[
c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]])))/((a^2 + b^2)^2*d*Cot[c + d*x]^(3/2))
```

3.77.3 Rubi [A] (warning: unable to verify)

Time = 1.45 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.89, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.840, Rules used = {3042, 4050, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4050}$$

$$-\frac{\int \frac{-a\cot^2(c+dx)e^2+ae^2-2b\cot(c+dx)e^2}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}-\frac{ae\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-a\cot^2(c+dx)e^2+ae^2-2b\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{2(a^2+b^2)}-\frac{ae\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-a\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+ae^2+2b\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2(a^2+b^2)}-\frac{ae\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{4136}$$

$$-\frac{\frac{\int \frac{2\left((a^2-b^2)e^2-2abe^2\cot(c+dx)\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}-\frac{ae^2(a^2-3b^2)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{2(a^2+b^2)}-\frac{ae\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{2\int \frac{(a^2-b^2)e^2-2abe^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}-\frac{ae^2(a^2-3b^2)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{2(a^2+b^2)}-\frac{ae\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2\int \frac{(a^2-b^2)e^2+2ab\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}-\frac{ae^2(a^2-3b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2(a^2+b^2)}-\frac{ae\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{4017}$$

$$-\frac{\frac{4\int -\frac{e^2\left((a^2-b^2)e-2abe\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}-\frac{ae^2(a^2-3b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2(a^2+b^2)}-\frac{ae\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}$$

3.77. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx$

$\downarrow$ 25

$$-\frac{-\frac{ae^2\left(a^2-3b^2\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4\int \frac{e^2\left(\left(a^2-b^2\right)e-2abe\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{2\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 27

$$-\frac{-\frac{ae^2\left(a^2-3b^2\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4e^2\int \frac{\left(a^2-b^2\right)e-2abe\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{2\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 1482

$$-\frac{-\frac{ae^2\left(a^2-3b^2\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4e^2\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{\cot(\cdots}{\cot^2(\cdots}\cdots\right.}{d(a^2+b^2)}}{2\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 1476

$$-\frac{-\frac{ae^2\left(a^2-3b^2\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4e^2\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{1}{2}\int \cdots\right.\right.}{\cdots}}{2\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 1082

$$-\frac{-\frac{ae^2\left(a^2-3b^2\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4e^2\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\int\cdots}{-e\cdots}\cdots\right.\right.}{d(a^2\cdots}}{2\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 217

3.77. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx$

$$-\frac{-\frac{4e^2\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d(a^2+b^2)}-\cdots}{2\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\big\downarrow$ 1479

$$-\frac{-\frac{4e^2\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(a^2-2ab-\cdots\right.\right.}{d(a^2+b^2)}\cdots}{2\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\big\downarrow$ 25

$$-\frac{-\frac{4e^2\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\cdots\right.\right.}{d(a^2+b^2)}\cdots}{2\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\big\downarrow$ 27

$$-\frac{-\frac{4e^2\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\cdots\right.\right.}{d(a^2+b^2)}\cdots}{2\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\big\downarrow$ 1103

$$-\frac{-\frac{ae^2\left(a^2-3b^2\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4e^2\left(\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\cdots\right.\right.}{\cdots}}{2\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

3.77. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx$

$\Big\downarrow$ 4117

$$-\frac{-\frac{ae^2(a^2-3b^2)\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}-\frac{4e^2\left(\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right.\right.}{}}{2(a^2+b^2)} \frac{ae\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}$$

$\Big\downarrow$ 73

$$-\frac{-\frac{2ae(a^2-3b^2)\int \frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}-\frac{4e^2\left(\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a^2+2a\right.}{d(a^2+}}{2(a^2+b^2)} \frac{ae\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}$$

$\Big\downarrow$ 218

$$-\frac{-\frac{2\sqrt{a}e^{3/2}(a^2-3b^2)\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}d(a^2+b^2)}-\frac{4e^2\left(\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a^2+2ab-\right.}{d(a^2+b^2)}}{2(a^2+b^2)} \frac{ae\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}$$

input
```
Int[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x])^2,x]
```

output
```
-((a*e*Sqrt[e*Cot[c + d*x]])/((a^2 + b^2)*d*(a + b*Cot[c + d*x]))) - ((-2*
Sqrt[a]*(a^2 - 3*b^2)*e^(3/2)*ArcTan[(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sqrt[
e])])/(Sqrt[b]*(a^2 + b^2)*d) - (4*e^2*(((a^2 - 2*a*b - b^2)*(-(ArcTan[1 -
 (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (
Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 + ((a^2 + 2*a
*b - b^2)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*
x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*C
ot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/((a^2 + b^2)*d))/(2*(a^2 + b^2))
```

3.77. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx$

3.77.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4050
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/((m
 + 1)*(a^2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^
(n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2
*a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[
2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.77. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx$

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.77.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{2e^3\left(\frac{a\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{\left(a^2-3b^2\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{\left(a^2+b^2\right)^2 e}+\frac{\left(-a^2e+b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\cdots}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\cdots}\right.\right.}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{2e^3\left(\frac{a\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{\left(a^2-3b^2\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{\left(a^2+b^2\right)^2 e}+\frac{\left(-a^2e+b^2e\right)\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\cdots}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\cdots}\right.\right.}{\cdots}\cdots\right.}{\cdots}$

input
```
int((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^3*(a/(a^2+b^2)^2/e*((1/2*a^2+1/2*b^2)*(e*cot(d*x+c))^(1/2)/(e*cot(d
*x+c)*b+a*e)+1/2*(a^2-3*b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(
a*e*b)^(1/2)))+1/e/(a^2+b^2)^2*(1/8*(-a^2*e+b^2*e)*(e^2)^(1/4)/e^2*2^(1/2)
*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(
e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arct
an(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/
4)*(e*cot(d*x+c))^(1/2)+1))+1/4*a*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(
1/2)+1))))
```

3.77.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3044 vs. $2(324) = 648$.

Time = 0.47 (sec) , antiderivative size = 6150, normalized size of antiderivative = 15.89

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.77.6 Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = \int \frac{(e\cot{(c+dx)})^{\frac{3}{2}}}{(a+b\cot{(c+dx)})^2}\,dx$$

input
```
integrate((e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**2,x)
```

output
```
Integral((e*cot(c + d*x))**(3/2)/(a + b*cot(c + d*x))**2, x)
```

3.77.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.77.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{(b\cot(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(3/2)/(b*cot(d*x + c) + a)^2, x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 16.25 (sec) , antiderivative size = 11953, normalized size of antiderivative = 30.89

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(3/2)/(a + b*cot(c + d*x))^2,x)
```

output
```
(atan((((a^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*(2*b^9*e^16 + a^8*b*e^16
 - 5*a^2*b^7*e^16 + 17*a^4*b^5*e^16 - 7*a^6*b^3*e^16))/(a^8*d^4 + b^8*d^4
+ 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + ((a^2 - 3*b^2)*((16*(2*
a^10*b*d^2*e^15 - 78*a^2*b^9*d^2*e^15 + 8*a^4*b^7*d^2*e^15 + 60*a^6*b^5*d^
2*e^15 - 24*a^8*b^3*d^2*e^15))/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*
b^4*d^5 + 4*a^6*b^2*d^5) - ((a^2 - 3*b^2)*((16*(e*cot(c + d*x))^(1/2)*(20*
a^3*b^10*d^2*e^13 - 60*a*b^12*d^2*e^13 + 168*a^5*b^8*d^2*e^13 + 40*a^7*b^6
*d^2*e^13 - 44*a^9*b^4*d^2*e^13 + 4*a^11*b^2*d^2*e^13))/(a^8*d^4 + b^8*d^4
 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4) + ((a^2 - 3*b^2)*((16*(4
0*a*b^14*d^4*e^12 + 192*a^3*b^12*d^4*e^12 + 360*a^5*b^10*d^4*e^12 + 320*a^
7*b^8*d^4*e^12 + 120*a^9*b^6*d^4*e^12 - 8*a^13*b^2*d^4*e^12))/(a^8*d^5 + b
^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (8*(e*cot(c + d*
x))^(1/2)*(a^2 - 3*b^2)*(-a*b*e^3)^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*
d^4*e^10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4
*e^10 - 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^
10))/((b^5*d + 2*a^2*b^3*d + a^4*b*d)*(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 +
 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*
d + a^4*b*d)))*(-a*b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*
b*e^3)^(1/2))/(2*(b^5*d + 2*a^2*b^3*d + a^4*b*d)))*(-a*b*e^3)^(1/2)*1i)/(2
*(b^5*d + 2*a^2*b^3*d + a^4*b*d)) + ((a^2 - 3*b^2)*((16*(e*cot(c + d*x)...
```

3.77. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^2}\,dx$

3.78 $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx$

3.78.1 Optimal result

Integrand size = 25, antiderivative size = 386

$$\begin{aligned}
\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx
&= \frac{\sqrt{b}(3a^2-b^2)\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\,(a^2+b^2)^2\,d} \\
&\quad + \frac{(a^2+2ab-b^2)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{(a^2+2ab-b^2)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} + \frac{b\sqrt{e\cot(c+dx)}}{(a^2+b^2)\,d(a+b\cot(c+dx))} \\
&\quad - \frac{(a^2-2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad + \frac{(a^2-2ab-b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^2\,d}
\end{aligned}$$

output
```
1/2*(a^2+2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)
/(a^2+b^2)^2/d*2^(1/2)-1/2*(a^2+2*a*b-b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))
^(1/2)/e^(1/2))*e^(1/2)/(a^2+b^2)^2/d*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(e^(1/
2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/(a^2+b^2)^2/d*
2^(1/2)+1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d
*x+c))^(1/2))*e^(1/2)/(a^2+b^2)^2/d*2^(1/2)+(3*a^2-b^2)*arctan(b^(1/2)*(e*
cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*b^(1/2)*e^(1/2)/(a^2+b^2)^2/d/a^(1/2)+b
*(e*cot(d*x+c))^(1/2)/(a^2+b^2)/d/(a+b*cot(d*x+c))
```

3.78.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.11 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx = -\frac{\sqrt{e\cot(c+dx)}\left(-\frac{4a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{(a^2+b^2)^2}+\frac{4ab\sqrt{\cot(c+dx)}}{(a^2+b^2)^2}-\frac{\sqrt{b}\left(-a\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)+\sqrt{a}\sqrt{b}\sqrt{\cot(c+dx)}-b\right.}{\sqrt{a}(a^2+b^2)(a+b\cot(c+\right.}}{}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x])^2,x]
```

output
```
-((Sqrt[e*Cot[c + d*x]]*((-4*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[Cot[c +
d*x]])/Sqrt[a]])/(a^2 + b^2)^2 + (4*a*b*Sqrt[Cot[c + d*x]])/(a^2 + b^2)^2
- (Sqrt[b]*(-(a*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]) + Sqrt[a]*Sq
rt[b]*Sqrt[Cot[c + d*x]] - b*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]*
Cot[c + d*x]))/(Sqrt[a]*(a^2 + b^2)*(a + b*Cot[c + d*x])) + (2*(a - b)*(a
+ b)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2])/(
3*(a^2 + b^2)^2) - (a*b*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]
- 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]]
+ Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log
[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(2*(a^2 + b^2)^2)))/(d*S
qrt[Cot[c + d*x]]))
```

3.78.3 Rubi [A] (warning: unable to verify)

Time = 1.40 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.88, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.840, Rules used = {3042, 4051, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4051}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}-\frac{\int -\frac{-be\cot^2(c+dx)+2ae\cot(c+dx)+be}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-be\cot^2(c+dx)+2ae\cot(c+dx)+be}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{2\left(a^2+b^2\right)}+\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-be\tan\left(c+dx+\frac{\pi}{2}\right)^2-2ae\tan\left(c+dx+\frac{\pi}{2}\right)+be}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\left(a^2+b^2\right)}+\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{4136}$$

$$\frac{\frac{\int \frac{2\left(2abe+\left(a^2-b^2\right)\cot(c+dx)e\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}-\frac{be\left(3a^2-b^2\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{2\left(a^2+b^2\right)}+\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{2abe+\left(a^2-b^2\right)\cot(c+dx)e}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}-\frac{be\left(3a^2-b^2\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{2\left(a^2+b^2\right)}+\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\int \frac{2abe-\left(a^2-b^2\right)e\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}-\frac{be\left(3a^2-b^2\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2\left(a^2+b^2\right)}+\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 4017$$

$$\frac{\frac{4\int -\frac{e\left(2abe+\left(a^2-b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{be\left(3a^2-b^2\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2\left(a^2+b^2\right)}+\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{4\int \frac{e\left(2abe+\left(a^2-b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{be\left(3a^2-b^2\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2\left(a^2+b^2\right)}+\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{4e\int \frac{2abe+\left(a^2-b^2\right)\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{be\left(3a^2-b^2\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2\left(a^2+b^2\right)}+\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 1482$$

$$\frac{-\frac{4e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d\left(a^2+b^2\right)}-\frac{be\left(3a^2-b^2\right)\int \frac{\tan\left(c+d\cdots\right.}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)\cdots}}}{a^2+b^2}}{2\left(a^2+b^2\right)}+\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 1476$$

3.78. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx$

$$-\frac{\frac{4e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}\right)-\frac{1}{2}\left(a^2-2ab-b \cdots\right.\right.}{d(a^2+b^2)}}{2\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$$\Big\downarrow \; 1082$$

$$-\frac{\frac{4e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{\cdots}\right.}{d(a^2+b^2)}}{2\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{4e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d(a^2+b^2)}-\frac{be(3 \cdots}{\cdots}}{2\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$$\Big\downarrow \; 1479$$

$$-\frac{\frac{4e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot \cdots}}{2\sqrt{2}\sqrt{e}}\cdots\right.\right.}{d(a^2+b^2)}}{2\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{4e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c \cdots}}{2\sqrt{2}\sqrt{e}}\cdots\right.\right.}{d(a^2+b^2)}}{2\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$$\Big\downarrow \; 27$$

3.78. $\displaystyle\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx$

$$-\frac{-\frac{4e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c-}{2\sqrt{2}\sqrt{e}}\right.\right.}{d(a^2+b^2)}}{2\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 1103

$$\frac{-\frac{be\left(3a^2-b^2\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\right.}{}}{2\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 4117

$$\frac{-\frac{be\left(3a^2-b^2\right)\int\frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}-\frac{4e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\right.}{}}{2\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 73

$$\frac{\frac{2b\left(3a^2-b^2\right)\int\frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}-\frac{4e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\left(a^2-2ab-b^2\right.\right.}{d(a^2+b^2)}}{2\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\downarrow$ 218

$$\frac{-\frac{2\sqrt{b}\sqrt{e}\left(3a^2-b^2\right)\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}d(a^2+b^2)}-\frac{4e\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\right.\right.}{d(a^2+b^2)}}{2\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

3.78. $\int\frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx$

input
```
Int[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x])^2,x]
```

output
```
(b*Sqrt[e*Cot[c + d*x]])/((a^2 + b^2)*d*(a + b*Cot[c + d*x])) + ((-2*Sqrt[
b]*(3*a^2 - b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sqrt[e])])
/(Sqrt[a]*(a^2 + b^2)*d) - (4*e*(((a^2 + 2*a*b - b^2)*(-(ArcTan[1 - (Sqrt[
2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]
*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 - ((a^2 - 2*a*b - b^
2)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(S
qrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c +
d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/((a^2 + b^2)*d))/(2*(a^2 + b^2))
```

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4051
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(a^2 + b^2
))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c
*(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Int
egerQ[2*m]
```

3.78. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx$

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.78.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{b\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{\left(3a^2-b^2\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{e^2\left(a^2+b^2\right)^2}+\frac{ab\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx\cdots}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx\cdots}}\right.\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2e^3\left(-\frac{b\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{\left(3a^2-b^2\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{e^2\left(a^2+b^2\right)^2}+\frac{ab\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx\cdots}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot(dx\cdots}}\right.\right.}{\cdots}\right.}{\cdots}$

input
```
int((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^3*(-b/e^2/(a^2+b^2)^2*((1/2*a^2+1/2*b^2)*(e*cot(d*x+c))^(1/2)/(e*co
t(d*x+c)*b+a*e)+1/2*(3*a^2-b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*
b/(a*e*b)^(1/2)))+1/e^2/(a^2+b^2)^2*(1/4*a/e*b*(e^2)^(1/4)*2^(1/2)*(ln((e*
cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*
x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/
2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*co
t(d*x+c))^(1/2)+1))+1/8*(a^2-b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e
^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4
)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)+1))))
```

3.78.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3031 vs. $2(323) = 646$.

Time = 0.45 (sec) , antiderivative size = 6104, normalized size of antiderivative = 15.81

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.78.6 Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx = \int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx$$

input
```
integrate((e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**2,x)
```

output
```
Integral(sqrt(e*cot(c + d*x))/(a + b*cot(c + d*x))**2, x)
```

3.78.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.78.8 Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{(b\cot(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*cot(d*x + c))/(b*cot(d*x + c) + a)^2, x)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 15.69 (sec) , antiderivative size = 11731, normalized size of antiderivative = 30.39

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(1/2)/(a + b*cot(c + d*x))^2,x)
```

output
```
(b*e*(e*cot(c + d*x))^(1/2))/((a*d*e + b*d*e*cot(c + d*x))*(a^2 + b^2)) -
atan((((((8*(320*a^6*b^9*d^4*e^11 - 96*a^2*b^13*d^4*e^11 - 32*b^15*d^4*e^
11 + 480*a^8*b^7*d^4*e^11 + 288*a^10*b^5*d^4*e^11 + 64*a^12*b^3*d^4*e^11))
/(a^8*d^5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5) - (16
*(e*cot(c + d*x))^(1/2)*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a
^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2)*(32*b^17*d^4*e^10 + 160*a^2*b^15*d^4*e^
10 + 288*a^4*b^13*d^4*e^10 + 160*a^6*b^11*d^4*e^10 - 160*a^8*b^9*d^4*e^10
- 288*a^10*b^7*d^4*e^10 - 160*a^12*b^5*d^4*e^10 - 32*a^14*b^3*d^4*e^10))/(
a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))*(e/(4*
(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(
1/2) + (16*(e*cot(c + d*x))^(1/2)*(68*a*b^12*d^2*e^11 + 20*a^3*b^10*d^2*e^
11 - 88*a^5*b^8*d^2*e^11 + 40*a^7*b^6*d^2*e^11 + 84*a^9*b^4*d^2*e^11 + 4*a
^11*b^2*d^2*e^11))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*
a^6*b^2*d^4))*(e/(4*(a^4*d^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 -
 a^2*b^2*d^2*6i)))^(1/2) + (8*(52*a*b^10*d^2*e^12 - 128*a^3*b^8*d^2*e^12 -
 24*a^5*b^6*d^2*e^12 + 160*a^7*b^4*d^2*e^12 + 4*a^9*b^2*d^2*e^12))/(a^8*d^
5 + b^8*d^5 + 4*a^2*b^6*d^5 + 6*a^4*b^4*d^5 + 4*a^6*b^2*d^5))*(e/(4*(a^4*d
^2*1i + b^4*d^2*1i + 4*a*b^3*d^2 - 4*a^3*b*d^2 - a^2*b^2*d^2*6i)))^(1/2) -
 (16*(e*cot(c + d*x))^(1/2)*(3*b^9*e^12 - 3*a^2*b^7*e^12 + 17*a^4*b^5*e^12
 - 9*a^6*b^3*e^12))/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 ...
```

3.78. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^2}\,dx$

3.79 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx$

3.79.1 Optimal result

Integrand size = 25, antiderivative size = 394

$$\begin{aligned}
&\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx \\
&= -\frac{b^{3/2}(5a^2+b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\left(a^2+b^2\right)^2 d\sqrt{e}} + \frac{(a^2-2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d\sqrt{e}} \\
&\quad - \frac{(a^2-2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^2 d\sqrt{e}} - \frac{b^2\sqrt{e\cot(c+dx)}}{a\left(a^2+b^2\right)de(a+b\cot(c+dx))} \\
&\quad + \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d\sqrt{e}} \\
&\quad - \frac{(a^2+2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^2 d\sqrt{e}}
\end{aligned}$$

output
```
-b^(3/2)*(5*a^2+b^2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))/
a^(3/2)/(a^2+b^2)^2/d/e^(1/2)+1/2*(a^2-2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot(
d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)/e^(1/2)-1/2*(a^2-2*a*b-b^2)*a
rctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d*2^(1/2)/e^(1/2
)+1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))
^(1/2))/(a^2+b^2)^2/d*2^(1/2)/e^(1/2)-1/4*(a^2+2*a*b-b^2)*ln(e^(1/2)+cot(d
*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d*2^(1/2)/e^(1/2)-
b^2*(e*cot(d*x+c))^(1/2)/a/(a^2+b^2)/d/e/(a+b*cot(d*x+c))
```

3.79.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.79 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx =$$

$$-\frac{\sqrt{\cot(c+dx)}\left(48\sqrt{a}b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)+\frac{12b^{3/2}(a^2+b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{12b^2(a^2+b^2)\sqrt{\cot(c+dx)}}{a(a+b\cot(c+dx))}\right.}{}$$

input
```
Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2),x]
```

output
```
-1/12*(Sqrt[Cot[c + d*x]]*(48*Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c +
 d*x]])/Sqrt[a]] + (12*b^(3/2)*(a^2 + b^2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*
x]])/Sqrt[a]])/a^(3/2) + (12*b^2*(a^2 + b^2)*Sqrt[Cot[c + d*x]])/(a*(a + b
*Cot[c + d*x])) - 16*a*b*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4,
 -Cot[c + d*x]^2] - 3*Sqrt[2]*(a - b)*(a + b)*(2*ArcTan[1 - Sqrt[2]*Sqrt[C
ot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + Log[1 - Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] +
 Cot[c + d*x]])))/((a^2 + b^2)^2*d*Sqrt[e*Cot[c + d*x]])
```

3.79.3 Rubi [A] (warning: unable to verify)

Time = 1.49 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.90, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4052, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4052}$$

$$-\frac{\int -\frac{b^2e\cot^2(c+dx)-2abe\cot(c+dx)+(2a^2+b^2)e}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae(a^2+b^2)}-\frac{b^2\sqrt{e\cot(c+dx)}}{ade(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^2e\cot^2(c+dx)-2abe\cot(c+dx)+(2a^2+b^2)e}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{2ae(a^2+b^2)}-\frac{b^2\sqrt{e\cot(c+dx)}}{ade(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b^2e\tan\left(c+dx+\frac{\pi}{2}\right)^2+2abe\tan\left(c+dx+\frac{\pi}{2}\right)+(2a^2+b^2)e}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2ae(a^2+b^2)}-\frac{b^2\sqrt{e\cot(c+dx)}}{ade(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{4136}$$

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{2\left(a\left(a^2-b^2\right)e-2a^2be\cot(c+dx)\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{b^2\sqrt{e\cot(c+dx)}}{ade(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}+\frac{2\int \frac{a\left(a^2-b^2\right)e-2a^2be\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{b^2\sqrt{e\cot(c+dx)}}{ade(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{2\int \frac{2be\tan\left(c+dx+\frac{\pi}{2}\right)a^2+\left(a^2-b^2\right)ea}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{b^2\sqrt{e\cot(c+dx)}}{ade(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\frac{4\int -\frac{ae\left(\left(a^2-b^2\right)e-2abe\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}+\frac{b^2e(5a^2+b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{b^2\sqrt{e\cot(c+dx)}}{ade(a^2+b^2)(a+b\cot(c+dx))}$$

3.79. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx$

$\Big\downarrow$ 25

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4\int \frac{ae\left(\left(a^2-b^2\right)e-2abe\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{2ae\left(a^2+b^2\right)}-\frac{b^2\sqrt{e\cot(c+dx)}}{ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4ae\int \frac{\left(a^2-b^2\right)e-2abe\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{2ae\left(a^2+b^2\right)}-\frac{b^2\sqrt{e\cot(c+dx)}}{ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\Big\downarrow$ 1482

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4ae\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2-2ab-b^2\right)\int \frac{\cot(c+dx\cdots}{\cot^2(c+dx)\cdots}\cdots\right.}{d(a^2+b^2)}}{2ae\left(a^2+b^2\right)}\cdots$$

$$-\frac{b^2\sqrt{e\cot(c+dx)}}{ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\Big\downarrow$ 1476

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4ae\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{1}{2}\int \frac{\cdots}{\cot(c+\cdots}\cdots\right.\right.}{\cdots}}{2ae\left(a^2+b^2\right)}\cdots$$

$$-\frac{b^2\sqrt{e\cot(c+dx)}}{ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\Big\downarrow$ 1082

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4ae\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\left(a^2-2ab-b^2\right)\left(\frac{\int \frac{1\cdots}{-e\cot(c\cdots}\cdots}{\cdots}\right.\right.}{d(a^2+b^2)\cdots}}{2ae\left(a^2+b^2\right)}\cdots$$

$$-\frac{b^2\sqrt{e\cot(c+dx)}}{ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\Big\downarrow$ 217

3.79. $\displaystyle\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx$

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4ae\left(\frac{1}{2}(a^2+2ab-b^2)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\arctan\left(\sqrt{2}\cdots\right.}{\cdots}\cdots\right.\right.}{d(a^2+b^2)}}{2ae\left(a^2+b^2\right)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\Big\downarrow$ 1479

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4ae\left(\frac{1}{2}(a^2+2ab-b^2)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\cdots}{\cot(c\cdots}}{\cdots}\cdots\right.\right.}{\cdots}}{2ae\left(a^2+b^2\right)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\Big\downarrow$ 25

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4ae\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{e}\cdots\right.}{\cot(c+dx)e\cdots}}{\cdots}\cdots\right.\right.}{\cdots}}{2ae\left(a^2+b^2\right)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4ae\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int\frac{\sqrt{e}+\cdots}{\cot(c+dx)e\cdots}}{\cdots}\cdots\right.\right.}{\cdots}}{2ae\left(a^2+b^2\right)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$\Big\downarrow$ 1103

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4ae\left(\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}\cdots\right.}{\cdots}}{2ae\left(a^2+b^2\right)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

3.79. $\displaystyle\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx$

$$\Big\downarrow \text{4117}$$

$$\frac{\frac{b^2e(5a^2+b^2)\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}-\frac{4ae\left(\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\cdots}{\cdots}}{2ae(a^2+b^2)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{ade(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{2b^2(5a^2+b^2)\int \frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}-\frac{4ae\left(\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a^2+2a\cdots}{d(a^2+\cdots}}{2ae(a^2+b^2)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{ade(a^2+b^2)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2b^{3/2}\sqrt{e}(5a^2+b^2)\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}d(a^2+b^2)}-\frac{4ae\left(\frac{1}{2}(a^2-2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a^2+2ab-b^2)\Big(\cdots}{d(a^2+b^2)}}{2ae(a^2+b^2)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{ade(a^2+b^2)(a+b\cot(c+dx))}$$

input
```
Int[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2),x]
```

output
```
-((b^2*Sqrt[e*Cot[c + d*x]])/(a*(a^2 + b^2)*d*e*(a + b*Cot[c + d*x]))) + (
(2*b^(3/2)*(5*a^2 + b^2)*Sqrt[e]*ArcTan[(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sq
rt[e])])/(Sqrt[a]*(a^2 + b^2)*d) - (4*a*e*(((a^2 - 2*a*b - b^2)*(-(ArcTan[
1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 + ((a^2 +
2*a*b - b^2)*(-1/2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c +
 d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[
e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))/2))/((a^2 + b^2)*d))/(2*a*(a^2 + b^
2)*e)
```

3.79. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx$

3.79.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.79. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx$

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.79.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{2e^3\left(\frac{b^2\left(\frac{(a^2+b^2)\sqrt{e\cot(dx+c)}}{2a(e\cot(dx+c)b+ae)}+\frac{(5a^2+b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2a\sqrt{aeb}}\right)}{e^3(a^2+b^2)^2}+\frac{(a^2e-b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cdots}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cdots}}\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
default	$-\frac{2e^3\left(\frac{b^2\left(\frac{(a^2+b^2)\sqrt{e\cot(dx+c)}}{2a(e\cot(dx+c)b+ae)}+\frac{(5a^2+b^2)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2a\sqrt{aeb}}\right)}{e^3(a^2+b^2)^2}+\frac{(a^2e-b^2e)(e^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+(e^2)^{\frac{1}{4}}\sqrt{e\cdots}}{e\cot(dx+c)-(e^2)^{\frac{1}{4}}\sqrt{e\cdots}}\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}$

input
```
int(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^3*(b^2/e^3/(a^2+b^2)^2*(1/2*(a^2+b^2)/a*(e*cot(d*x+c))^(1/2)/(e*cot
(d*x+c)*b+a*e)+1/2*(5*a^2+b^2)/a/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)
*b/(a*e*b)^(1/2)))+1/e^3/(a^2+b^2)^2*(1/8*(a^2*e-b^2*e)*(e^2)^(1/4)/e^2*2^
(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2
*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)+1))-1/4*a*b/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*
x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(
e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^
2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1))))
```

3.79.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3102 vs. $2(331) = 662$.

Time = 0.61 (sec) , antiderivative size = 6248, normalized size of antiderivative = 15.86

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.79.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\cot(c+dx)}\,(a+b\cot(c+dx))^2}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**2,x)
```

output
```
Integral(1/(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))**2), x)
```

3.79.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.79.8 Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx = \int \frac{1}{(b\cot(dx+c)+a)^2\sqrt{e\cot(dx+c)}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*cot(d*x + c) + a)^2*sqrt(e*cot(d*x + c))), x)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 20.82 (sec) , antiderivative size = 9400, normalized size of antiderivative = 23.86

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))^2),x)
```

output
```
(log(- (((((((((128*b^2*e^10*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a^4*b^2))/(a*d)
- 256*b^3*e^10*(e*cot(c + d*x))^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(1i/(d^2*e
*(a*1i - b)^4))^(1/2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 - (64*b^2*e^9*(e
*cot(c + d*x))^(1/2)*(2*b^8 - a^8 + 5*a^2*b^6 + 67*a^4*b^4 - a^6*b^2))/(a*
d^2*(a^2 + b^2)^2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 - (32*b^5*e^9*(25*a
^6 + b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(a^2*d^3*(a^2 + b^2)^3))*(1i/(d^2*e*(
a*1i - b)^4))^(1/2))/2 - (16*b^5*e^8*(e*cot(c + d*x))^(1/2)*(b^6 - 27*a^6
+ 7*a^2*b^4 + 11*a^4*b^2))/(a^2*d^4*(a^2 + b^2)^4))*(1i/(d^2*e*(a*1i - b)^
4))^(1/2))/2 - (16*b^6*e^8*(5*a^2 + b^2))/(a*d^5*(a^2 + b^2)^4))*(-1/(a^4*
d^2*e*1i + b^4*d^2*e*1i - a^2*b^2*d^2*e*6i + 4*a*b^3*d^2*e - 4*a^3*b*d^2*e
))^(1/2))/2 - log(- (((((((((128*b^2*e^10*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a^4
*b^2))/(a*d) + 256*b^3*e^10*(e*cot(c + d*x))^(1/2)*(a^2 - b^2)*(a^2 + b^2)
^2*(1i/(d^2*e*(a*1i - b)^4))^(1/2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 + (
64*b^2*e^9*(e*cot(c + d*x))^(1/2)*(2*b^8 - a^8 + 5*a^2*b^6 + 67*a^4*b^4 -
a^6*b^2))/(a*d^2*(a^2 + b^2)^2))*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 - (32*
b^5*e^9*(25*a^6 + b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(a^2*d^3*(a^2 + b^2)^3))
*(1i/(d^2*e*(a*1i - b)^4))^(1/2))/2 + (16*b^5*e^8*(e*cot(c + d*x))^(1/2)*(
b^6 - 27*a^6 + 7*a^2*b^4 + 11*a^4*b^2))/(a^2*d^4*(a^2 + b^2)^4))*(1i/(d^2*
e*(a*1i - b)^4))^(1/2))/2 - (16*b^6*e^8*(5*a^2 + b^2))/(a*d^5*(a^2 + b^2)^
4))*(-1/(4*(a^4*d^2*e*1i + b^4*d^2*e*1i - a^2*b^2*d^2*e*6i + 4*a*b^3*d^...
```

3.79. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}\,dx$

3.80 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx$

3.80.1 Optimal result

Integrand size = 25, antiderivative size = 437

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \frac{b^{5/2}(7a^2+3b^2)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{a^{5/2}(a^2+b^2)^2 de^{3/2}}$$
$$-\frac{(a^2+2ab-b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^2 de^{3/2}}+\frac{(a^2+2ab-b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^2 de^{3/2}}$$
$$+\frac{2a^2+3b^2}{a^2(a^2+b^2)de\sqrt{e\cot(c+dx)}}-\frac{b^2}{a(a^2+b^2)de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$
$$+\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^2 de^{3/2}}$$
$$-\frac{(a^2-2ab-b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^2 de^{3/2}}$$

output
```
b^(5/2)*(7*a^2+3*b^2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))
/a^(5/2)/(a^2+b^2)^2/d/e^(3/2)-1/2*(a^2+2*a*b-b^2)*arctan(1-2^(1/2)*(e*cot
(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d/e^(3/2)*2^(1/2)+1/2*(a^2+2*a*b-b^2)*
arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^2/d/e^(3/2)*2^(1/
2)+1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c)
)^(1/2))/(a^2+b^2)^2/d/e^(3/2)*2^(1/2)-1/4*(a^2-2*a*b-b^2)*ln(e^(1/2)+cot(
d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^2/d/e^(3/2)*2^(1/2)
+(2*a^2+3*b^2)/a^2/(a^2+b^2)/d/e/(e*cot(d*x+c))^(1/2)-b^2/a/(a^2+b^2)/d/e/
(a+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2)
```

3.80.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.56

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \frac{8a^2b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{b\cot(c+dx)}{a}\right)+4b^2(a^2+}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^2),x]
```

output
```
(8*a^2*b^2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*Cot[c + d*x])/a)] + 4*b^2*
(a^2 + b^2)*Hypergeometric2F1[-1/2, 2, 1/2, -((b*Cot[c + d*x])/a)] + a^2*(
4*(a^2 - b^2)*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[c + d*x]^2] + Sqrt[2]*a
*b*Sqrt[Cot[c + d*x]]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] + 2*ArcTa
n[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]])))/(2*a^
2*(a^2 + b^2)^2*d*e*Sqrt[e*Cot[c + d*x]])
```

3.80.3 Rubi [A] (warning: unable to verify)

Time = 2.09 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.92, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.960, Rules used = {3042, 4052, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4052}$$

$$-\frac{\int -\frac{3b^2e\cot^2(c+dx)-2abe\cot(c+dx)+(2a^2+3b^2)e}{2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}dx}{ae\left(a^2+b^2\right)}-\frac{b^2}{ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \, 27$$

$$\frac{\int \frac{3b^2e\cot^2(c+dx)-2abe\cot(c+dx)+(2a^2+3b^2)e}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}dx}{2ae\,(a^2+b^2)}-\frac{b^2}{ade\,(a^2+b^2)\,\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \, 3042$$

$$\frac{\int \frac{3b^2e\tan\left(c+dx+\frac{\pi}{2}\right)^2+2abe\tan\left(c+dx+\frac{\pi}{2}\right)+(2a^2+3b^2)e}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2ae\,(a^2+b^2)}-\frac{b^2}{ade\,(a^2+b^2)\,\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \, 4132$$

$$\frac{\frac{2\int -\frac{b\left(2a^2+3b^2\right)\cot^2(c+dx)e^3+b\left(4a^2+3b^2\right)e^3+2a^3\cot(c+dx)e^3}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae^3}+\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}}{2ae\,(a^2+b^2)}-\frac{b^2}{ade\,(a^2+b^2)\,\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \, 27$$

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\int \frac{b\left(2a^2+3b^2\right)\cot^2(c+dx)e^3+b\left(4a^2+3b^2\right)e^3+2a^3\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae^3}}{2ae\,(a^2+b^2)}-\frac{b^2}{ade\,(a^2+b^2)\,\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \, 3042$$

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\int \frac{b\left(2a^2+3b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3+b\left(4a^2+3b^2\right)e^3-2a^3\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}}{2ae\,(a^2+b^2)}-\frac{b^2}{ade\,(a^2+b^2)\,\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \, 4136$$

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3\left(7a^2+3b^2\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{2\left(2a^3be^3+a^2\left(a^2-b^2\right)\cot(c+dx)e^3\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{ae^3}}{2ae\,(a^2+b^2)}-\frac{b^2}{ade\,(a^2+b^2)\,\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \, 27$$

3.80. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx$

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}+\frac{2\int \frac{2a^3be^3+a^2(a^2-b^2)\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{ae^3}}{2ae(a^2+b^2)}-\frac{b^2}{ade(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$\big\downarrow$ 3042

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{2\int \frac{2a^3be^3-a^2(a^2-b^2)e^3\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{ae^3}}{2ae(a^2+b^2)}-\frac{b^2}{ade(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$\big\downarrow$ 4017

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{4\int -\frac{a^2e^3\left(2abe+(a^2-b^2)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}+\frac{b^3e^3(7a^2+3b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{ae^3}}{2ae(a^2+b^2)}-\frac{b^2}{ade(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$\big\downarrow$ 25

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4\int \frac{a^2e^3\left(2abe+(a^2-b^2)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{ae^3}}{2ae(a^2+b^2)}-\frac{b^2}{ade(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$\big\downarrow$ 27

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2e^3\int \frac{2abe+(a^2-b^2)\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}}{ae^3}}{2ae(a^2+b^2)}-\frac{b^2}{ade(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

3.80. $\displaystyle\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx$

$$\Big\downarrow \; 1482$$

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2e^3\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}\left(a^2-2ab \cdots\right.\right.}{d\left(a^2+b^2\right)}}{ae^3}}{2ae\left(a^2+b^2\right)}$$

$$\frac{b^2}{ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2e^3\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot \cdots}\right.\right.}{\cdots}}{ae^3}}{2ae\left(a^2+b^2\right)}$$

$$\frac{b^2}{ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2e^3\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\cdots\right.\right.}{\cdots}}{ae^3}}{2ae\left(a^2+b^2\right)}$$

$$\frac{b^2}{ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2e^3\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{\cdots}}\right.\right.}{d\left(a^2 \cdots\right)}}{ae^3}}{2ae\left(a^2+b^2\right)}$$

$$\frac{b^2}{ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \; 1479$$

3.80. $\displaystyle\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx$

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2e^3\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{\cdots}}\cdots\right.\right.}{\cdots}}{\cdots}}{2ae\left(a^2+\cdots\right.}-\frac{b^2}{ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$\Big\downarrow$ 25

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2e^3\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{\cdots}}\cdots\right.\right.}{\cdots}}{\cdots}}{2ae\left(a^2+b\cdots\right.}-\frac{b^2}{ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$\Big\downarrow$ 27

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2e^3\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{\cdots}}\cdots\right.\right.}{\cdots}}{\cdots}}{2ae\left(a^2+b\cdots\right.}-\frac{b^2}{ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$\Big\downarrow$ 1103

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2e^3\left(\frac{1}{2}\left(a^2+2ab-b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{\cdots}}\cdots\right.\right.}{\cdots}}{ae^3\cdots}}{2ae\left(a^2+b^2\right)}-\frac{b^2}{ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$\Big\downarrow$ 4117

3.80. $\displaystyle\int\frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx$

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^3(7a^2+3b^2)\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}-\frac{4a^2e^3\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\ldots\right)}{\ldots}\right.\right.}{\ldots}}{ae^3}}{2ae(a^2+b^2)}$$

$$\frac{b^2}{ade(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{-\frac{2b^3e^2(7a^2+3b^2)\int \frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d(a^2+b^2)}-\frac{4a^2e^3\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+\ldots}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right.\right.}{\ldots}}{ae^3}}{2ae(a^2+b^2)}$$

$$\frac{b^2}{ade(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2(2a^2+3b^2)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{2b^{5/2}e^{5/2}(7a^2+3b^2)\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}d(a^2+b^2)}-\frac{4a^2e^3\left(\frac{1}{2}(a^2+2ab-b^2)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\ldots\right.}{\ldots}}{ae^3}}{2ae(a^2+b^2)}$$

$$\frac{b^2}{ade(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}$$

input
```
Int[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^2),x]
```

output
```
-(b^2/(a*(a^2 + b^2)*d*e*Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x]))) + ((2
*(2*a^2 + 3*b^2))/(a*d*Sqrt[e*Cot[c + d*x]]) - ((2*b^(5/2)*(7*a^2 + 3*b^2)
*e^(5/2)*ArcTan[(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*(a^2 +
 b^2)*d) - (4*a^2*e^3*(((a^2 + 2*a*b - b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*
Cot[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Co
t[c + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 - ((a^2 - 2*a*b - b^2)*(-1/2*L
og[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqr
t[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*
Sqrt[2]*Sqrt[e])))/2))/((a^2 + b^2)*d))/(a*e^3))/(2*a*(a^2 + b^2)*e)
```

3.80. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx$

3.80.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.80. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx$

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.80.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{2e^3\left(-\frac{b^3\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{\left(7a^2+3b^2\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{a^2e^4\left(a^2+b^2\right)^2}+\frac{-ab\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot}}\right.\right.}{}\right.}{}$
default	$-\frac{2e^3\left(-\frac{b^3\left(\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)\sqrt{e\cot(dx+c)}}{e\cot(dx+c)b+ae}+\frac{\left(7a^2+3b^2\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{2\sqrt{aeb}}\right)}{a^2e^4\left(a^2+b^2\right)^2}+\frac{-ab\left(e^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{e\cot(dx+c)+\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot}}{e\cot(dx+c)-\left(e^2\right)^{\frac{1}{4}}\sqrt{e\cot}}\right.\right.}{}\right.}{}$

input
```
int(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^3*(-b^3/a^2/e^4/(a^2+b^2)^2*((1/2*a^2+1/2*b^2)*(e*cot(d*x+c))^(1/2)
/(e*cot(d*x+c)*b+a*e)+1/2*(7*a^2+3*b^2)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c)
)^(1/2)*b/(a*e*b)^(1/2)))+1/(a^2+b^2)^2/e^4*(-1/4*a/e*b*(e^2)^(1/4)*2^(1/2
)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/
(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arc
tan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1
/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-a^2+b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot
(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c
)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d
*x+c))^(1/2)+1)))-1/a^2/e^4/(e*cot(d*x+c))^(1/2))
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3239 vs. $2(372) = 744$.

Time = 0.72 (sec) , antiderivative size = 6519, normalized size of antiderivative = 14.92

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.80.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \int \frac{1}{(e\cot{(c+dx)})^{\frac{3}{2}}\,(a+b\cot{(c+dx)})^2}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**2,x)
```

output
```
Integral(1/((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**2), x)
```

3.80.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.80.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

output
```
Timed out
```

3.80.9 Mupad [B] (verification not implemented)

Time = 16.75 (sec) , antiderivative size = 15251, normalized size of antiderivative = 34.90

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))^2),x)
```

output
```
(2/a + (b*cot(c + d*x)*(2*a^2 + 3*b^2))/(a^2*(a^2 + b^2)))/(b*d*(e*cot(c +
 d*x))^(3/2) + a*d*e*(e*cot(c + d*x))^(1/2)) - atan((((e*cot(c + d*x))^(1/
2)*(144*a^14*b^23*d^5*e^13 + 1248*a^16*b^21*d^5*e^13 + 4224*a^18*b^19*d^5*
e^13 + 6720*a^20*b^17*d^5*e^13 + 3872*a^22*b^15*d^5*e^13 - 2816*a^24*b^13*
d^5*e^13 - 5632*a^26*b^11*d^5*e^13 - 3136*a^28*b^9*d^5*e^13 - 560*a^30*b^7
*d^5*e^13 + 32*a^32*b^5*d^5*e^13) + (1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*
b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i - 6*a^2*b^2*d^2*e^3)))^(1/2)*(26496*a^25
*b^14*d^6*e^15 - 1152*a^15*b^24*d^6*e^15 - 8448*a^17*b^22*d^6*e^15 - 23776
*a^19*b^20*d^6*e^15 - 29664*a^21*b^18*d^6*e^15 - 6528*a^23*b^16*d^6*e^15 -
 ((e*cot(c + d*x))^(1/2)*(1152*a^15*b^26*d^7*e^16 + 13440*a^17*b^24*d^7*e^
16 + 69056*a^19*b^22*d^7*e^16 + 202752*a^21*b^20*d^7*e^16 + 372800*a^23*b^
18*d^7*e^16 + 443136*a^25*b^16*d^7*e^16 + 337792*a^27*b^14*d^7*e^16 + 1561
60*a^29*b^12*d^7*e^16 + 37632*a^31*b^10*d^7*e^16 + 3200*a^33*b^8*d^7*e^16
+ 704*a^35*b^6*d^7*e^16 + 512*a^37*b^4*d^7*e^16 + 64*a^39*b^2*d^7*e^16) +
(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i -
6*a^2*b^2*d^2*e^3)))^(1/2)*(768*a^16*b^27*d^8*e^18 - (e*cot(c + d*x))^(1/2
)*(1i/(4*(a^4*d^2*e^3 + b^4*d^2*e^3 + a*b^3*d^2*e^3*4i - a^3*b*d^2*e^3*4i
- 6*a^2*b^2*d^2*e^3)))^(1/2)*(512*a^18*b^27*d^9*e^19 + 5120*a^20*b^25*d^9*
e^19 + 22528*a^22*b^23*d^9*e^19 + 56320*a^24*b^21*d^9*e^19 + 84480*a^26*b^
19*d^9*e^19 + 67584*a^28*b^17*d^9*e^19 - 67584*a^32*b^13*d^9*e^19 - 844...
```

3.80. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}\,dx$

3.81 $\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx$

3.81.1 Optimal result

Integrand size = 25, antiderivative size = 529

$$
\begin{aligned}
\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx &= \frac{a^{5/2}(15a^4+46a^2b^2+63b^4)\,e^{9/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{7/2}\,(a^2+b^2)^3\,d} \\
&+ \frac{(a-b)\,(a^2+4ab+b^2)\,e^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&- \frac{(a-b)\,(a^2+4ab+b^2)\,e^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&- \frac{(15a^4+31a^2b^2+8b^4)\,e^4\sqrt{e\cot(c+dx)}}{4b^3\,(a^2+b^2)^2\,d} \\
&+ \frac{a^2e^2(e\cot(c+dx))^{5/2}}{2b\,(a^2+b^2)\,d(a+b\cot(c+dx))^2} + \frac{a^2(5a^2+13b^2)\,e^3(e\cot(c+dx))^{3/2}}{4b^2\,(a^2+b^2)^2\,d(a+b\cot(c+dx))} \\
&- \frac{(a+b)\,(a^2-4ab+b^2)\,e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d} \\
&+ \frac{(a+b)\,(a^2-4ab+b^2)\,e^{9/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}
\end{aligned}
$$

output
```
1/4*a^(5/2)*(15*a^4+46*a^2*b^2+63*b^4)*e^(9/2)*arctan(b^(1/2)*(e*cot(d*x+c
))^(1/2)/a^(1/2)/e^(1/2))/b^(7/2)/(a^2+b^2)^3/d+1/2*a^2*e^2*(e*cot(d*x+c))
^(5/2)/b/(a^2+b^2)/d/(a+b*cot(d*x+c))^2+1/4*a^2*(5*a^2+13*b^2)*e^3*(e*cot(
d*x+c))^(3/2)/b^2/(a^2+b^2)^2/d/(a+b*cot(d*x+c))+1/2*(a-b)*(a^2+4*a*b+b^2)
*e^(9/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1
/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*e^(9/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2
)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*e^(9/2)*ln(e^(1
/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)
+1/4*(a+b)*(a^2-4*a*b+b^2)*e^(9/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(
e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(15*a^4+31*a^2*b^2+8*b^4)*e
^4*(e*cot(d*x+c))^(1/2)/b^3/(a^2+b^2)^2/d
```

3.81.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.32 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.18

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx =$$

$$-\frac{(e\cot(c+dx))^{9/2}\left(-\frac{2a^{9/2}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{b^{7/2}(a^2+b^2)^3}+\frac{2a^4(3a^2-b^2)\sqrt{\cot(c+dx)}}{b^3(a^2+b^2)^3}-\frac{2a^3(3a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3b^2(a^2+b^2)^3}+\frac{2a^2(3a^2-}{5b}\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(9/2)/(a + b*Cot[c + d*x])^3,x]
```

output
```
-(((e*Cot[c + d*x])^(9/2)*((-2*a^(9/2)*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[
Cot[c + d*x]])/Sqrt[a]])/(b^(7/2)*(a^2 + b^2)^3) + (2*a^4*(3*a^2 - b^2)*Sq
rt[Cot[c + d*x]])/(b^3*(a^2 + b^2)^3) - (2*a^3*(3*a^2 - b^2)*Cot[c + d*x]^
(3/2))/(3*b^2*(a^2 + b^2)^3) + (2*a^2*(3*a^2 - b^2)*Cot[c + d*x]^(5/2))/(5
*b*(a^2 + b^2)^3) - (2*a*(3*a^2 - b^2)*Cot[c + d*x]^(7/2))/(7*(a^2 + b^2)^
3) + (2*b*(3*a^2 - b^2)*Cot[c + d*x]^(9/2))/(9*(a^2 + b^2)^3) - (2*a*(a^2
- 3*b^2)*(7*Cot[c + d*x]^(3/2) - 3*Cot[c + d*x]^(7/2) - 7*Cot[c + d*x]^(3/
2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(21*(a^2 + b^2)^3) +
(4*b^2*Cot[c + d*x]^(11/2)*Hypergeometric2F1[2, 11/2, 13/2, -((b*Cot[c + d
*x])/a)])/(11*a*(a^2 + b^2)^2) + (2*b^2*Cot[c + d*x]^(11/2)*Hypergeometric
2F1[3, 11/2, 13/2, -((b*Cot[c + d*x])/a)])/(11*a^3*(a^2 + b^2)) - (b*(3*a^
2 - b^2)*(90*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 90*Sqrt[2]*A
rcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 360*Sqrt[Cot[c + d*x]] - 72*Cot[c
+ d*x]^(5/2) + 40*Cot[c + d*x]^(9/2) + 45*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot
[c + d*x]] + Cot[c + d*x]] - 45*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]]))/(180*(a^2 + b^2)^3)))/(d*Cot[c + d*x]^(9/2)))
```

3.81.3 Rubi [A] (warning: unable to verify)

Time = 2.85 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.93, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.080$, Rules used = {3042, 4048, 27, 3042, 4128, 27, 3042, 4130, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4048$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}-\frac{\int -\frac{(e\cot(c+dx))^{3/2}\left(5a^2e^3+\left(5a^2+4b^2\right)\cot^2(c+dx)e^3-4ab\cot(c+dx)e^3\right)}{2(a+b\cot(c+dx))^2}dx}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(e\cot(c+dx))^{3/2}\left(5a^2e^3+\left(5a^2+4b^2\right)\cot^2(c+dx)e^3-4ab\cot(c+dx)e^3\right)}{(a+b\cot(c+dx))^2}dx}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(5a^2e^3+\left(5a^2+4b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3+4ab\tan\left(c+dx+\frac{\pi}{2}\right)e^3\right)}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 4128

$$\frac{\frac{a^2e^3\left(5a^2+13b^2\right)(e\cot(c+dx))^{3/2}}{bd\left(a^2+b^2\right)(a+b\cot(c+dx))}-\frac{\int -\frac{\sqrt{e\cot(c+dx)}\left(\left(15a^4+31b^2a^2+8b^4\right)\cot^2(c+dx)e^4+3a^2\left(5a^2+13b^2\right)e^4-16ab^3\cot(c+dx)e^4\right)}{2(a+b\cot(c+dx))}dx}{b\left(a^2+b^2\right)}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\sqrt{e\cot(c+dx)}\left(\left(15a^4+31b^2a^2+8b^4\right)\cot^2(c+dx)e^4+3a^2\left(5a^2+13b^2\right)e^4-16ab^3\cot(c+dx)e^4\right)}{a+b\cot(c+dx)}dx}{2b\left(a^2+b^2\right)}+\frac{a^2e^3\left(5a^2+13b^2\right)(e\cot(c+dx))^{3/2}}{bd\left(a^2+b^2\right)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(\left(15a^4+31b^2a^2+8b^4\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^4+3a^2\left(5a^2+13b^2\right)e^4+16ab^3\tan\left(c+dx+\frac{\pi}{2}\right)e^4\right)}{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2+b^2\right)}+\frac{a^2e^3\left(5a^2+13b^2\right)(e\cot(c+dx))^{3/2}}{bd\left(a^2+b^2\right)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 4130

$$\frac{\frac{-\frac{2\int \frac{a\left(15a^4+31b^2a^2+24b^4\right)\cot^2(c+dx)e^5+a\left(15a^4+31b^2a^2+8b^4\right)e^5-8b^3\left(a^2-b^2\right)\cot(c+dx)e^5}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{b}-\frac{2e^4\left(15a^4+31a^2b^2+8b^4\right)\sqrt{e\cot(c+dx)}}{bd}}{2b\left(a^2+b^2\right)}+\frac{a^2e^3\left(5a^2+13\right.}{bd\left(a^2+b^2\right)}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 27

3.81. $\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx$

$$-\frac{\frac{\int \frac{a\left(15a^4+31b^2a^2+24b^4\right)\cot^2(c+dx)e^5+a\left(15a^4+31b^2a^2+8b^4\right)e^5-8b^3\left(a^2-b^2\right)\cot(c+dx)e^5}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{b}-\frac{2e^4\left(15a^4+31a^2b^2+8b^4\right)\sqrt{e\cot(c+dx)}}{bd}}{2b(a^2+b^2)}+\frac{a^2e^3\left(5a^2+13b^2 \cdots\right.}{bd(a^2+b^2)(\cdots}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{a\left(15a^4+31b^2a^2+24b^4\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^5+a\left(15a^4+31b^2a^2+8b^4\right)e^5+8b^3\left(a^2-b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)e^5}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2e^4\left(15a^4+31a^2b^2+8b^4\right)\sqrt{e\cot(c+dx)}}{bd}}{2b(a^2+b^2)}+\frac{a^2e^3\left(\cdots\right.}{bd(\cdots}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow \; 4136$$

$$-\frac{\frac{\frac{\int -\frac{8\left(b^4\left(3a^2-b^2\right)e^5+ab^3\left(a^2-3b^2\right)\cot(c+dx)e^5\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}+\frac{a^3e^5\left(15a^4+46a^2b^2+63b^4\right)\int\frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{b}-\frac{2e^4\left(15a^4+31a^2b^2+8b^4\right)\sqrt{e\cot(c+dx \cdots}}{bd}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{a^3e^5\left(15a^4+46a^2b^2+63b^4\right)\int\frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}-\frac{8\int\frac{b^4\left(3a^2-b^2\right)e^5+ab^3\left(a^2-3b^2\right)\cot(c+dx)e^5}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{b}-\frac{2e^4\left(15a^4+31a^2b^2+8b^4\right)\sqrt{e\cot(c+dx)}}{bd}}{2b(a^2+b^2)}+\cdots}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{a^3e^5\left(15a^4+46a^2b^2+63b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{8\int\frac{b^4\left(3a^2-b^2\right)e^5-ab^3\left(a^2-3b^2\right)e^5\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{b}-\frac{2e^4\left(15a^4+31a^2b^2+8b^4 \cdots\right.}{bd}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow \; 4017$$

3.81. $\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{\frac{a^3e^5\left(15a^4+46a^2b^2+63b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16\int -\frac{b^3e^5\left(b\left(3a^2-b^2\right)e+a\left(a^2-3b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}}{b}-2e^4\left(15a^4+3\right.}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 25

$$-\frac{\frac{\frac{\frac{16\int \frac{b^3e^5\left(b\left(3a^2-b^2\right)e+a\left(a^2-3b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a^3e^5\left(15a^4+46a^2b^2+63b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{b}-2e^4\left(15a^4+31a\right.}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\frac{16b^3e^5\int \frac{b\left(3a^2-b^2\right)e+a\left(a^2-3b^2\right)\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a^3e^5\left(15a^4+46a^2b^2+63b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{b}-2e^4\left(15a^4+31a^2b\right.}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1482

$$-\frac{\frac{\frac{\frac{16b^3e^5\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d\left(a^2+b^2\right)}+\frac{a^3e^5\left(15a^4+46a^2b^2+63b^4\right)\int \cdots}{\cdots}}{b}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1476

3.81. $\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx$

$$-\frac{\frac{16b^3e^5\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int\frac{1}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}\right)-\frac{1}{2}(a+b)\left(a^2-\right.}{d\left(a^2+b^2\right)}}{b}}{2b(}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1082

$$-\frac{\frac{16b^3e^5\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int\frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int\frac{e-e\cot}{\cot^2(c+d}\right.}{d\left(a^2+b^2\right)}}{b}}{2b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 217

$$-\frac{\frac{16b^3e^5\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d\left(a^2+b^2\right)}+}{b}}{2b\left(a^2+b^2\right)}$$

$4b\left(a^2\right.$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1479

$$-\frac{\frac{16b^3e^5\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx}}}{2\sqrt{2}\sqrt{e}}\right.\right.}{d\left(a^2+b^2\right)}}{b}}{}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 25

3.81. $\int\frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx$

$$-\frac{\frac{16b^3e^5\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{\cdots}}\cdots}{2\sqrt{2}\sqrt{e}}\cdots\right)\right)}{d\left(a^2+b^2\right)}\cdots}{b}\cdots$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 27

$$-\frac{\frac{16b^3e^5\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{\cdots}}\cdots}{2\sqrt{2}\sqrt{e}}\cdots\right)\right)}{d\left(a^2+b^2\right)}\cdots}{b}\cdots$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1103

$$-\frac{\frac{a^3e^5\left(15a^4+46a^2b^2+63b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{16b^3e^5\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cdots}}{\cdots}\right)}{\sqrt{2}\sqrt{\cdots}}\cdots\right)\cdots\right)}{\cdots}}{b}\cdots}{2\cdots}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 4117

$$-\frac{\frac{a^3e^5\left(15a^4+46a^2b^2+63b^4\right)\int\frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}+\frac{16b^3e^5\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cdots}}{\cdots}\right)}{\sqrt{2}\sqrt{\cdots}}\cdots\right)\cdots\right)}{\cdots}}{b}\cdots}{2\cdots}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 73

3.81. $\int\frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx$

$$-\frac{\frac{16b^3e^5\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}\right.}{2\sqrt{2}\sqrt{e}}\right.}{d\left(a^2+b^2\right)}}{b}}{2b(a^2+}$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\downarrow 218$$

$$\frac{a^2e^2(e\cot(c+dx))^{5/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}+$$

$$\frac{a^2e^3\left(5a^2+13b^2\right)(e\cot(c+dx))^{3/2}}{bd(a^2+b^2)(a+b\cot(c+dx))}+\frac{-\frac{2e^4\left(15a^4+31a^2b^2+8b^4\right)\sqrt{e\cot(c+dx)}}{bd}-\frac{16b^3e^5\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\right.}{}\right.\right.}{}}{}$$

input
```
Int[(e*Cot[c + d*x])^(9/2)/(a + b*Cot[c + d*x])^3,x]
```

output
```
(a^2*e^2*(e*Cot[c + d*x])^(5/2))/(2*b*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2
) + ((a^2*(5*a^2 + 13*b^2)*e^3*(e*Cot[c + d*x])^(3/2))/(b*(a^2 + b^2)*d*(a
 + b*Cot[c + d*x])) + ((-2*(15*a^4 + 31*a^2*b^2 + 8*b^4)*e^4*Sqrt[e*Cot[c
+ d*x]])/(b*d) - ((2*a^(5/2)*(15*a^4 + 46*a^2*b^2 + 63*b^4)*e^(9/2)*ArcTan
[(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt[b]*(a^2 + b^2)*d) + (16*
b^3*e^5*(((a - b)*(a^2 + 4*a*b + b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c
+ d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c +
d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 - ((a + b)*(a^2 - 4*a*b + b^2)*(-1/2
*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*S
qrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(
2*Sqrt[2]*Sqrt[e])))/2))/((a^2 + b^2)*d))/b)/(2*b*(a^2 + b^2)))/(4*b*(a^2
+ b^2))
```

3.81. $\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx$

3.81.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.81. $\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx$

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.81. $\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx$

3.81.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{2e^4\left(\frac{\sqrt{e\cot(dx+c)}}{b^3}-\frac{a^3e\left(\frac{\left(-\frac{9}{8}a^4b-\frac{13}{4}a^2b^3-\frac{17}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}-\frac{ae\left(7a^4+22a^2b^2+15b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(15a^4+46a^2b^2+\cdots\right.}{\cdots}\right)}{b^3\left(a^2+b^2\right)^3}\cdots\right)}{\cdots}$
default	$-\frac{2e^4\left(\frac{\sqrt{e\cot(dx+c)}}{b^3}-\frac{a^3e\left(\frac{\left(-\frac{9}{8}a^4b-\frac{13}{4}a^2b^3-\frac{17}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}-\frac{ae\left(7a^4+22a^2b^2+15b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(15a^4+46a^2b^2+\cdots\right.}{\cdots}\right)}{b^3\left(a^2+b^2\right)^3}\cdots\right)}{\cdots}$

input
```
int((e*cot(d*x+c))^(9/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^4*((e*cot(d*x+c))^(1/2)/b^3-a^3*e/b^3/(a^2+b^2)^3*(((-9/8*a^4*b-13/
4*a^2*b^3-17/8*b^5)*(e*cot(d*x+c))^(3/2)-1/8*a*e*(7*a^4+22*a^2*b^2+15*b^4)
*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)*b+a*e)^2+1/8*(15*a^4+46*a^2*b^2+63*b^
4)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2)))+e/(a^2+b^2)
^3*(1/8*(3*a^2*b*e-b^3*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^
(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*
(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e
*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
))+1/8*(a^3-3*a*b^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*
cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*
x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+
c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))
```

3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4519 vs. 2(454) = 908.

Time = 2.08 (sec) , antiderivative size = 9101, normalized size of antiderivative = 17.20

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(9/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))**(9/2)/(a+b*cot(d*x+c))**3,x)
```

output
```
Timed out
```

3.81.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(9/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.81.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx=\int \frac{(e\cot(dx+c))^{\frac{9}{2}}}{(b\cot(dx+c)+a)^3}\,dx$$

input
```
integrate((e*cot(d*x+c))^(9/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(9/2)/(b*cot(d*x + c) + a)^3, x)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 23.64 (sec) , antiderivative size = 20651, normalized size of antiderivative = 39.04

$$\int \frac{(e\cot(c+dx))^{9/2}}{(a+b\cot(c+dx))^3}\,dx=\text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(9/2)/(a + b*cot(c + d*x))^3,x)
```

output
```
atan(((((((128*a*b^26*d^4*e^15 + 3648*a^3*b^24*d^4*e^15 + 25536*a^5*b^22*d
^4*e^15 + 88320*a^7*b^20*d^4*e^15 + 182784*a^9*b^18*d^4*e^15 + 244608*a^11
*b^16*d^4*e^15 + 217728*a^13*b^14*d^4*e^15 + 128256*a^15*b^12*d^4*e^15 + 4
8000*a^17*b^10*d^4*e^15 + 10304*a^19*b^8*d^4*e^15 + 960*a^21*b^6*d^4*e^15)
/(b^21*d^5 + 8*a^2*b^19*d^5 + 28*a^4*b^17*d^5 + 56*a^6*b^15*d^5 + 70*a^8*b
^13*d^5 + 56*a^10*b^11*d^5 + 28*a^12*b^9*d^5 + 8*a^14*b^7*d^5 + a^16*b^5*d
^5) + ((e*cot(c + d*x))^(1/2)*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2
*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^
(1/2)*(512*b^30*d^4*e^10 + 4608*a^2*b^28*d^4*e^10 + 17920*a^4*b^26*d^4*e^1
0 + 38400*a^6*b^24*d^4*e^10 + 46080*a^8*b^22*d^4*e^10 + 21504*a^10*b^20*d^
4*e^10 - 21504*a^12*b^18*d^4*e^10 - 46080*a^14*b^16*d^4*e^10 - 38400*a^16*
b^14*d^4*e^10 - 17920*a^18*b^12*d^4*e^10 - 4608*a^20*b^10*d^4*e^10 - 512*a
^22*b^8*d^4*e^10))/(b^21*d^4 + 8*a^2*b^19*d^4 + 28*a^4*b^17*d^4 + 56*a^6*b
^15*d^4 + 70*a^8*b^13*d^4 + 56*a^10*b^11*d^4 + 28*a^12*b^9*d^4 + 8*a^14*b^
7*d^4 + a^16*b^5*d^4))*(-(e^9*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i + a
^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(1/2) -
 ((e*cot(c + d*x))^(1/2)*(1800*a^23*b*d^2*e^19 - 1472*a*b^23*d^2*e^19 - 10
24*a^3*b^21*d^2*e^19 + 8448*a^5*b^19*d^2*e^19 + 46088*a^7*b^17*d^2*e^19 +
177344*a^9*b^15*d^2*e^19 + 402912*a^11*b^13*d^2*e^19 + 541632*a^13*b^11*d^
2*e^19 + 455472*a^15*b^9*d^2*e^19 + 248064*a^17*b^7*d^2*e^19 + 87008*a^...
```

3.82 $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx$

3.82.1 Optimal result

Integrand size = 25, antiderivative size = 476

$$\begin{aligned}\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx &= -\frac{a^{3/2}(3a^4+6a^2b^2+35b^4)\,e^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{5/2}\left(a^2+b^2\right)^3 d}\\
&+\frac{(a+b)\left(a^2-4ab+b^2\right)e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}\\
&-\frac{(a+b)\left(a^2-4ab+b^2\right)e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}\\
&+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2b\left(a^2+b^2\right)d(a+b\cot(c+dx))^2}+\frac{a^2(3a^2+11b^2)\,e^3\sqrt{e\cot(c+dx)}}{4b^2\left(a^2+b^2\right)^2 d(a+b\cot(c+dx))}\\
&+\frac{(a-b)\left(a^2+4ab+b^2\right)e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d}\\
&-\frac{(a-b)\left(a^2+4ab+b^2\right)e^{7/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 d}\end{aligned}$$

output
```
-1/4*a^(3/2)*(3*a^4+6*a^2*b^2+35*b^4)*e^(7/2)*arctan(b^(1/2)*(e*cot(d*x+c)
)^(1/2)/a^(1/2)/e^(1/2))/b^(5/2)/(a^2+b^2)^3/d+1/2*a^2*e^2*(e*cot(d*x+c))^
(3/2)/b/(a^2+b^2)/d/(a+b*cot(d*x+c))^2+1/2*(a+b)*(a^2-4*a*b+b^2)*e^(7/2)*a
rctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/2*(a
+b)*(a^2-4*a*b+b^2)*e^(7/2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))
/(a^2+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*e^(7/2)*ln(e^(1/2)+cot(d*
x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)
*(a^2+4*a*b+b^2)*e^(7/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+
c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*a^2*(3*a^2+11*b^2)*e^3*(e*cot(d*x+c))
^(1/2)/b^2/(a^2+b^2)^2/d/(a+b*cot(d*x+c))
```

3.82.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.25 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.21

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx =$$

$$-\frac{(e\cot(c+dx))^{7/2}\left(\frac{2a^{7/2}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}(a^2+b^2)^3}-\frac{2a^3(3a^2-b^2)\sqrt{\cot(c+dx)}}{b^2(a^2+b^2)^3}+\frac{2a^2(3a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3b(a^2+b^2)^3}-\frac{2a(3a^2-b^2)}{5(a^2}\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(7/2)/(a + b*Cot[c + d*x])^3,x]
```

output
```
-(((e*Cot[c + d*x])^(7/2)*((2*a^(7/2)*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[C
ot[c + d*x]])/Sqrt[a]])/(b^(5/2)*(a^2 + b^2)^3) - (2*a^3*(3*a^2 - b^2)*Sqr
t[Cot[c + d*x]])/(b^2*(a^2 + b^2)^3) + (2*a^2*(3*a^2 - b^2)*Cot[c + d*x]^(
3/2))/(3*b*(a^2 + b^2)^3) - (2*a*(3*a^2 - b^2)*Cot[c + d*x]^(5/2))/(5*(a^2
 + b^2)^3) + (2*b*(3*a^2 - b^2)*Cot[c + d*x]^(7/2))/(7*(a^2 + b^2)^3) + (2
*b*(3*a^2 - b^2)*(7*Cot[c + d*x]^(3/2) - 3*Cot[c + d*x]^(7/2) - 7*Cot[c +
d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(21*(a^2 + b^
2)^3) + (4*b^2*Cot[c + d*x]^(9/2)*Hypergeometric2F1[2, 9/2, 11/2, -((b*Cot
[c + d*x])/a)])/(9*a*(a^2 + b^2)^2) + (2*b^2*Cot[c + d*x]^(9/2)*Hypergeome
tric2F1[3, 9/2, 11/2, -((b*Cot[c + d*x])/a)])/(9*a^3*(a^2 + b^2)) - (a*(a^
2 - 3*b^2)*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 10*Sqrt[2]
*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 40*Sqrt[Cot[c + d*x]] - 8*Cot[c
+ d*x]^(5/2) + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]
] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(20*(a^
2 + b^2)^3)))/(d*Cot[c + d*x]^(7/2)))
```

3.82.3 Rubi [A] (warning: unable to verify)

Time = 2.13 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.94, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4048, 27, 3042, 4128, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \ 4048$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}-\frac{\int -\frac{\sqrt{e\cot(c+dx)}\left(3a^2e^3+\left(3a^2+4b^2\right)\cot^2(c+dx)e^3-4ab\cot(c+dx)e^3\right)}{2(a+b\cot(c+dx))^2}dx}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{e\cot(c+dx)}\left(3a^2e^3+\left(3a^2+4b^2\right)\cot^2(c+dx)e^3-4ab\cot(c+dx)e^3\right)}{(a+b\cot(c+dx))^2}dx}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(3a^2e^3+\left(3a^2+4b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3+4ab\tan\left(c+dx+\frac{\pi}{2}\right)e^3\right)}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \ 4128$$

$$\frac{\frac{a^2e^3\left(3a^2+11b^2\right)\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}-\frac{\int -\frac{\left(3a^4+3b^2a^2+8b^4\right)\cot^2(c+dx)e^4+a^2\left(3a^2+11b^2\right)e^4-16ab^3\cot(c+dx)e^4}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{b(a^2+b^2)}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\left(3a^4+3b^2a^2+8b^4\right)\cot^2(c+dx)e^4+a^2\left(3a^2+11b^2\right)e^4-16ab^3\cot(c+dx)e^4}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{2b(a^2+b^2)}+\frac{a^2e^3\left(3a^2+11b^2\right)\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(3a^4+3b^2a^2+8b^4\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^4+a^2\left(3a^2+11b^2\right)e^4+16ab^3\tan\left(c+dx+\frac{\pi}{2}\right)e^4}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2+b^2)}+\frac{a^2e^3\left(3a^2+11b^2\right)\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 4136

$$\frac{\frac{\frac{\int \frac{8\left(ab^2\left(a^2-3b^2\right)e^4-b^3\left(3a^2-b^2\right)e^4\cot(c+dx)\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}+\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{2b(a^2+b^2)}+\frac{a^2e^3\left(3a^2+11b^2\right)\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{8\int \frac{ab^2\left(a^2-3b^2\right)e^4-b^3\left(3a^2-b^2\right)e^4\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}+\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{2b(a^2+b^2)}+\frac{a^2e^3\left(3a^2+11b^2\right)\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{8\int \frac{ab^2\left(a^2-3b^2\right)e^4+b^3\left(3a^2-b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)e^4}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}+\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2b(a^2+b^2)}+\frac{a^2e^3\left(3a^2+11b^2\right)\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 4017

3.82. $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{16\int -\frac{b^2e^4\left(a\left(a^2-3b^2\right)e-b\left(3a^2-b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2b(a^2+b^2)}+\frac{a^2e^3(3a^2+11\ldots}{bd(a^2+b^2)(\ldots}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 25

$$\frac{\frac{\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16\int \frac{b^2e^4\left(a\left(a^2-3b^2\right)e-b\left(3a^2-b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}}{2b(a^2+b^2)}+\frac{a^2e^3(3a^2+11b^2\ldots}{bd(a^2+b^2)(a\ldots}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16b^2e^4\int \frac{a\left(a^2-3b^2\right)e-b\left(3a^2-b^2\right)e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}}{2b(a^2+b^2)}+\frac{a^2e^3(3a^2+11b^2)\ldots}{bd(a^2+b^2)(a+b\ldots}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1482

$$\frac{\frac{\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16b^2e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(a^2-4ab+\ldots\right.\right.}{d\left(a^2+b^2\right)}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1476

3.82. $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16b^2e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(a^2-4ab+\right.\right.}{}}{2b(a^2+b^2)}}{4b\left(a^2\right.}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1082

$$\frac{\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16b^2e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(a^2-4ab+\right.\right.}{}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 217

$$\frac{\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16b^2e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(a^2-4ab+\right.\right.}{d\left(a^2+b^2\right)}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1479

$$\frac{\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16b^2e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+\right.}{2\sqrt{2}\sqrt{e}}\right.\right.}{}}{2b(a^2+b^2}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 25

3.82. $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16b^2e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right.\right.}{2b(a^2+b^2)}}{}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16b^2e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right.\right.}{2b(a^2+b^2)}}{}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1103

$$\frac{\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16b^2e^4\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{2b(a^2+b^2)}}{4b\,(a}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 4117

$$\frac{\frac{a^2e^4\left(3a^4+6a^2b^2+35b^4\right)\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}-\frac{16b^2e^4\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{2b(a^2+b^2)}}{4b\,(a}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 73

3.82. $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx$

$$-\frac{-\frac{2a^2e^3\left(3a^4+6a^2b^2+35b^4\right)\int \frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{16b^2e^4\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right.}{2b(a^2+b^2)}}{4b\,(a^2+}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\,(a^2+b^2)\,(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{a^2e^2(e\cot(c+dx))^{3/2}}{2bd\,(a^2+b^2)\,(a+b\cot(c+dx))^2}+$$

$$\frac{\frac{a^2e^3(3a^2+11b^2)\sqrt{e\cot(c+dx)}}{bd(a^2+b^2)(a+b\cot(c+dx))}+\frac{\frac{2a^{3/2}e^{7/2}\left(3a^4+6a^2b^2+35b^4\right)\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{b}d\left(a^2+b^2\right)}-\frac{16b^2e^4\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\right.\right.}{}}{}}{4b\,(a^2+b^2)}$$

input
```
Int[(e*Cot[c + d*x])^(7/2)/(a + b*Cot[c + d*x])^3,x]
```

output
```
(a^2*e^2*(e*Cot[c + d*x])^(3/2))/(2*b*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2
) + ((a^2*(3*a^2 + 11*b^2)*e^3*Sqrt[e*Cot[c + d*x]])/(b*(a^2 + b^2)*d*(a +
 b*Cot[c + d*x])) + ((2*a^(3/2)*(3*a^4 + 6*a^2*b^2 + 35*b^4)*e^(7/2)*ArcTa
n[(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt[b]*(a^2 + b^2)*d) - (16
*b^2*e^4*(((a + b)*(a^2 - 4*a*b + b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c
 + d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c +
 d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 + ((a - b)*(a^2 + 4*a*b + b^2)*(-1/
2*Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*
Sqrt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/
(2*Sqrt[2]*Sqrt[e])))/2))/((a^2 + b^2)*d))/(2*b*(a^2 + b^2)))/(4*b*(a^2 +
b^2))
```

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.82. $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx$

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.82. $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx$

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.82.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{a^2\left(\frac{\frac{\left(5a^4+18a^2b^2+13b^4\right)(e\cot(dx+c))^{\frac{3}{2}}}{8b}+\frac{ae\left(3a^4+14a^2b^2+11b^4\right)\sqrt{e\cot(dx+c)}}{8b^2}}{(e\cot(dx+c)b+ae)^2}-\frac{\left(3a^4+6a^2b^2+35b^4\right)\arctan\left(\frac{\sqrt{e\cot(}}{\sqrt{a}}\right.}{8b^2\sqrt{aeb}}\right.}{\left(a^2+b^2\right)^3}\right.}{}$
default	$-\frac{2e^4\left(-\frac{a^2\left(\frac{\frac{\left(5a^4+18a^2b^2+13b^4\right)(e\cot(dx+c))^{\frac{3}{2}}}{8b}+\frac{ae\left(3a^4+14a^2b^2+11b^4\right)\sqrt{e\cot(dx+c)}}{8b^2}}{(e\cot(dx+c)b+ae)^2}-\frac{\left(3a^4+6a^2b^2+35b^4\right)\arctan\left(\frac{\sqrt{e\cot(}}{\sqrt{a}}\right.}{8b^2\sqrt{aeb}}\right.}{\left(a^2+b^2\right)^3}\right.}{}$

input
```
int((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^4*(-a^2/(a^2+b^2)^3*((1/8*(5*a^4+18*a^2*b^2+13*b^4)/b*(e*cot(d*x+c)
)^(3/2)+1/8*a*e*(3*a^4+14*a^2*b^2+11*b^4)/b^2*(e*cot(d*x+c))^(1/2))/(e*cot
(d*x+c)*b+a*e)^2-1/8*(3*a^4+6*a^2*b^2+35*b^4)/b^2/(a*e*b)^(1/2)*arctan((e*
cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2)))+1/(a^2+b^2)^3*(1/8*(a^3*e-3*a*b^2*e)*(
e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*
2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2
)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arc
tan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-3*a^2*b+b^3)/(e^2)
^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(
1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4483 vs. $2(405) = 810$.

Time = 63.17 (sec) , antiderivative size = 9029, normalized size of antiderivative = 18.97

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.82.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))**(7/2)/(a+b*cot(d*x+c))**3,x)
```

output
```
Timed out
```

3.82.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.82.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{(e\cot(dx+c))^{\frac{7}{2}}}{(b\cot(dx+c)+a)^3}\,dx$$

input
```
integrate((e*cot(d*x+c))^(7/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(7/2)/(b*cot(d*x + c) + a)^3, x)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 20.49 (sec) , antiderivative size = 20089, normalized size of antiderivative = 42.20

$$\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(7/2)/(a + b*cot(c + d*x))^3,x)
```

output
```
(((e*cot(c + d*x))^(1/2)*(3*a^5*e^5 + 11*a^3*b^2*e^5))/(4*b^2*(a^4 + b^4 +
 2*a^2*b^2)) + (e^4*(e*cot(c + d*x))^(3/2)*(5*a^4 + 13*a^2*b^2))/(4*b*(a^4
 + b^4 + 2*a^2*b^2)))/(a^2*d*e^2 + b^2*d*e^2*cot(c + d*x)^2 + 2*a*b*d*e^2*
cot(c + d*x)) - atan(((((32*a*b^18*d^2*e^21 - 18*a^19*d^2*e^21 - 6528*a^3*
b^16*d^2*e^21 + 2758*a^5*b^14*d^2*e^21 + 26482*a^7*b^12*d^2*e^21 + 21582*a
^9*b^10*d^2*e^21 + 7594*a^11*b^8*d^2*e^21 + 3314*a^13*b^6*d^2*e^21 + 246*a
^15*b^4*d^2*e^21 + 90*a^17*b^2*d^2*e^21)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a
^4*b^15*d^5 + 56*a^6*b^13*d^5 + 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^1
2*b^7*d^5 + 8*a^14*b^5*d^5 + a^16*b^3*d^5) + (((1600*a^2*b^23*d^4*e^14 + 1
2864*a^4*b^21*d^4*e^14 + 45312*a^6*b^19*d^4*e^14 + 91392*a^8*b^17*d^4*e^14
 + 115584*a^10*b^15*d^4*e^14 + 94080*a^12*b^13*d^4*e^14 + 48384*a^14*b^11*
d^4*e^14 + 14592*a^16*b^9*d^4*e^14 + 2112*a^18*b^7*d^4*e^14 + 64*a^20*b^5*
d^4*e^14)/(b^19*d^5 + 8*a^2*b^17*d^5 + 28*a^4*b^15*d^5 + 56*a^6*b^13*d^5 +
 70*a^8*b^11*d^5 + 56*a^10*b^9*d^5 + 28*a^12*b^7*d^5 + 8*a^14*b^5*d^5 + a^
16*b^3*d^5) + ((e*cot(c + d*x))^(1/2)*((e^7*1i)/(4*(b^6*d^2 - a^6*d^2 + a*
b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*
d^2)))^(1/2)*(512*b^28*d^4*e^10 + 4608*a^2*b^26*d^4*e^10 + 17920*a^4*b^24*
d^4*e^10 + 38400*a^6*b^22*d^4*e^10 + 46080*a^8*b^20*d^4*e^10 + 21504*a^10*
b^18*d^4*e^10 - 21504*a^12*b^16*d^4*e^10 - 46080*a^14*b^14*d^4*e^10 - 3840
0*a^16*b^12*d^4*e^10 - 17920*a^18*b^10*d^4*e^10 - 4608*a^20*b^8*d^4*e^1...
```

3.82. $\int \frac{(e\cot(c+dx))^{7/2}}{(a+b\cot(c+dx))^3}\,dx$

3.83 $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx$

3.83.1 Optimal result

Integrand size = 25, antiderivative size = 470

$$\begin{aligned}\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx =& -\frac{\sqrt{a}(a^4+18a^2b^2-15b^4)\,e^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4b^{3/2}\,(a^2+b^2)^3\,d}\\
&-\frac{(a-b)\,(a^2+4ab+b^2)\,e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}\\
&+\frac{(a-b)\,(a^2+4ab+b^2)\,e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}\\
&+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2b\,(a^2+b^2)\,d(a+b\cot(c+dx))^2}-\frac{a(a^2+9b^2)\,e^2\sqrt{e\cot(c+dx)}}{4b\,(a^2+b^2)^2\,d(a+b\cot(c+dx))}\\
&+\frac{(a+b)\,(a^2-4ab+b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}\\
&-\frac{(a+b)\,(a^2-4ab+b^2)\,e^{5/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}\end{aligned}$$

output
```
-1/2*(a-b)*(a^2+4*a*b+b^2)*e^(5/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e
^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/2*(a-b)*(a^2+4*a*b+b^2)*e^(5/2)*arctan(1+2
^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-
4*a*b+b^2)*e^(5/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1
/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4*a*b+b^2)*e^(5/2)*ln(e^(1/2)+co
t(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(
a^4+18*a^2*b^2-15*b^4)*e^(5/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)
/e^(1/2))*a^(1/2)/b^(3/2)/(a^2+b^2)^3/d+1/2*a^2*e^2*(e*cot(d*x+c))^(1/2)/b
/(a^2+b^2)/d/(a+b*cot(d*x+c))^2-1/4*a*(a^2+9*b^2)*e^2*(e*cot(d*x+c))^(1/2)
/b/(a^2+b^2)^2/d/(a+b*cot(d*x+c))
```

3.83.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.23 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.11

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx =$$

$$-\frac{(e\cot(c+dx))^{5/2}\left(-\frac{2a^{5/2}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}(a^2+b^2)^3}+\frac{2a^2(3a^2-b^2)\sqrt{\cot(c+dx)}}{b(a^2+b^2)^3}-\frac{2a(3a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3(a^2+b^2)^3}+\frac{2b(3a^2-b^2}{5(a}\right.}{}$$

input
```
Integrate[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^3,x]
```

output
```
-(((e*Cot[c + d*x])^(5/2)*((-2*a^(5/2)*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[
Cot[c + d*x]])/Sqrt[a]])/(b^(3/2)*(a^2 + b^2)^3) + (2*a^2*(3*a^2 - b^2)*Sq
rt[Cot[c + d*x]])/(b*(a^2 + b^2)^3) - (2*a*(3*a^2 - b^2)*Cot[c + d*x]^(3/2
))/(3*(a^2 + b^2)^3) + (2*b*(3*a^2 - b^2)*Cot[c + d*x]^(5/2))/(5*(a^2 + b^
2)^3) + (2*a*(a^2 - 3*b^2)*(Cot[c + d*x]^(3/2) - Cot[c + d*x]^(3/2)*Hyperg
eometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(3*(a^2 + b^2)^3) + (4*b^2*Cot
[c + d*x]^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*Cot[c + d*x])/a)])/(7*
a*(a^2 + b^2)^2) + (2*b^2*Cot[c + d*x]^(7/2)*Hypergeometric2F1[3, 7/2, 9/2
, -((b*Cot[c + d*x])/a)])/(7*a^3*(a^2 + b^2)) + (b*(3*a^2 - b^2)*(10*Sqrt[
2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*
Sqrt[Cot[c + d*x]]] + 40*Sqrt[Cot[c + d*x]] - 8*Cot[c + d*x]^(5/2) + 5*Sqr
t[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - 5*Sqrt[2]*Log[1
+ Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(20*(a^2 + b^2)^3)))/(d*Cot
[c + d*x]^(5/2)))
```

3.83.3 Rubi [A] (warning: unable to verify)

Time = 2.13 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.94, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.960, Rules used = {3042, 4048, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4048}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}-\frac{\int -\frac{a^2e^3+\left(a^2+4b^2\right)\cot^2(c+dx)e^3-4ab\cot(c+dx)e^3}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}dx}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2e^3+\left(a^2+4b^2\right)\cot^2(c+dx)e^3-4ab\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}dx}{4b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2e^3+\left(a^2+4b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3+4ab\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{4132}$$

$$\frac{-\frac{\int -\frac{a^2\left(a^2+9b^2\right)\cot^2(c+dx)e^4+a^2\left(a^2-7b^2\right)e^4-8ab\left(a^2-b^2\right)\cot(c+dx)e^4}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae\left(a^2+b^2\right)}-\frac{ae^2\left(a^2+9b^2\right)\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a^2\left(a^2+9b^2\right)\cot^2(c+dx)e^4+a^2\left(a^2-7b^2\right)e^4-8ab\left(a^2-b^2\right)\cot(c+dx)e^4}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{2ae(a^2+b^2)}-\frac{ae^2\left(a^2+9b^2\right)\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{a^2\left(a^2+9b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^4+a^2\left(a^2-7b^2\right)e^4+8ab\left(a^2-b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)e^4}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2ae(a^2+b^2)}-\frac{ae^2\left(a^2+9b^2\right)\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 4136$$

$$\frac{\frac{\frac{\int -\frac{8\left(ab^2\left(3a^2-b^2\right)e^4+a^2b\left(a^2-3b^2\right)\cot(c+dx)e^4\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}+\frac{a^2e^4\left(a^4+18a^2b^2-15b^4\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{ae^2\left(a^2+9b^2\right)\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{a^2e^4\left(a^4+18a^2b^2-15b^4\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}-\frac{8\int \frac{ab^2\left(3a^2-b^2\right)e^4+a^2b\left(a^2-3b^2\right)\cot(c+dx)e^4}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{ae^2\left(a^2+9b^2\right)\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{a^2e^4\left(a^4+18a^2b^2-15b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{8\int \frac{ab^2\left(3a^2-b^2\right)e^4-a^2b\left(a^2-3b^2\right)e^4\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{ae^2\left(a^2+9b^2\right)\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}}{4b\left(a^2+b^2\right)}+\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 4017$$

3.83. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{a^2e^4\left(a^4+18a^2b^2-15b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16\int -\frac{abe^4\left(b\left(3a^2-b^2\right)e+a\left(a^2-3b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}}{2ae(a^2+b^2)}-\frac{ae^2\left(a^2+9b^2\right)}{d(a^2+b^2)(a+}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{16\int \frac{abe^4\left(b\left(3a^2-b^2\right)e+a\left(a^2-3b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a^2e^4\left(a^4+18a^2b^2-15b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{ae^2\left(a^2+9b^2\right)\sqrt{}}{d(a^2+b^2)(a+b}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{16abe^4\int \frac{b\left(3a^2-b^2\right)e+a\left(a^2-3b^2\right)\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a^2e^4\left(a^4+18a^2b^2-15b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{ae^2\left(a^2+9b^2\right)\sqrt{e}}{d(a^2+b^2)(a+b\cot}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\big\downarrow$ 1482

$$\frac{\frac{\frac{16abe^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d\left(a^2+b^2\right)}+\frac{a^2e^4\left(a^4+18a^2b^2-15b^4\right)\int \frac{}{\sqrt{-e}}}{}}{2ae(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\big\downarrow$ 1476

3.83. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx$

$$16abe^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}\right)-\frac{1}{2}(a+b)\left(a^2-4a\right.\right.$$

$$d\left(a^2+b^2\right)$$

$$2ae(a^2+b^2)$$

$$4b\left(a^2+\right.$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 1082$$

$$16abe^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int \frac{e-e\cot(c\ }{\cot^2(c+dx)\right.$$

$$d\left(a^2+b^2\right)$$

$$2ae(a^2+b^2)$$

$$4b\left(a^2+b^2\right)$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 217$$

$$16abe^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right) \quad a$$

$$d\left(a^2+b^2\right) \quad +-$$

$$2ae(a^2+b^2)$$

$$4b\left(a^2+b^2\right)$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 1479$$

$$16abe^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{}}}{2\sqrt{2}\sqrt{e}}\right.\right.$$

$$d\left(a^2+b^2\right)$$

$$2ae(a^2+b$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 25$$

3.83. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{16abe^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d}{2\sqrt{2}\sqrt{e}}\right.\right.}{d\left(a^2+b^2\right)}}{2ae(a^2+b^2)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{16abe^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d}{2\sqrt{2}\sqrt{e}}\right.\right.}{d\left(a^2+b^2\right)}}{2ae(a^2+b^2)}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1103

$$\frac{\frac{\frac{a^2e^4\left(a^4+18a^2b^2-15b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{16abe^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{}}{2ae(a^2+b^2)}}{4b\left(a^2\right.}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 4117

$$\frac{\frac{\frac{a^2e^4\left(a^4+18a^2b^2-15b^4\right)\int\frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}+\frac{16abe^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{}}{2ae(a^2+b^2)}}{4b\left(a^2\right.}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 73

3.83. $\int\frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{16abe^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+\cdots\right.}{2\sqrt{2}\sqrt{e}}\cdots\right.}{d\left(a^2+b^2\right)}\cdots}{2ae(a^2+b^2)}\cdots}{4b\left(a^2+b^2\cdots\right.}$$

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\big\downarrow$ 218

$$\frac{a^2e^2\sqrt{e\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\cot(c+dx))^2}+$$

$$\frac{\frac{\frac{16abe^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\log\left(e\cot(c+dx)+\sqrt{2}\sqrt{e}\sqrt{e\cot(c+dx)}+\cdots\right.}{2\sqrt{2}\sqrt{e}}\cdots\right.}{d\left(a^2+b^2\right)}\cdots}{2ae(a^2+b^2)}\cdots}{4b\left(a^2+b^2\right)}$$

input
```
Int[(e*Cot[c + d*x])^(5/2)/(a + b*Cot[c + d*x])^3,x]
```

output
```
(a^2*e^2*Sqrt[e*Cot[c + d*x]])/(2*b*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2)
+ (-((a*(a^2 + 9*b^2)*e^2*Sqrt[e*Cot[c + d*x]])/((a^2 + b^2)*d*(a + b*Cot[
c + d*x]))) + ((2*a^(3/2)*(a^4 + 18*a^2*b^2 - 15*b^4)*e^(7/2)*ArcTan[(Sqrt
[b]*Cot[c + d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt[b]*(a^2 + b^2)*d) + (16*a*b*e^
4*(((a - b)*(a^2 + 4*a*b + b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]
])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])
/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 - ((a + b)*(a^2 - 4*a*b + b^2)*(-1/2*Log[e
 + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]
) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt
[2]*Sqrt[e])))/2))/((a^2 + b^2)*d))/(2*a*(a^2 + b^2)*e))/(4*b*(a^2 + b^2))
```

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.83. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx$

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4048
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)^2*(a + b*Tan[e + f*x])^(m
 - 2)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1
/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e +
f*x])^(n + 1)*Simp[a^2*d*(b*d*(m - 2) - a*c*(n + 1)) + b*(b*c - 2*a*d)*(b*c
*(m - 2) + a*d*(n + 1)) - d*(n + 1)*(3*a^2*b*c - b^3*c - a^3*d + 3*a*b^2*d)
*Tan[e + f*x] - b*(a*d*(2*b*c - a*d)*(m + n - 1) - b^2*(c^2*(m - 2) - d^2*(
n + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 2] && LtQ
[n, -1] && IntegerQ[2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.83. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx$

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.83.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{2e^4\left(\frac{a\left(\frac{\left(\frac{1}{8}a^4+\frac{5}{4}a^2b^2+\frac{9}{8}b^4\right)(e\cot(dx+c))^{\frac{3}{2}}-\frac{ae\left(a^4-6a^2b^2-7b^4\right)\sqrt{e\cot(dx+c)}}{8b}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(a^4+18a^2b^2-15b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{8b\sqrt{aeb}}\right.}{\left(a^2+b^2\right)^3e}\right.}{}$
default	$-\frac{2e^4\left(\frac{a\left(\frac{\left(\frac{1}{8}a^4+\frac{5}{4}a^2b^2+\frac{9}{8}b^4\right)(e\cot(dx+c))^{\frac{3}{2}}-\frac{ae\left(a^4-6a^2b^2-7b^4\right)\sqrt{e\cot(dx+c)}}{8b}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(a^4+18a^2b^2-15b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{8b\sqrt{aeb}}\right.}{\left(a^2+b^2\right)^3e}\right.}{}$

input
```
int((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^4*(a/(a^2+b^2)^3/e*(((1/8*a^4+5/4*a^2*b^2+9/8*b^4)*(e*cot(d*x+c))^(
3/2)-1/8*a*e*(a^4-6*a^2*b^2-7*b^4)/b*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)*b
+a*e)^2+1/8*(a^4+18*a^2*b^2-15*b^4)/b/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^
(1/2)*b/(a*e*b)^(1/2)))+1/e/(a^2+b^2)^3*(1/8*(-3*a^2*b*e+b^3*e)*(e^2)^(1/4
)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(
e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(
1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1
/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-a^3+3*a*b^2)/(e^2)^(1/4)*2^
(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2
*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4446 vs. $2(399) = 798$.

Time = 39.25 (sec) , antiderivative size = 8955, normalized size of antiderivative = 19.05

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*cot(d*x+c))**(5/2)/(a+b*cot(d*x+c))**3,x)
```

output
```
Timed out
```

3.83.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.83.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{(e\cot(dx+c))^{\frac{5}{2}}}{(b\cot(dx+c)+a)^3}\,dx$$

input
```
integrate((e*cot(d*x+c))^(5/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(5/2)/(b*cot(d*x + c) + a)^3, x)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 19.08 (sec) , antiderivative size = 19256, normalized size of antiderivative = 40.97

$$\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(5/2)/(a + b*cot(c + d*x))^3,x)
```

output
```
atan((((10*a^16*b*d^2*e^18 - 2398*a^2*b^15*d^2*e^18 + 5238*a^4*b^13*d^2*e
^18 + 7386*a^6*b^11*d^2*e^18 - 8322*a^8*b^9*d^2*e^18 - 5498*a^10*b^7*d^2*e
^18 + 2946*a^12*b^5*d^2*e^18 + 382*a^14*b^3*d^2*e^18)/(b^17*d^5 + a^16*b*d
^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 + 56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 +
 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a^14*b^3*d^5) - (((832*a*b^22*d^4*e
^13 + 5952*a^3*b^20*d^4*e^13 + 17664*a^5*b^18*d^4*e^13 + 26880*a^7*b^16*d^
4*e^13 + 18816*a^9*b^14*d^4*e^13 - 2688*a^11*b^12*d^4*e^13 - 16128*a^13*b^
10*d^4*e^13 - 13056*a^15*b^8*d^4*e^13 - 4800*a^17*b^6*d^4*e^13 - 704*a^19*
b^4*d^4*e^13)/(b^17*d^5 + a^16*b*d^5 + 8*a^2*b^15*d^5 + 28*a^4*b^13*d^5 +
56*a^6*b^11*d^5 + 70*a^8*b^9*d^5 + 56*a^10*b^7*d^5 + 28*a^12*b^5*d^5 + 8*a
^14*b^3*d^5) + ((e*cot(c + d*x))^(1/2)*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 +
a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^
2*d^2)))^(1/2)*(512*b^26*d^4*e^10 + 4608*a^2*b^24*d^4*e^10 + 17920*a^4*b^2
2*d^4*e^10 + 38400*a^6*b^20*d^4*e^10 + 46080*a^8*b^18*d^4*e^10 + 21504*a^1
0*b^16*d^4*e^10 - 21504*a^12*b^14*d^4*e^10 - 46080*a^14*b^12*d^4*e^10 - 38
400*a^16*b^10*d^4*e^10 - 17920*a^18*b^8*d^4*e^10 - 4608*a^20*b^6*d^4*e^10
- 512*a^22*b^4*d^4*e^10))/(b^17*d^4 + a^16*b*d^4 + 8*a^2*b^15*d^4 + 28*a^4
*b^13*d^4 + 56*a^6*b^11*d^4 + 70*a^8*b^9*d^4 + 56*a^10*b^7*d^4 + 28*a^12*b
^5*d^4 + 8*a^14*b^3*d^4))*(-(e^5*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^5*d^2*6i
+ a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2*d^2)))^(...
```

3.83. $\int \frac{(e\cot(c+dx))^{5/2}}{(a+b\cot(c+dx))^3}\,dx$

3.84 $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx$

3.84.1 Optimal result

Integrand size = 25, antiderivative size = 461

$$\begin{aligned}\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx &= -\frac{(3a^4-26a^2b^2+3b^4)\,e^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4\sqrt{a}\sqrt{b}\,(a^2+b^2)^3\,d} \\ &\quad -\frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\ &\quad +\frac{(a+b)\,(a^2-4ab+b^2)\,e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\ &\quad -\frac{ae\sqrt{e\cot(c+dx)}}{2\,(a^2+b^2)\,d(a+b\cot(c+dx))^2}-\frac{(3a^2-5b^2)\,e\sqrt{e\cot(c+dx)}}{4\,(a^2+b^2)^2\,d(a+b\cot(c+dx))} \\ &\quad -\frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d} \\ &\quad +\frac{(a-b)\,(a^2+4ab+b^2)\,e^{3/2}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\,(a^2+b^2)^3\,d}\end{aligned}$$

output
```
-1/2*(a+b)*(a^2-4*a*b+b^2)*e^(3/2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e
^(1/2))/(a^2+b^2)^3/d*2^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*e^(3/2)*arctan(1+2
^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(a-b)*(a^2+
4*a*b+b^2)*e^(3/2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1
/2))/(a^2+b^2)^3/d*2^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*e^(3/2)*ln(e^(1/2)+co
t(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)-1/4*(
3*a^4-26*a^2*b^2+3*b^4)*e^(3/2)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2
)/e^(1/2))/(a^2+b^2)^3/d/a^(1/2)/b^(1/2)-1/2*a*e*(e*cot(d*x+c))^(1/2)/(a^2
+b^2)/d/(a+b*cot(d*x+c))^2-1/4*(3*a^2-5*b^2)*e*(e*cot(d*x+c))^(1/2)/(a^2+b
^2)^2/d/(a+b*cot(d*x+c))
```

3.84.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.20 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.16

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx =$$

$$-\frac{(e\cot(c+dx))^{3/2}\left(\frac{2a^{3/2}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}(a^2+b^2)^3}-\frac{2a(3a^2-b^2)\sqrt{\cot(c+dx)}}{(a^2+b^2)^3}+\frac{2b(3a^2-b^2)\cot^{\frac{3}{2}}(c+dx)}{3(a^2+b^2)^3}-\frac{-\frac{3\sqrt{b}\arctan\left(\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x])^3,x]
```

output
```
-(((e*Cot[c + d*x])^(3/2)*((2*a^(3/2)*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[C
ot[c + d*x]])/Sqrt[a]])/(Sqrt[b]*(a^2 + b^2)^3) - (2*a*(3*a^2 - b^2)*Sqrt[
Cot[c + d*x]])/(a^2 + b^2)^3 + (2*b*(3*a^2 - b^2)*Cot[c + d*x]^(3/2))/(3*(
a^2 + b^2)^3) - ((-3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]*
Sqrt[Cot[c + d*x]])/Sqrt[a] + (2*b^2*Cot[c + d*x]^2)/(a + b*Cot[c + d*x])^
2 + (3*b*Cot[c + d*x])/(a + b*Cot[c + d*x]))/(4*b*(a^2 + b^2)*Sqrt[Cot[c +
 d*x]]) - (2*b*(3*a^2 - b^2)*(Cot[c + d*x]^(3/2) - Cot[c + d*x]^(3/2)*Hype
rgeometric2F1[3/4, 1, 7/4, -Cot[c + d*x]^2]))/(3*(a^2 + b^2)^3) + (4*b^2*C
ot[c + d*x]^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*Cot[c + d*x])/a)])/(
5*a*(a^2 + b^2)^2) + (a*(a^2 - 3*b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[C
ot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[
Cot[c + d*x]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]
 - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(4*(a^2 +
b^2)^3)))/(d*Cot[c + d*x]^(3/2)))
```

3.84.3 Rubi [A] (warning: unable to verify)

Time = 2.12 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.93, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.960, Rules used = {3042, 4050, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4050}$$

$$-\frac{\int \frac{-3a\cot^2(c+dx)e^2+ae^2-4b\cot(c+dx)e^2}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}dx}{2\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-3a\cot^2(c+dx)e^2+ae^2-4b\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}dx}{4\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-3a\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+ae^2+4b\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{e(3a^2-5b^2)\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}-\frac{\int -\frac{-a\left(3a^2-5b^2\right)\cot^2(c+dx)e^3+a\left(5a^2-3b^2\right)e^3-16a^2b\cot(c+dx)e^3}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae(a^2+b^2)}}{4\left(a^2+b^2\right)}-\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{-a(3a^2-5b^2)\cot^2(c+dx)e^3+a(5a^2-3b^2)e^3-16a^2b\cot(c+dx)e^3}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{2ae(a^2+b^2)}+\frac{e(3a^2-5b^2)\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}}{4(a^2+b^2)}-\frac{ae\sqrt{e\cot(c+dx)}}{2d(a^2+b^2)(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{-a(3a^2-5b^2)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^3+a(5a^2-3b^2)e^3+16a^2b\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2ae(a^2+b^2)}+\frac{e(3a^2-5b^2)\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}}{4(a^2+b^2)}-\frac{ae\sqrt{e\cot(c+dx)}}{2d(a^2+b^2)(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 4136

$$-\frac{\frac{\frac{\int \frac{8\left(a^2(a^2-3b^2)e^3-ab(3a^2-b^2)e^3\cot(c+dx)\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}-\frac{ae^3(3a^4-26a^2b^2+3b^4)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{2ae(a^2+b^2)}+\frac{e(3a^2-5b^2)\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}}{4(a^2+b^2)}-\frac{ae\sqrt{e\cot(c+dx)}}{2d(a^2+b^2)(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{8\int \frac{a^2(a^2-3b^2)e^3-ab(3a^2-b^2)e^3\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}-\frac{ae^3(3a^4-26a^2b^2+3b^4)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{2ae(a^2+b^2)}+\frac{e(3a^2-5b^2)\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}}{4(a^2+b^2)}-\frac{ae\sqrt{e\cot(c+dx)}}{2d(a^2+b^2)(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{8\int \frac{a^2(a^2-3b^2)e^3+ab(3a^2-b^2)\tan\left(c+dx+\frac{\pi}{2}\right)e^3}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}-\frac{ae^3(3a^4-26a^2b^2+3b^4)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2ae(a^2+b^2)}+\frac{e(3a^2-5b^2)\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}}{4(a^2+b^2)}-\frac{ae\sqrt{e\cot(c+dx)}}{2d(a^2+b^2)(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 4017

3.84. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{16\int -\frac{ae^3\left(a\left(a^2-3b^2\right)e-b\left(3a^2-b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{ae^3\left(3a^4-26a^2b^2+3b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2ae(a^2+b^2)}+\frac{e(3a^2-5b^2)\sqrt{\cdots}}{d(a^2+b^2)(a+\cdots}}{4\left(a^2+b^2\right)}$$
$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\Big\downarrow$ 25

$$-\frac{\frac{-\frac{16\int\frac{ae^3\left(a\left(a^2-3b^2\right)e-b\left(3a^2-b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{ae^3\left(3a^4-26a^2b^2+3b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2ae(a^2+b^2)}+\frac{e(3a^2-5b^2)\sqrt{\cdots}}{d(a^2+b^2)(a+\cdots}}{4\left(a^2+b^2\right)}$$
$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{-\frac{16ae^3\int\frac{a\left(a^2-3b^2\right)e-b\left(3a^2-b^2\right)e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{ae^3\left(3a^4-26a^2b^2+3b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2ae(a^2+b^2)}+\frac{e(3a^2-5b^2)\sqrt{e\cdots}}{d(a^2+b^2)(a+b\cdots}}{4\left(a^2+b^2\right)}$$
$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\Big\downarrow$ 1482

$$-\frac{\frac{-\frac{16ae^3\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int\frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d\left(a^2+b^2\right)}-\frac{ae^3\left(3a^4-26a^2b^2+3b^4\right)\int\cdots}{\sqrt{\cdots}}}{2ae(a^2+b^2)}\cdots}{4\left(a^2+b^2\right)}$$
$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\Big\downarrow$ 1476

3.84. $\int\frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx$

$$-\frac{-\frac{\frac{16ae^3\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int\frac{}{\cot(c+}\right.}{d\left(a^2+b^2\right)}}{2ae(a^2+b^2)}}{4\left(a^2+\right.}$$

$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 1082$$

$$-\frac{-\frac{\frac{16ae^3\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int\frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int\frac{1}{-e\cot(c+dx)-1}d}{\sqrt{}}\right.\right.}{d\left(a^2+b^2\right)}}{2ae(a^2+b^2)}}{4\left(a^2+b^2\right)}$$

$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 217$$

$$-\frac{-\frac{\frac{16ae^3\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int\frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)\right)}{d\left(a^2+b^2\right)}}{2ae(a^2+b^2)}}{4\left(a^2+b^2\right)}$$

$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 1479$$

$$-\frac{-\frac{\frac{16ae^3\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(-\frac{\int-\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(\right.\right.}{d\left(a^2+b^2\right)}}{2ae(a^2}}{}$$

$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 25$$

3.84. $\int\frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx$

$$-\frac{-\frac{16ae^3\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(a^2-\cdots\right.\right.}{d\left(a^2+b^2\right)}}{2ae(a^2+b\cdots}$$

$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{16ae^3\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}+\frac{\int \frac{\sqrt{e}+\sqrt{2}\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{e}}\right)+\frac{1}{2}(a+b)\left(a^2-\cdots\right.\right.}{d\left(a^2+b^2\right)}}{2ae(a^2+b\cdots}$$

$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \ 1103$$

$$-\frac{-\frac{ae^3\left(3a^4-26a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16ae^3\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\,c\cdots}}{\cdots}\right)}{\sqrt{2}\sqrt{e}}\cdots\right.\right.}{\cdots}}{2ae(a^2+b^2)}}{4\left(a^2\cdots\right.}$$

$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \ 4117$$

$$-\frac{-\frac{ae^3\left(3a^4-26a^2b^2+3b^4\right)\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}-\frac{16ae^3\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\,c\cdots}}{\cdots}\right)}{\sqrt{2}\sqrt{e}}\cdots\right.\right.}{\cdots}}{2ae(a^2+b^2)}}{4\left(a^2\cdots\right.}$$

$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \ 73$$

3.84. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{2ae^2\left(3a^4-26a^2b^2+3b^4\right)\int \frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{16ae^3\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\cdots}{\cdots}}{2ae(a^2+b^2)}}{4\left(a^2+b^2\right)\cdots}$$

$$\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 218$$

$$-\frac{ae\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}-$$

$$\frac{\frac{e\left(3a^2-5b^2\right)\sqrt{e\cot(c+dx)}}{d(a^2+b^2)(a+b\cot(c+dx))}+\frac{-\frac{16ae^3\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\log(\cdots}{\cdots}\right.\cdots\right)}{d\left(a^2+b^2\right)}\cdots}{\cdots}}{4\left(a^2+b^2\right)}$$

input
```
Int[(e*Cot[c + d*x])^(3/2)/(a + b*Cot[c + d*x])^3,x]
```

output
```
-1/2*(a*e*Sqrt[e*Cot[c + d*x]])/((a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) - (
((3*a^2 - 5*b^2)*e*Sqrt[e*Cot[c + d*x]])/((a^2 + b^2)*d*(a + b*Cot[c + d*x
])) + ((-2*Sqrt[a]*(3*a^4 - 26*a^2*b^2 + 3*b^4)*e^(5/2)*ArcTan[(Sqrt[b]*Co
t[c + d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt[b]*(a^2 + b^2)*d) - (16*a*e^3*(((a +
 b)*(a^2 - 4*a*b + b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[
e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]
]/(Sqrt[2]*Sqrt[e])))/2 + ((a - b)*(a^2 + 4*a*b + b^2)*(-1/2*Log[e + e*Cot
[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[
e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt
[e])))/2))/((a^2 + b^2)*d))/(2*a*(a^2 + b^2)*e))/(4*(a^2 + b^2))
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.84. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx$

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4050
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(a + b*Tan[e + f*x])^(m +
 1)*((c + d*Tan[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/((m
 + 1)*(a^2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^
(n - 2)*Simp[a*c^2*(m + 1) + a*d^2*(n - 1) + b*c*d*(m - n + 2) - (b*c^2 - 2
*a*c*d - b*d^2)*(m + 1)*Tan[e + f*x] - d*(b*c - a*d)*(m + n)*Tan[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[
2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.84. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx$

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.84.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.99

method	result
derivativedivides	$-\frac{2e^4\left(\frac{\frac{\left(\frac{3}{8}a^4b-\frac{1}{4}a^2b^3-\frac{5}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}+\frac{ae\left(5a^4+2a^2b^2-3b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(3a^4-26a^2b^2+3b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{8\sqrt{aeb}}}{\left(a^2+b^2\right)^3e^2}\cdots\right)}{\cdots}$
default	$-\frac{2e^4\left(\frac{\frac{\left(\frac{3}{8}a^4b-\frac{1}{4}a^2b^3-\frac{5}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}+\frac{ae\left(5a^4+2a^2b^2-3b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(3a^4-26a^2b^2+3b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right)}{8\sqrt{aeb}}}{\left(a^2+b^2\right)^3e^2}\cdots\right)}{\cdots}$

input
```
int((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^4*(1/(a^2+b^2)^3/e^2*(((3/8*a^4*b-1/4*a^2*b^3-5/8*b^5)*(e*cot(d*x+c
))^(3/2)+1/8*a*e*(5*a^4+2*a^2*b^2-3*b^4)*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+
c)*b+a*e)^2+1/8*(3*a^4-26*a^2*b^2+3*b^4)/(a*e*b)^(1/2)*arctan((e*cot(d*x+c
))^(1/2)*b/(a*e*b)^(1/2)))+1/(a^2+b^2)^3/e^2*(1/8*(-a^3*e+3*a*b^2*e)*(e^2)
^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1
/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e
^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(
-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(3*a^2*b-b^3)/(e^2)^(1/4
)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)
^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)
))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/
(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4432 vs. $2(390) = 780$.

Time = 0.78 (sec) , antiderivative size = 8913, normalized size of antiderivative = 19.33

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.84.6 Sympy [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{(e\cot{(c+dx)})^{\frac{3}{2}}}{(a+b\cot{(c+dx)})^3}\,dx$$

input
```
integrate((e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**3,x)
```

output
```
Integral((e*cot(c + d*x))**(3/2)/(a + b*cot(c + d*x))**3, x)
```

3.84.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.84.8 Giac [F]

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{(e\cot(dx+c))^{\frac{3}{2}}}{(b\cot(dx+c)+a)^3}\,dx$$

input
```
integrate((e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*cot(d*x + c))^(3/2)/(b*cot(d*x + c) + a)^3, x)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 19.03 (sec) , antiderivative size = 19000, normalized size of antiderivative = 41.21

$$\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(3/2)/(a + b*cot(c + d*x))^3,x)
```

output

```
atan(((((518*a*b^15*d^2*e^15 - 18*a^15*b*d^2*e^15 - 4494*a^3*b^13*d^2*e^15
 + 3022*a^5*b^11*d^2*e^15 + 17194*a^7*b^9*d^2*e^15 + 5298*a^9*b^7*d^2*e^15
 - 3338*a^11*b^5*d^2*e^15 + 506*a^13*b^3*d^2*e^15)/(a^16*d^5 + b^16*d^5 +
8*a^2*b^14*d^5 + 28*a^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a
^10*b^6*d^5 + 28*a^12*b^4*d^5 + 8*a^14*b^2*d^5) + (((4224*a^4*b^18*d^4*e^1
2 - 320*a^2*b^20*d^4*e^12 - 192*b^22*d^4*e^12 + 22272*a^6*b^16*d^4*e^12 +
51072*a^8*b^14*d^4*e^12 + 67200*a^10*b^12*d^4*e^12 + 53760*a^12*b^10*d^4*e
^12 + 25344*a^14*b^8*d^4*e^12 + 5952*a^16*b^6*d^4*e^12 + 192*a^18*b^4*d^4*
e^12 - 128*a^20*b^2*d^4*e^12)/(a^16*d^5 + b^16*d^5 + 8*a^2*b^14*d^5 + 28*a
^4*b^12*d^5 + 56*a^6*b^10*d^5 + 70*a^8*b^8*d^5 + 56*a^10*b^6*d^5 + 28*a^12
*b^4*d^5 + 8*a^14*b^2*d^5) + ((e*cot(c + d*x))^(1/2)*((e^3*1i)/(4*(b^6*d^2
 - a^6*d^2 + a*b^5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20
i + 15*a^4*b^2*d^2)))^(1/2)*(512*b^25*d^4*e^10 + 4608*a^2*b^23*d^4*e^10 +
17920*a^4*b^21*d^4*e^10 + 38400*a^6*b^19*d^4*e^10 + 46080*a^8*b^17*d^4*e^1
0 + 21504*a^10*b^15*d^4*e^10 - 21504*a^12*b^13*d^4*e^10 - 46080*a^14*b^11*
d^4*e^10 - 38400*a^16*b^9*d^4*e^10 - 17920*a^18*b^7*d^4*e^10 - 4608*a^20*b
^5*d^4*e^10 - 512*a^22*b^3*d^4*e^10))/(a^16*d^4 + b^16*d^4 + 8*a^2*b^14*d^
4 + 28*a^4*b^12*d^4 + 56*a^6*b^10*d^4 + 70*a^8*b^8*d^4 + 56*a^10*b^6*d^4 +
 28*a^12*b^4*d^4 + 8*a^14*b^2*d^4))*((e^3*1i)/(4*(b^6*d^2 - a^6*d^2 + a*b^
5*d^2*6i + a^5*b*d^2*6i - 15*a^2*b^4*d^2 - a^3*b^3*d^2*20i + 15*a^4*b^2...
```

3.84. $\int \frac{(e\cot(c+dx))^{3/2}}{(a+b\cot(c+dx))^3}\,dx$

3.85 $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$

3.85.1 Optimal result

Integrand size = 25, antiderivative size = 463

$$
\begin{aligned}
\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx
&= \frac{\sqrt{b}(15a^4-18a^2b^2-b^4)\sqrt{e}\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{3/2}(a^2+b^2)^3 d} \\
&\quad+\frac{(a-b)(a^2+4ab+b^2)\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d} \\
&\quad-\frac{(a-b)(a^2+4ab+b^2)\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d} \\
&\quad+\frac{b\sqrt{e\cot(c+dx)}}{2(a^2+b^2)d(a+b\cot(c+dx))^2}+\frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{4a(a^2+b^2)^2 d(a+b\cot(c+dx))} \\
&\quad-\frac{(a+b)(a^2-4ab+b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d} \\
&\quad+\frac{(a+b)(a^2-4ab+b^2)\sqrt{e}\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d}
\end{aligned}
$$

output
```
1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))*e
^(1/2)/(a^2+b^2)^3/d*2^(1/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e
*cot(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/(a^2+b^2)^3/d*2^(1/2)-1/4*(a+b)*(a^2-4
*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1
/2)/(a^2+b^2)^3/d*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*
e^(1/2)+2^(1/2)*(e*cot(d*x+c))^(1/2))*e^(1/2)/(a^2+b^2)^3/d*2^(1/2)+1/4*(1
5*a^4-18*a^2*b^2-b^4)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/a^(1/2)/e^(1/2))
*b^(1/2)*e^(1/2)/a^(3/2)/(a^2+b^2)^3/d+1/2*b*(e*cot(d*x+c))^(1/2)/(a^2+b^2
)/d/(a+b*cot(d*x+c))^2+1/4*b*(7*a^2-b^2)*(e*cot(d*x+c))^(1/2)/a/(a^2+b^2)^
2/d/(a+b*cot(d*x+c))
```

3.85.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.22 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx =$$

$$-\frac{\sqrt{e\cot(c+dx)}\left(-\frac{2\sqrt{a}\sqrt{b}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{(a^2+b^2)^3}+\frac{2b(3a^2-b^2)\sqrt{\cot(c+dx)}}{(a^2+b^2)^3}-\frac{2\sqrt{a}\sqrt{b}\left(-a\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)+\cdots\right.}{(a^2\cdots}\right.}{}$$

input
```
Integrate[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x])^3,x]
```

output
```
-((Sqrt[e*Cot[c + d*x]]*((-2*Sqrt[a]*Sqrt[b]*(3*a^2 - b^2)*ArcTan[(Sqrt[b]
*Sqrt[Cot[c + d*x]])/Sqrt[a]])/(a^2 + b^2)^3 + (2*b*(3*a^2 - b^2)*Sqrt[Cot
[c + d*x]])/(a^2 + b^2)^3 - (2*Sqrt[a]*Sqrt[b]*(-(a*ArcTan[(Sqrt[b]*Sqrt[C
ot[c + d*x]])/Sqrt[a]]) + Sqrt[a]*Sqrt[b]*Sqrt[Cot[c + d*x]] - b*ArcTan[(S
qrt[b]*Sqrt[Cot[c + d*x]])/Sqrt[a]]*Cot[c + d*x]))/((a^2 + b^2)^2*(a + b*C
ot[c + d*x])) + (2*a*(a^2 - 3*b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/
4, 1, 7/4, -Cot[c + d*x]^2])/(3*(a^2 + b^2)^3) + (2*b^2*Cot[c + d*x]^(3/2)
*Hypergeometric2F1[3/2, 3, 5/2, -((b*Cot[c + d*x])/a)])/(3*a^3*(a^2 + b^2)
) - (b*(3*a^2 - b^2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2
*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]] + 8*Sqrt[Cot[c + d*x]] + S
qrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*Log[1
+ Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(4*(a^2 + b^2)^3)))/(d*Sqrt
[Cot[c + d*x]]))
```

3.85.3 Rubi [A] (warning: unable to verify)

Time = 2.03 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.93, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4051, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4051$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}-\frac{\int -\frac{-3be\cot^2(c+dx)+4ae\cot(c+dx)+be}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}dx}{2\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-3be\cot^2(c+dx)+4ae\cot(c+dx)+be}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}dx}{4\left(a^2+b^2\right)}+\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{-3be\tan\left(c+dx+\frac{\pi}{2}\right)^2-4ae\tan\left(c+dx+\frac{\pi}{2}\right)+be}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2+b^2\right)}+\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 4132$$

$$\frac{\frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}-\frac{\int -\frac{-b\left(7a^2-b^2\right)\cot^2(c+dx)e^2+b\left(9a^2+b^2\right)e^2+8a\left(a^2-b^2\right)\cot(c+dx)e^2}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae(a^2+b^2)}}{4\left(a^2+b^2\right)}+\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{-b(7a^2-b^2)\cot^2(c+dx)e^2+b(9a^2+b^2)e^2+8a(a^2-b^2)\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{2ae(a^2+b^2)}+\frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}}{4(a^2+b^2)}+\frac{b\sqrt{e\cot(c+dx)}}{2d(a^2+b^2)(a+b\cot(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{-b(7a^2-b^2)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^2+b(9a^2+b^2)e^2-8a(a^2-b^2)\tan\left(c+dx+\frac{\pi}{2}\right)e^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2ae(a^2+b^2)}+\frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}}{4(a^2+b^2)}+\frac{b\sqrt{e\cot(c+dx)}}{2d(a^2+b^2)(a+b\cot(c+dx))^2}$$

$\downarrow$ 4136

$$\frac{\frac{\frac{\int \frac{8\left(ab(3a^2-b^2)e^2+a^2(a^2-3b^2)\cot(c+dx)e^2\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}-\frac{be^2(15a^4-18a^2b^2-b^4)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{2ae(a^2+b^2)}+\frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}}{4(a^2+b^2)}+\frac{b\sqrt{e\cot(c+dx)}}{2d(a^2+b^2)(a+b\cot(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{8\int \frac{ab(3a^2-b^2)e^2+a^2(a^2-3b^2)\cot(c+dx)e^2}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}-\frac{be^2(15a^4-18a^2b^2-b^4)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}}{2ae(a^2+b^2)}+\frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}}{4(a^2+b^2)}+\frac{b\sqrt{e\cot(c+dx)}}{2d(a^2+b^2)(a+b\cot(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{8\int \frac{ab(3a^2-b^2)e^2-a^2(a^2-3b^2)e^2\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}-\frac{be^2(15a^4-18a^2b^2-b^4)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2ae(a^2+b^2)}+\frac{b(7a^2-b^2)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}}{4(a^2+b^2)}+\frac{b\sqrt{e\cot(c+dx)}}{2d(a^2+b^2)(a+b\cot(c+dx))^2}$$

$\downarrow$ 4017

3.85. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{16\int -\frac{ae^2\left(b\left(3a^2-b^2\right)e+a\left(a^2-3b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{be^2\left(15a^4-18a^2b^2-b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2ae(a^2+b^2)}+\frac{b(7a^2-b^2)\sqrt{e}}{ad(a^2+b^2)(a+b}}{4\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\Big\downarrow$ 25

$$\frac{-\frac{16\int \frac{ae^2\left(b\left(3a^2-b^2\right)e+a\left(a^2-3b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{be^2\left(15a^4-18a^2b^2-b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2ae(a^2+b^2)}+\frac{b(7a^2-b^2)\sqrt{e}}{ad(a^2+b^2)(a+b}$$

$$4\left(a^2+b^2\right)$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\Big\downarrow$ 27

$$\frac{-\frac{16ae^2\int \frac{b\left(3a^2-b^2\right)e+a\left(a^2-3b^2\right)\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{be^2\left(15a^4-18a^2b^2-b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2ae(a^2+b^2)}+\frac{b(7a^2-b^2)\sqrt{e\cot}}{ad(a^2+b^2)(a+b\cot}$$

$$4\left(a^2+b^2\right)$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\Big\downarrow$ 1482

$$\frac{-\frac{16ae^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d\left(a^2+b^2\right)}-\frac{be^2\left(15a^4-18a^2b^2-b^4\right)\int}{\sqrt{-e}}}{2ae(a^2+b^2)}$$

$$4\left(a^2+b^2\right)$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\Big\downarrow$ 1476

3.85. $\displaystyle\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$

$$-\frac{\frac{16ae^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e+\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}\right)-\frac{1}{2}(a+b)\left(a^2-4\right.}{d\left(a^2+b^2\right)}}{2ae(a^2+b^2)}}{4\left(a^2+\right.}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1082

$$-\frac{\frac{16ae^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}-\frac{\int \frac{1}{-e\cot(c+dx)-1}d\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int \frac{e-e\cot(\ldots}{\cot^2(c+dx\ldots}\right.}{d\left(a^2+b^2\right)}}{2ae(a^2+b^2)}}{4\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 217

$$-\frac{\frac{16ae^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}\right)}{d\left(a^2+b^2\right)}-\ldots}{2ae(a^2+b^2)}}{4\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 1479

$$-\frac{\frac{16ae^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\ldots}}{2\sqrt{2}\sqrt{e}}\right.}{d\left(a^2+b^2\right)}}{2ae(a^2+b\ldots}}{\ldots}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 25

3.85. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$

$$-\frac{\frac{16ae^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}\cdots}{2\sqrt{2}\sqrt{e}}\cdots\right)\cdots\right)}{d\left(a^2+b^2\right)}\cdots}{2ae(a^2+b^2)}\cdots$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow\ 27$$

$$-\frac{\frac{16ae^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\int\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}\cdots}{2\sqrt{2}\sqrt{e}}\cdots\right)\cdots\right)}{d\left(a^2+b^2\right)}\cdots}{2ae(a^2+b^2)}\cdots$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow\ 1103$$

$$-\frac{\frac{be^2\left(15a^4-18a^2b^2-b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16ae^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{\sqrt{e}}\cdots\right)}{\sqrt{2}\sqrt{e}}\cdots\right)\cdots\right)}{\cdots}}{2ae(a^2+b^2)}\cdots \over 4\left(a^2\cdots\right.}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow\ 4117$$

$$-\frac{\frac{be^2\left(15a^4-18a^2b^2-b^4\right)\int\frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}-\frac{16ae^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(\cdots}}{\sqrt{e}}\cdots\right)}{\sqrt{2}\sqrt{e}}\cdots\right)\cdots\right)}{\cdots}}{2ae(a^2+b^2)}\cdots \over 4\left(a^2\cdots\right.}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow\ 73$$

3.85. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{2be\left(15a^4-18a^2b^2-b^4\right)\int \frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{16ae^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a \cdots}{d \cdots}}{2ae(a^2+b^2)}}{4\left(a^2+b^2\right)}$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow 218$$

$$\frac{b\sqrt{e\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}+$$

$$\frac{\frac{b\left(7a^2-b^2\right)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}+\frac{-\frac{16ae^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)-\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\log \cdots}{\cdots}\right.}{d\left(a^2+b^2\right)}}{\cdots}}{4\left(a^2+b^2\right)}$$

input
```
Int[Sqrt[e*Cot[c + d*x]]/(a + b*Cot[c + d*x])^3,x]
```

output
```
(b*Sqrt[e*Cot[c + d*x]])/(2*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) + ((b*(7
*a^2 - b^2)*Sqrt[e*Cot[c + d*x]])/(a*(a^2 + b^2)*d*(a + b*Cot[c + d*x])) +
 ((-2*Sqrt[b]*(15*a^4 - 18*a^2*b^2 - b^4)*e^(3/2)*ArcTan[(Sqrt[b]*Cot[c +
d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*(a^2 + b^2)*d) - (16*a*e^2*(((a - b)*(a
^2 + 4*a*b + b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(S
qrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sqrt[e]]/(Sqr
t[2]*Sqrt[e])))/2 - ((a + b)*(a^2 - 4*a*b + b^2)*(-1/2*Log[e + e*Cot[c + d
*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) + Log[e + e*
Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]*Sqrt[e])))
/2))/((a^2 + b^2)*d))/(2*a*(a^2 + b^2)*e))/(4*(a^2 + b^2))
```

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.85. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4051
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(a + b*Tan[e + f*x])^(m + 1)*((c +
d*Tan[e + f*x])^n/(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(a^2 + b^2
))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[a*c
*(m + 1) - b*d*n - (b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(m + n + 1)*Tan[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] && GtQ[n, 0] && Int
egerQ[2*m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.85. $\int \frac{\sqrt{e \cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.85.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.99

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{b\left(\frac{\frac{b\left(7a^4+6a^2b^2-b^4\right)(e\cot(dx+c))^{\frac{3}{2}}}{8a}+\frac{e\left(9a^4+10a^2b^2+b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(15a^4-18a^2b^2-b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right.}{8a\sqrt{aeb}}\right.}{e^3\left(a^2+b^2\right)^3}\right.}{}$
default	$-\frac{2e^4\left(-\frac{b\left(\frac{\frac{b\left(7a^4+6a^2b^2-b^4\right)(e\cot(dx+c))^{\frac{3}{2}}}{8a}+\frac{e\left(9a^4+10a^2b^2+b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(15a^4-18a^2b^2-b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx+c)}\,b}{\sqrt{aeb}}\right.}{8a\sqrt{aeb}}\right.}{e^3\left(a^2+b^2\right)^3}\right.}{}$

input
```
int((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^4*(-b/e^3/(a^2+b^2)^3*((1/8*b*(7*a^4+6*a^2*b^2-b^4)/a*(e*cot(d*x+c)
)^(3/2)+1/8*e*(9*a^4+10*a^2*b^2+b^4)*(e*cot(d*x+c))^(1/2))/(e*cot(d*x+c)*b
+a*e)^2+1/8*(15*a^4-18*a^2*b^2-b^4)/a/(a*e*b)^(1/2)*arctan((e*cot(d*x+c))^
(1/2)*b/(a*e*b)^(1/2)))+1/(a^2+b^2)^3/e^3*(1/8*(3*a^2*b*e-b^3*e)*(e^2)^(1/
4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+
(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^
(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(
1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(a^3-3*a*b^2)/(e^2)^(1/4)*2^
(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/
2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)+(e^2)^(1/2)))+2
*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(e^2
)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.85.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4405 vs. $2(392) = 784$.

Time = 0.93 (sec) , antiderivative size = 8853, normalized size of antiderivative = 19.12

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.85.6 Sympy [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{\sqrt{e\cot{(c+dx)}}}{(a+b\cot{(c+dx)})^3}\,dx$$

input
```
integrate((e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**3,x)
```

output
```
Integral(sqrt(e*cot(c + d*x))/(a + b*cot(c + d*x))**3, x)
```

3.85.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.85.8 Giac [F]

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx = \int \frac{\sqrt{e\cot(dx+c)}}{(b\cot(dx+c)+a)^3}\,dx$$

input
```
integrate((e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(e*cot(d*x + c))/(b*cot(d*x + c) + a)^3, x)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 18.85 (sec) , antiderivative size = 19534, normalized size of antiderivative = 42.19

$$\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((e*cot(c + d*x))^(1/2)/(a + b*cot(c + d*x))^3,x)
```

output
```
(((e*cot(c + d*x))^(1/2)*(b^3*e^2 + 9*a^2*b*e^2))/(4*(a^4 + b^4 + 2*a^2*b^
2)) + (b^2*e*(e*cot(c + d*x))^(3/2)*(7*a^2 - b^2))/(4*a*(a^4 + b^4 + 2*a^2
*b^2)))/(a^2*d*e^2 + b^2*d*e^2*cot(c + d*x)^2 + 2*a*b*d*e^2*cot(c + d*x))
- atan((((((64*a*b^23*d^4*e^11 + 1472*a^3*b^21*d^4*e^11 + 8832*a^5*b^19*d
^4*e^11 + 25344*a^7*b^17*d^4*e^11 + 40320*a^9*b^15*d^4*e^11 + 34944*a^11*b
^13*d^4*e^11 + 10752*a^13*b^11*d^4*e^11 - 8448*a^15*b^9*d^4*e^11 - 10176*a
^17*b^7*d^4*e^11 - 4160*a^19*b^5*d^4*e^11 - 640*a^21*b^3*d^4*e^11)/(a^18*d
^5 + a^2*b^16*d^5 + 8*a^4*b^14*d^5 + 28*a^6*b^12*d^5 + 56*a^8*b^10*d^5 + 7
0*a^10*b^8*d^5 + 56*a^12*b^6*d^5 + 28*a^14*b^4*d^5 + 8*a^16*b^2*d^5) + ((e
*cot(c + d*x))^(1/2)*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5
*b*d^2 - a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2)*(512*
a^2*b^25*d^4*e^10 + 4608*a^4*b^23*d^4*e^10 + 17920*a^6*b^21*d^4*e^10 + 384
00*a^8*b^19*d^4*e^10 + 46080*a^10*b^17*d^4*e^10 + 21504*a^12*b^15*d^4*e^10
 - 21504*a^14*b^13*d^4*e^10 - 46080*a^16*b^11*d^4*e^10 - 38400*a^18*b^9*d^
4*e^10 - 17920*a^20*b^7*d^4*e^10 - 4608*a^22*b^5*d^4*e^10 - 512*a^24*b^3*d
^4*e^10))/(a^18*d^4 + a^2*b^16*d^4 + 8*a^4*b^14*d^4 + 28*a^6*b^12*d^4 + 56
*a^8*b^10*d^4 + 70*a^10*b^8*d^4 + 56*a^12*b^6*d^4 + 28*a^14*b^4*d^4 + 8*a^
16*b^2*d^4))*(-e/(4*(b^6*d^2*1i - a^6*d^2*1i + 6*a*b^5*d^2 + 6*a^5*b*d^2 -
 a^2*b^4*d^2*15i - 20*a^3*b^3*d^2 + a^4*b^2*d^2*15i)))^(1/2) - ((e*cot(c +
 d*x))^(1/2)*(8*a*b^20*d^2*e^11 - 1152*a^3*b^18*d^2*e^11 + 2528*a^5*b^1...
```

3.85. $\int \frac{\sqrt{e\cot(c+dx)}}{(a+b\cot(c+dx))^3}\,dx$

3.86 $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx$

3.86.1 Optimal result

Integrand size = 25, antiderivative size = 476

$$\begin{aligned}
\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx &= -\frac{b^{3/2}(35a^4+6a^2b^2+3b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{5/2}(a^2+b^2)^3 d\sqrt{e}} \\
&+ \frac{(a+b)(a^2-4ab+b^2)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d\sqrt{e}} \\
&- \frac{(a+b)(a^2-4ab+b^2)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}(a^2+b^2)^3 d\sqrt{e}} \\
&- \frac{b^2\sqrt{e\cot(c+dx)}}{2a(a^2+b^2)de(a+b\cot(c+dx))^2} - \frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{4a^2(a^2+b^2)^2 de(a+b\cot(c+dx))} \\
&+ \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d\sqrt{e}} \\
&- \frac{(a-b)(a^2+4ab+b^2)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}(a^2+b^2)^3 d\sqrt{e}}
\end{aligned}$$

output
```
-1/4*b^(3/2)*(35*a^4+6*a^2*b^2+3*b^4)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)/
a^(1/2)/e^(1/2))/a^(5/2)/(a^2+b^2)^3/d/e^(1/2)+1/2*(a+b)*(a^2-4*a*b+b^2)*a
rctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d*2^(1/2)/e^(1/2
)-1/2*(a+b)*(a^2-4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))
/(a^2+b^2)^3/d*2^(1/2)/e^(1/2)+1/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*
x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)/e^(1/2)-1
/4*(a-b)*(a^2+4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x+
c))^(1/2))/(a^2+b^2)^3/d*2^(1/2)/e^(1/2)-1/2*b^2*(e*cot(d*x+c))^(1/2)/a/(a
^2+b^2)/d/e/(a+b*cot(d*x+c))^2-1/4*b^2*(11*a^2+3*b^2)*(e*cot(d*x+c))^(1/2)
/a^2/(a^2+b^2)^2/d/e/(a+b*cot(d*x+c))
```

3.86.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.17 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx = -\frac{\sqrt{\cot(c+dx)}\left(\frac{2b^{3/2}(3a^2-b^2)\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}(a^2+b^2)^3}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{\cot(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}(a^2+b^2)^2}+\frac{2b^2\sqrt{\cot(c+dx)}}{(a^2+b^2)^2(a+b\cot(c+dx))}+\frac{2b^2\sqrt{\cot(}\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^3),x]
```

output
```
-((Sqrt[Cot[c + d*x]]*((2*b^(3/2)*(3*a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[Cot[c
 + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)^3) + (2*b^(3/2)*ArcTan[(Sqrt[b]*S
qrt[Cot[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)^2) + (2*b^2*Sqrt[Cot[c +
 d*x]])/((a^2 + b^2)^2*(a + b*Cot[c + d*x])) + (2*b^2*Sqrt[Cot[c + d*x]]*H
ypergeometric2F1[1/2, 3, 3/2, -((b*Cot[c + d*x])/a)])/(a^3*(a^2 + b^2)) -
(2*b*(3*a^2 - b^2)*Cot[c + d*x]^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[
c + d*x]^2])/(3*(a^2 + b^2)^3) - (a*(a^2 - 3*b^2)*(2*Sqrt[2]*ArcTan[1 - Sq
rt[2]*Sqrt[Cot[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Sqrt[2]*
Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(4*(a^2 + b^2)^3)))/(
d*Sqrt[e*Cot[c + d*x]]))
```

3.86.3 Rubi [A] (warning: unable to verify)

Time = 2.11 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.95, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.960, Rules used = {3042, 4052, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 4052}$$

$$-\frac{\int -\frac{3b^2e\cot^2(c+dx)-4abe\cot(c+dx)+(4a^2+3b^2)e}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}dx}{2ae\left(a^2+b^2\right)}-\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3b^2e\cot^2(c+dx)-4abe\cot(c+dx)+(4a^2+3b^2)e}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}dx}{4ae\left(a^2+b^2\right)}-\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{3b^2e\tan\left(c+dx+\frac{\pi}{2}\right)^2+4abe\tan\left(c+dx+\frac{\pi}{2}\right)+(4a^2+3b^2)e}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4ae\left(a^2+b^2\right)}-\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{ 4132}$$

$$\frac{-\frac{\int -\frac{-16be^2\cot(c+dx)a^3+\left(8a^4+3b^2a^2+3b^4\right)e^2+b^2\left(11a^2+3b^2\right)e^2\cot^2(c+dx)}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae(a^2+b^2)}-\frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}}{4ae\left(a^2+b^2\right)}-\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{-16be^2\cot(c+dx)a^3+\left(8a^4+3b^2a^2+3b^4\right)e^2+b^2\left(11a^2+3b^2\right)e^2\cot^2(c+dx)}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{2ae(a^2+b^2)}-\frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}}{4ae\left(a^2+b^2\right)}-\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{16be^2\tan\left(c+dx+\frac{\pi}{2}\right)a^3+\left(8a^4+3b^2a^2+3b^4\right)e^2+b^2\left(11a^2+3b^2\right)e^2\tan\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2ae(a^2+b^2)}-\frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}}{4ae\left(a^2+b^2\right)}-\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 4136

$$\frac{\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{8\left(a^3\left(a^2-3b^2\right)e^2-a^2b\left(3a^2-b^2\right)e^2\cot(c+dx)\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}}{4ae\left(a^2+b^2\right)}-\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}+\frac{8\int \frac{a^3\left(a^2-3b^2\right)e^2-a^2b\left(3a^2-b^2\right)e^2\cot(c+dx)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}}{4ae\left(a^2+b^2\right)}-\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{8\int \frac{\left(a^2-3b^2\right)e^2a^3+b\left(3a^2-b^2\right)e^2\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{b^2(11a^2+3b^2)\sqrt{e\cot(c+dx)}}{ad(a^2+b^2)(a+b\cot(c+dx))}}{4ae\left(a^2+b^2\right)}-\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$\downarrow$ 4017

3.86. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{16\int -\frac{a^2e^2\left(a\left(a^2-3b^2\right)e-b\left(3a^2-b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2ae(a^2+b^2)}-\frac{b^2(11a^2+3b^2\cdots}{ad(a^2+b^2)(a\cdots}}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16\int \frac{a^2e^2\left(a\left(a^2-3b^2\right)e-b\left(3a^2-b^2\right)e\cot(c+dx)\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}}{2ae(a^2+b^2)}-\frac{b^2(11a^2+3b^2)\cdots}{ad(a^2+b^2)(a+\cdots}}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^2e^2\int \frac{a\left(a^2-3b^2\right)e-b\left(3a^2-b^2\right)e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}}{2ae(a^2+b^2)}-\frac{b^2(11a^2+3b^2)\sqrt{e\cdots}}{ad(a^2+b^2)(a+b\,c\cdots}}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\big\downarrow$ 1482

$$\frac{\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(a^2-4ab+\cdots\right.\right.}{d\left(a^2+b^2\right)}}{2ae(a^2+b^2)}\cdots}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\big\downarrow$ 1476

3.86. $\displaystyle\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(a^2-4ab+\cdots\right.\right.}{\cdots}}{2ae(a^2+b^2)}}{4ae\left(a^2+\cdots\right.}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(a^2-4ab+\cdots\right.\right.}{\cdots}}{2ae(a^2+b^2)}}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\Big\downarrow$ 217

$$\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{e-e\cot(c+dx)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c+dx)}+\frac{1}{2}(a+b)\left(a^2-4ab+\cdots\right.\right.}{d\left(a^2+b^2\right)}}{2ae(a^2+b^2)}}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\Big\downarrow$ 1479

$$\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+\cdots}}{2\sqrt{2}\sqrt{e}}\cdots\right.\right.}{\cdots}}{2ae(a^2+b\cdots}}{\cdots}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$\Big\downarrow$ 25

3.86. $\displaystyle\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right.\right.}{}}{2ae(a^2+b^2)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^2e^2\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{e}-2\sqrt{e\cot(c+dx)}}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e\cot(c+dx)}\sqrt{e}}d\sqrt{e\cot(c+dx)}}{2\sqrt{2}\sqrt{e}}\right.\right.}{}}{2ae(a^2+b^2)}$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^2e^2\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{}}{2ae(a^2+b^2)}$$

$4ae\,(a$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow \; 4117$$

$$\frac{\frac{b^2e^2\left(35a^4+6a^2b^2+3b^4\right)\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}-\frac{16a^2e^2\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot}}{\sqrt{e}}\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{}}{2ae(a^2+b^2)}$$

$4ae\,(a$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow \; 73$$

3.86. $\displaystyle\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx$

$$-\frac{2b^2e\left(35a^4+6a^2b^2+3b^4\right)\int \frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{16a^2e^2\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\right.}{2ae(a^2+b^2)}$$

$$4ae\left(a^2+b\right.$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

$$\Big\downarrow 218$$

$$\frac{2b^{3/2}e^{3/2}\left(35a^4+6a^2b^2+3b^4\right)\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}d\left(a^2+b^2\right)}-\frac{16a^2e^2\left(\frac{1}{2}(a+b)\left(a^2-4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\sqrt{e}}\right)+\frac{1}{2}(a-\right.}{d\left(a\right.}$$

$$2ae(a^2+b^2)$$

$$4ae\left(a^2+b^2\right)$$

$$\frac{b^2\sqrt{e\cot(c+dx)}}{2ade\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^2}$$

input
```
Int[1/(Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^3),x]
```

output
```
-1/2*(b^2*Sqrt[e*Cot[c + d*x]])/(a*(a^2 + b^2)*d*e*(a + b*Cot[c + d*x])^2)
 + (-((b^2*(11*a^2 + 3*b^2)*Sqrt[e*Cot[c + d*x]])/(a*(a^2 + b^2)*d*(a + b*
Cot[c + d*x]))) + ((2*b^(3/2)*(35*a^4 + 6*a^2*b^2 + 3*b^4)*e^(3/2)*ArcTan[
(Sqrt[b]*Cot[c + d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*(a^2 + b^2)*d) - (16*a
^2*e^2*(((a + b)*(a^2 - 4*a*b + b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c +
 d*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d
*x]])/Sqrt[e]]/(Sqrt[2]*Sqrt[e])))/2 + ((a - b)*(a^2 + 4*a*b + b^2)*(-1/2*
Log[e + e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sq
rt[e]) + Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2
*Sqrt[2]*Sqrt[e])))/2))/((a^2 + b^2)*d))/(2*a*(a^2 + b^2)*e))/(4*a*(a^2 +
b^2)*e)
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.86. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx$

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.86. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx$

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.86.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{2e^4\left(\frac{b^2\left(\frac{\frac{b\left(11a^4+14a^2b^2+3b^4\right)(e\cot(dx+c))^{\frac{3}{2}}}{8a^2}+\frac{e\left(13a^4+18a^2b^2+5b^4\right)\sqrt{e\cot(dx+c)}}{8a}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(35a^4+6a^2b^2+3b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx}}{\sqrt{aeb}}\right)}{8a^2\sqrt{aeb}}\right.}{e^4\left(a^2+b^2\right)^3}\right.}{}$
default	$-\frac{2e^4\left(\frac{b^2\left(\frac{\frac{b\left(11a^4+14a^2b^2+3b^4\right)(e\cot(dx+c))^{\frac{3}{2}}}{8a^2}+\frac{e\left(13a^4+18a^2b^2+5b^4\right)\sqrt{e\cot(dx+c)}}{8a}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(35a^4+6a^2b^2+3b^4\right)\arctan\left(\frac{\sqrt{e\cot(dx}}{\sqrt{aeb}}\right)}{8a^2\sqrt{aeb}}\right.}{e^4\left(a^2+b^2\right)^3}\right.}{}$

input
```
int(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^4*(b^2/e^4/(a^2+b^2)^3*((1/8*b*(11*a^4+14*a^2*b^2+3*b^4)/a^2*(e*cot
(d*x+c))^(3/2)+1/8*e*(13*a^4+18*a^2*b^2+5*b^4)/a*(e*cot(d*x+c))^(1/2))/(e*
cot(d*x+c)*b+a*e)^2+1/8*(35*a^4+6*a^2*b^2+3*b^4)/a^2/(a*e*b)^(1/2)*arctan(
(e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2)))+1/e^4/(a^2+b^2)^3*(1/8*(a^3*e-3*a*b
^2*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))
^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1
)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-3*a^2*b+b^3
)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2
^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/2)
+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*arct
an(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))))
```

3.86.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4473 vs. $2(405) = 810$.

Time = 1.58 (sec) , antiderivative size = 8991, normalized size of antiderivative = 18.89

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.86.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx = \int \frac{1}{\sqrt{e\cot{(c+dx)}}\,(a+b\cot{(c+dx)})^3}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))**(1/2)/(a+b*cot(d*x+c))**3,x)
```

output
```
Integral(1/(sqrt(e*cot(c + d*x))*(a + b*cot(c + d*x))**3), x)
```

3.86.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.86.8 Giac [F]

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx = \int \frac{1}{(b\cot(dx+c)+a)^3\sqrt{e\cot(dx+c)}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(1/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*cot(d*x + c) + a)^3*sqrt(e*cot(d*x + c))), x)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 19.45 (sec) , antiderivative size = 20155, normalized size of antiderivative = 42.34

$$\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/((e*cot(c + d*x))^(1/2)*(a + b*cot(c + d*x))^3),x)
```

output
```
atan(((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i
 + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^
6*d^2*e - a^6*d^2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^
2*e + a*b^5*d^2*e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d^
2*e - 15*a^2*b^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*
e*6i + a^5*b*d^2*e*6i)))^(1/2)*((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b^4
*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d^2
*e*6i)))^(1/2)*((192*a^2*b^24*d^4*e^10 + 1728*a^4*b^22*d^4*e^10 + 8320*a^6
*b^20*d^4*e^10 + 27264*a^8*b^18*d^4*e^10 + 62592*a^10*b^16*d^4*e^10 + 9945
6*a^12*b^14*d^4*e^10 + 107520*a^14*b^12*d^4*e^10 + 76800*a^16*b^10*d^4*e^1
0 + 33984*a^18*b^8*d^4*e^10 + 7872*a^20*b^6*d^4*e^10 + 384*a^22*b^4*d^4*e^
10 - 128*a^24*b^2*d^4*e^10)/(a^20*d^5 + a^4*b^16*d^5 + 8*a^6*b^14*d^5 + 28
*a^8*b^12*d^5 + 56*a^10*b^10*d^5 + 70*a^12*b^8*d^5 + 56*a^14*b^6*d^5 + 28*
a^16*b^4*d^5 + 8*a^18*b^2*d^5) - ((1i/(4*(b^6*d^2*e - a^6*d^2*e - 15*a^2*b
^4*d^2*e - a^3*b^3*d^2*e*20i + 15*a^4*b^2*d^2*e + a*b^5*d^2*e*6i + a^5*b*d
^2*e*6i)))^(1/2)*(e*cot(c + d*x))^(1/2)*(512*a^4*b^25*d^4*e^10 + 4608*a^6*
b^23*d^4*e^10 + 17920*a^8*b^21*d^4*e^10 + 38400*a^10*b^19*d^4*e^10 + 46080
*a^12*b^17*d^4*e^10 + 21504*a^14*b^15*d^4*e^10 - 21504*a^16*b^13*d^4*e^10
- 46080*a^18*b^11*d^4*e^10 - 38400*a^20*b^9*d^4*e^10 - 17920*a^22*b^7*d^4*
e^10 - 4608*a^24*b^5*d^4*e^10 - 512*a^26*b^3*d^4*e^10))/(a^20*d^4 + a^4...
```

3.86. $\int \frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^3}\,dx$

3.87 $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx$

3.87.1 Optimal result

Integrand size = 25, antiderivative size = 529

$$\begin{aligned}
\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx &= \frac{b^{5/2}(63a^4+46a^2b^2+15b^4)\arctan\left(\frac{\sqrt{b}\sqrt{e\cot(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{4a^{7/2}\left(a^2+b^2\right)^3 de^{3/2}} \\
&- \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 de^{3/2}} \\
&+ \frac{(a-b)\left(a^2+4ab+b^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 de^{3/2}} \\
&+ \frac{8a^4+31a^2b^2+15b^4}{4a^3\left(a^2+b^2\right)^2 de\sqrt{e\cot(c+dx)}} - \frac{b^2}{2a\left(a^2+b^2\right) de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2} \\
&- \frac{b^2(13a^2+5b^2)}{4a^2\left(a^2+b^2\right)^2 de\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))} \\
&+ \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)-\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 de^{3/2}} \\
&- \frac{(a+b)\left(a^2-4ab+b^2\right)\log\left(\sqrt{e}+\sqrt{e}\cot(c+dx)+\sqrt{2}\sqrt{e\cot(c+dx)}\right)}{2\sqrt{2}\left(a^2+b^2\right)^3 de^{3/2}}
\end{aligned}$$

output
```
1/4*b^(5/2)*(63*a^4+46*a^2*b^2+15*b^4)*arctan(b^(1/2)*(e*cot(d*x+c))^(1/2)
/a^(1/2)/e^(1/2))/a^(7/2)/(a^2+b^2)^3/d/e^(3/2)-1/2*(a-b)*(a^2+4*a*b+b^2)*
arctan(1-2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2))/(a^2+b^2)^3/d/e^(3/2)*2^(1/
2)+1/2*(a-b)*(a^2+4*a*b+b^2)*arctan(1+2^(1/2)*(e*cot(d*x+c))^(1/2)/e^(1/2)
)/(a^2+b^2)^3/d/e^(3/2)*2^(1/2)+1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d
*x+c)*e^(1/2)-2^(1/2)*(e*cot(d*x+c))^(1/2))/(a^2+b^2)^3/d/e^(3/2)*2^(1/2)-
1/4*(a+b)*(a^2-4*a*b+b^2)*ln(e^(1/2)+cot(d*x+c)*e^(1/2)+2^(1/2)*(e*cot(d*x
+c))^(1/2))/(a^2+b^2)^3/d/e^(3/2)*2^(1/2)+1/4*(8*a^4+31*a^2*b^2+15*b^4)/a^
3/(a^2+b^2)^2/d/e/(e*cot(d*x+c))^(1/2)-1/2*b^2/a/(a^2+b^2)/d/e/(a+b*cot(d*
x+c))^2/(e*cot(d*x+c))^(1/2)-1/4*b^2*(13*a^2+5*b^2)/a^2/(a^2+b^2)^2/d/e/(a
+b*cot(d*x+c))/(e*cot(d*x+c))^(1/2)
```

3.87.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.75 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.57

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx =$$
$$-\frac{-8a^2b^2(3a^2-b^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{b\cot(c+dx)}{a}\right)-16a^2b^2(a^2+b^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\right.}{}$$

input
```
Integrate[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3),x]
```

output
```
-1/4*(-8*a^2*b^2*(3*a^2 - b^2)*Hypergeometric2F1[-1/2, 1, 1/2, -((b*Cot[c
+ d*x])/a)] - 16*a^2*b^2*(a^2 + b^2)*Hypergeometric2F1[-1/2, 2, 1/2, -((b*
Cot[c + d*x])/a)] - 8*b^2*(a^2 + b^2)^2*Hypergeometric2F1[-1/2, 3, 1/2, -(
(b*Cot[c + d*x])/a)] - 8*a^4*(a^2 - 3*b^2)*Hypergeometric2F1[-1/4, 1, 3/4,
 -Cot[c + d*x]^2] + Sqrt[2]*a^3*b*(3*a^2 - b^2)*Sqrt[Cot[c + d*x]]*(2*ArcT
an[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x
]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(a^3*(a^2 + b^2)^3*d*e*Sqrt[e*Cot[c
+ d*x]])
```

3.87.3 Rubi [A] (warning: unable to verify)

Time = 2.81 (sec) , antiderivative size = 499, normalized size of antiderivative = 0.94, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.080$, Rules used = {3042, 4052, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4052}$$

$$-\frac{\int -\frac{5b^2e\cot^2(c+dx)-4abe\cot(c+dx)+(4a^2+5b^2)e}{2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}dx}{2ae\left(a^2+b^2\right)}-\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5b^2e\cot^2(c+dx)-4abe\cot(c+dx)+(4a^2+5b^2)e}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^2}dx}{4ae\left(a^2+b^2\right)}-\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5b^2e\tan\left(c+dx+\frac{\pi}{2}\right)^2+4abe\tan\left(c+dx+\frac{\pi}{2}\right)+(4a^2+5b^2)e}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4ae\left(a^2+b^2\right)}-\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{4132}$$

$$\frac{-\frac{\int -\frac{-16be^2\cot(c+dx)a^3+\left(8a^4+31b^2a^2+15b^4\right)e^2+3b^2\left(13a^2+5b^2\right)e^2\cot^2(c+dx)}{2(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}dx}{ae(a^2+b^2)}-\frac{b^2\left(13a^2+5b^2\right)}{ad(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}}{4ae\left(a^2+b^2\right)}-\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{-16be^2\cot(c+dx)a^3+\left(8a^4+31b^2a^2+15b^4\right)e^2+3b^2\left(13a^2+5b^2\right)e^2\cot^2(c+dx)}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))}dx}{2ae(a^2+b^2)}-\frac{b^2(13a^2+5b^2)}{ad(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}}{4ae\left(a^2+b^2\right)}-\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{16be^2\tan\left(c+dx+\frac{\pi}{2}\right)a^3+\left(8a^4+31b^2a^2+15b^4\right)e^2+3b^2\left(13a^2+5b^2\right)e^2\tan\left(c+dx+\frac{\pi}{2}\right)^2}{\left(-e\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2ae(a^2+b^2)}-\frac{b^2(13a^2+5b^2)}{ad(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}}{4ae\left(a^2+b^2\right)}-\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 4132

$$\frac{\frac{\frac{2\int -\frac{b\left(8a^4+31b^2a^2+15b^4\right)\cot^2(c+dx)e^4+b\left(24a^4+31b^2a^2+15b^4\right)e^4+8a^3\left(a^2-b^2\right)\cot(c+dx)e^4}{2\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae^3}+\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}}{2ae(a^2+b^2)}-\frac{b^2(13a^2+5b^2)}{ad(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}}{4ae\left(a^2+b^2\right)}-\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\int \frac{b\left(8a^4+31b^2a^2+15b^4\right)\cot^2(c+dx)e^4+b\left(24a^4+31b^2a^2+15b^4\right)e^4+8a^3\left(a^2-b^2\right)\cot(c+dx)e^4}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{ae^3}}{2ae(a^2+b^2)}-\frac{b^2(13a^2+5b^2)}{ad(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}}{4ae\left(a^2+b^2\right)}-\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\int \frac{b\left(8a^4+31b^2a^2+15b^4\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2e^4+b\left(24a^4+31b^2a^2+15b^4\right)e^4-8a^3\left(a^2-b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)e^4}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{ae^3}}{2ae(a^2+b^2)}-\frac{b^2(13a^2+5b^2)}{ad(a^2+b^2)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}}{4ae\left(a^2+b^2\right)}-\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 4136

3.87. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int\frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}+\frac{\int\frac{8\left(a^3b\left(3a^2-b^2\right)e^4+a^4\left(a^2-3b^2\right)\cot(c+dx)e^4\right)}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{ae^3}}{2ae(a^2+b^2)}-\frac{}{ad(a^2+b^2)\cdots}}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int\frac{\cot^2(c+dx)+1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}dx}{a^2+b^2}+\frac{8\int\frac{a^3b\left(3a^2-b^2\right)e^4+a^4\left(a^2-3b^2\right)\cot(c+dx)e^4}{\sqrt{e\cot(c+dx)}}dx}{a^2+b^2}}{ae^3}}{2ae(a^2+b^2)}-\frac{}{ad(a^2+b^2)\sqrt{\cdots}}}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{8\int\frac{a^3b\left(3a^2-b^2\right)e^4-a^4\left(a^2-3b^2\right)e^4\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{ae^3}}{2ae(a^2+b^2)}-\cdots}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\downarrow$ 4017

$$\frac{\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{16\int-\frac{a^3e^4\left(b\left(3a^2-b^2\right)e+a\left(a^2-3b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot\cdots}}{d\left(a^2+b^2\right)}}{ae^3}}{2ae(a^2+b^2)}\cdots}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\downarrow$ 25

3.87. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16\int \frac{a^3e^4\left(b\left(3a^2-b^2\right)e+a\left(a^2-3b^2\right)\cot(c+dx)e\right)}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot}}{d\left(a^2+b^2\right)}}{ae^3}}{2ae(a^2+b^2)}}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3e^4\int \frac{b\left(3a^2-b^2\right)e+a\left(a^2-3b^2\right)\cot(c+dx)e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot(c}}{d\left(a^2+b^2\right)}}{ae^3}}{2ae(a^2+b^2)}}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 1482

$$\frac{\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-16a^3e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\int \frac{\cot(c+dx)e+e}{\cot^2(c+dx)e^2+e^2}d\sqrt{e\cot}\right.}{ae^3}}{2ae(a^2+b^2)}}{4ae\left(a^2+b^2\right)}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 1476

$$\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-16a^3e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)e+e-\sqrt{2}\sqrt{e}}\right.\right.}{}}{2ae(a^2+b^2)}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 1082

3.87. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\int\frac{1}{-e\cot(c+dx)-1}d\left(1-\cdots\right.}{\sqrt{2}\sqrt{e}\cdots}\right.\right.}{ae^3}}{2ae(a^2+b^2)}}{4\cdots}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 217

$$\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\cdots\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{ae^3}}{2ae(a^2+b^2)}}{4ae\left(a^2+\cdots\right.}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 1479

$$\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\cdots\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\cdots\right.}{\sqrt{2}\sqrt{e}}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$\Big\downarrow$ 27

3.87. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\cdots\right)}{\sqrt{2}\sqrt{e}}\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-e\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\cdots\right)}{\sqrt{2}\sqrt{e}}\cdots\right.\right.}{\cdots}}{\cdots}}{2ae(a^2+\cdots}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 4117$$

$$\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{b^3e^4\left(63a^4+46a^2b^2+15b^4\right)\int\frac{1}{\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}-\frac{16a^3e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}\cdots\right)}{\sqrt{2}\sqrt{e}}\cdots\right.\right.}{\cdots}}{\cdots}}{2ae(a^2+\cdots}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{-\frac{2b^3e^3\left(63a^4+46a^2b^2+15b^4\right)\int\frac{1}{\frac{b\cot^2(c+dx)}{e}+a}d\sqrt{e\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{16a^3e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\mathrm{arc}\cdots}{\cdots}\right.\right.}{\cdots}}{\cdots}}{2ae(a^2+b^2)\cdots}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 218$$

3.87. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx$

$$\frac{\frac{2e\left(8a^4+31a^2b^2+15b^4\right)}{ad\sqrt{e\cot(c+dx)}}-\frac{\frac{2b^{5/2}e^{7/2}\left(63a^4+46a^2b^2+15b^4\right)\arctan\left(\frac{\sqrt{b}\cot(c+dx)}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}d\left(a^2+b^2\right)}-\frac{16a^3e^4\left(\frac{1}{2}(a-b)\left(a^2+4ab+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{e\cot(c+dx)}}{\sqrt{e}}+1\right)}{\sqrt{2}\sqrt{e}}-\frac{\arctan\left(1-\right.}{}\right.\right.}{a}}{2ae(a^2+b^2)}$$

$$\frac{b^2}{2ade\left(a^2+b^2\right)\sqrt{e\cot(c+dx)}(a+b\cot(c+dx))^2}$$

input
```
Int[1/((e*Cot[c + d*x])^(3/2)*(a + b*Cot[c + d*x])^3),x]
```

output
```
-1/2*b^2/(a*(a^2 + b^2)*d*e*Sqrt[e*Cot[c + d*x]]*(a + b*Cot[c + d*x])^2) +
 (-((b^2*(13*a^2 + 5*b^2))/(a*(a^2 + b^2)*d*Sqrt[e*Cot[c + d*x]]*(a + b*Co
t[c + d*x]))) + ((2*(8*a^4 + 31*a^2*b^2 + 15*b^4)*e)/(a*d*Sqrt[e*Cot[c + d
*x]]) - ((2*b^(5/2)*(63*a^4 + 46*a^2*b^2 + 15*b^4)*e^(7/2)*ArcTan[(Sqrt[b]
*Cot[c + d*x])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*(a^2 + b^2)*d) - (16*a^3*e^4*(
((a - b)*(a^2 + 4*a*b + b^2)*(-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/
Sqrt[e]]/(Sqrt[2]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e*Cot[c + d*x]])/Sq
rt[e]]/(Sqrt[2]*Sqrt[e])))/2 - ((a + b)*(a^2 - 4*a*b + b^2)*(-1/2*Log[e +
e*Cot[c + d*x] - Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(Sqrt[2]*Sqrt[e]) +
 Log[e + e*Cot[c + d*x] + Sqrt[2]*Sqrt[e]*Sqrt[e*Cot[c + d*x]]]/(2*Sqrt[2]
*Sqrt[e])))/2))/((a^2 + b^2)*d))/(a*e^3))/(2*a*(a^2 + b^2)*e))/(4*a*(a^2 +
 b^2)*e)
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.87. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx$

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.87. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx$

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.87.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.91

method	result
derivativedivides	$-\frac{2e^4\left(-\frac{b^3\left(\frac{\left(\frac{15}{8}a^4b+\frac{11}{4}a^2b^3+\frac{7}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}+\frac{ae\left(17a^4+26a^2b^2+9b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(63a^4+46a^2b^2+15b^4\right)\arctan\left(\sqrt{e\cdots}\right.}{8\sqrt{aeb}}\cdots\right)}{a^3e^5\left(a^2+b^2\right)^3}\cdots\right)}{\cdots}$
default	$-\frac{2e^4\left(-\frac{b^3\left(\frac{\left(\frac{15}{8}a^4b+\frac{11}{4}a^2b^3+\frac{7}{8}b^5\right)(e\cot(dx+c))^{\frac{3}{2}}+\frac{ae\left(17a^4+26a^2b^2+9b^4\right)\sqrt{e\cot(dx+c)}}{8}}{(e\cot(dx+c)b+ae)^2}+\frac{\left(63a^4+46a^2b^2+15b^4\right)\arctan\left(\sqrt{e\cdots}\right.}{8\sqrt{aeb}}\cdots\right)}{a^3e^5\left(a^2+b^2\right)^3}\cdots\right)}{\cdots}$

input
```
int(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/d*e^4*(-b^3/a^3/e^5/(a^2+b^2)^3*(((15/8*a^4*b+11/4*a^2*b^3+7/8*b^5)*(e*
cot(d*x+c))^(3/2)+1/8*a*e*(17*a^4+26*a^2*b^2+9*b^4)*(e*cot(d*x+c))^(1/2))/
(e*cot(d*x+c)*b+a*e)^2+1/8*(63*a^4+46*a^2*b^2+15*b^4)/(a*e*b)^(1/2)*arctan
((e*cot(d*x+c))^(1/2)*b/(a*e*b)^(1/2)))+1/(a^2+b^2)^3/e^5*(1/8*(-3*a^2*b*e
+b^3*e)*(e^2)^(1/4)/e^2*2^(1/2)*(ln((e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c
))^(1/2)*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/
2)*2^(1/2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
+1)-2*arctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1))+1/8*(-a^3+3*a*b
^2)/(e^2)^(1/4)*2^(1/2)*(ln((e*cot(d*x+c)-(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)
*2^(1/2)+(e^2)^(1/2))/(e*cot(d*x+c)+(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)*2^(1/
2)+(e^2)^(1/2)))+2*arctan(2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)-2*ar
ctan(-2^(1/2)/(e^2)^(1/4)*(e*cot(d*x+c))^(1/2)+1)))-1/a^3/e^5/(e*cot(d*x+c
))^(1/2))
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5133 vs. $2(454) = 908$.

Time = 2.31 (sec) , antiderivative size = 10308, normalized size of antiderivative = 19.49

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.87.6 Sympy [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx = \int \frac{1}{(e\cot(c+dx))^{\frac{3}{2}}\,(a+b\cot(c+dx))^3}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))**(3/2)/(a+b*cot(d*x+c))**3,x)
```

output
```
Integral(1/((e*cot(c + d*x))**(3/2)*(a + b*cot(c + d*x))**3), x)
```

3.87.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.87.8 Giac [F]

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx = \int \frac{1}{(b\cot(dx+c)+a)^3(e\cot(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*cot(d*x+c))^(3/2)/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*cot(d*x + c) + a)^3*(e*cot(d*x + c))^(3/2)), x)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 21158, normalized size of antiderivative = 40.00

$$\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/((e*cot(c + d*x))^(3/2)*(a + b*cot(c + d*x))^3),x)
```

output

```
((2*e)/a + (e*cot(c + d*x)*(16*a^4*b + 25*b^5 + 49*a^2*b^3))/(4*a^2*(a^4 +
 b^4 + 2*a^2*b^2)) + (b^2*e^2*cot(c + d*x)^2*(8*a^4 + 15*b^4 + 31*a^2*b^2)
)/(4*a^3*(a^4*e + b^4*e + 2*a^2*b^2*e)))/(b^2*d*(e*cot(c + d*x))^(5/2) + a
^2*d*e^2*(e*cot(c + d*x))^(1/2) + 2*a*b*d*e*(e*cot(c + d*x))^(3/2)) + atan
((((-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5*b*d^2*e^3*6
i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i + 15*a^4*b^2*d^2*e^3)))^(1/2)
*(((e*cot(c + d*x))^(1/2)*(471859200*a^22*b^44*d^7*e^16 + 9500098560*a^24*
b^42*d^7*e^16 + 91857354752*a^26*b^40*d^7*e^16 + 564502986752*a^28*b^38*d^
7*e^16 + 2464648527872*a^30*b^36*d^7*e^16 + 8104469069824*a^32*b^34*d^7*e^
16 + 20769933361152*a^34*b^32*d^7*e^16 + 42351565209600*a^36*b^30*d^7*e^16
 + 69534945902592*a^38*b^28*d^7*e^16 + 92434029608960*a^40*b^26*d^7*e^16 +
 99508717355008*a^42*b^24*d^7*e^16 + 86342935511040*a^44*b^22*d^7*e^16 + 5
9767095558144*a^46*b^20*d^7*e^16 + 32432589897728*a^48*b^18*d^7*e^16 + 134
11815522304*a^50*b^16*d^7*e^16 + 4030457708544*a^52*b^14*d^7*e^16 + 805425
905664*a^54*b^12*d^7*e^16 + 86608183296*a^56*b^10*d^7*e^16 + 1612709888*a^
58*b^8*d^7*e^16 + 16777216*a^60*b^6*d^7*e^16 + 167772160*a^62*b^4*d^7*e^16
 + 16777216*a^64*b^2*d^7*e^16) + (-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^
5*d^2*e^3*6i + a^5*b*d^2*e^3*6i - 15*a^2*b^4*d^2*e^3 - a^3*b^3*d^2*e^3*20i
 + 15*a^4*b^2*d^2*e^3)))^(1/2)*(251658240*a^24*b^45*d^8*e^18 - (e*cot(c +
d*x))^(1/2)*(-1i/(4*(b^6*d^2*e^3 - a^6*d^2*e^3 + a*b^5*d^2*e^3*6i + a^5...
```

3.87. $\int \frac{1}{(e\cot(c+dx))^{3/2}(a+b\cot(c+dx))^3}\,dx$

3.88 $\int (a+b\cot(c+dx))^n\,dx$

3.88.1 Optimal result

Integrand size = 12, antiderivative size = 167

$$\int (a+b\cot(c+dx))^n\,dx = -\frac{b(a+b\cot(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\cot(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)d(1+n)} + \frac{b(a+b\cot(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\cot(c+dx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2}\left(a+\sqrt{-b^2}\right)d(1+n)}$$

output
```
-1/2*b*(a+b*cot(d*x+c))^(1+n)*hypergeom([1, 1+n],[2+n],(a+b*cot(d*x+c))/(a
-(-b^2)^(1/2)))/d/(1+n)/(a-(-b^2)^(1/2))/(-b^2)^(1/2)+1/2*b*(a+b*cot(d*x+c
))^(1+n)*hypergeom([1, 1+n],[2+n],(a+b*cot(d*x+c))/(a+(-b^2)^(1/2)))/d/(1+
n)/(-b^2)^(1/2)/(a+(-b^2)^(1/2))
```

3.88.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.71

$$\int (a+b\cot(c+dx))^n\,dx = \frac{(a+b\cot(c+dx))^{1+n}\left((a+ib)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\cot(c+dx)}{a-ib}\right)-(a-ib)\,\text{Hypergeom}\right.}{2(a-ib)(-ia+b)d(1+n)}$$

input
```
Integrate[(a + b*Cot[c + d*x])^n,x]
```

output
```
((a + b*Cot[c + d*x])^(1 + n)*((a + I*b)*Hypergeometric2F1[1, 1 + n, 2 + n
, (a + b*Cot[c + d*x])/(a - I*b)] - (a - I*b)*Hypergeometric2F1[1, 1 + n,
2 + n, (a + b*Cot[c + d*x])/(a + I*b)]))/(2*(a - I*b)*((-I)*a + b)*d*(1 +
n))
```

3.88.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3966, 485, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cot(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3966}$$

$$-\frac{b\int \frac{(a+b\cot(c+dx))^n}{\cot^2(c+dx)b^2+b^2}d(b\cot(c+dx))}{d}$$

$$\Big\downarrow \text{485}$$

$$-\frac{b\int \left(\frac{\sqrt{-b^2}(a+b\cot(c+dx))^n}{2b^2\left(\sqrt{-b^2}-b\cot(c+dx)\right)}+\frac{\sqrt{-b^2}(a+b\cot(c+dx))^n}{2b^2\left(b\cot(c+dx)+\sqrt{-b^2}\right)}\right)d(b\cot(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{(a+b\cot(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\cot(c+dx)}{a-\sqrt{-b^2}}\right)}{2\sqrt{-b^2}(n+1)\left(a-\sqrt{-b^2}\right)}-\frac{(a+b\cot(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\cot(c+dx)}{a+\sqrt{-b^2}}\right)}{2\sqrt{-b^2}(n+1)\left(a+\sqrt{-b^2}\right)}\right)}{d}$$

input
```
Int[(a + b*Cot[c + d*x])^n,x]
```

output
```
-((b*(((a + b*Cot[c + d*x])^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a
+ b*Cot[c + d*x])/(a - Sqrt[-b^2])])/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])*(1 + n
)) - ((a + b*Cot[c + d*x])^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a +
 b*Cot[c + d*x])/(a + Sqrt[-b^2])])/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*(1 + n)
)))/d)
```

3.88.3.1 Defintions of rubi rules used

rule 485
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(c + d*x)^n, 1/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d, n}, x] &
&  !IntegerQ[2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

3.88.4 Maple [F]

$$\int (a+b\cot (dx+c))^n\,dx$$

input
```
int((a+b*cot(d*x+c))^n,x)
```

output
```
int((a+b*cot(d*x+c))^n,x)
```

3.88.5 Fricas [F]

$$\int (a+b\cot(c+dx))^n\,dx = \int (b\cot(dx+c)+a)^n\,dx$$

input
```
integrate((a+b*cot(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*cot(d*x + c) + a)^n, x)
```

3.88.6 Sympy [F]

$$\int (a+b\cot(c+dx))^n\,dx = \int (a+b\cot(c+dx))^n\,dx$$

input
```
integrate((a+b*cot(d*x+c))**n,x)
```

output
```
Integral((a + b*cot(c + d*x))**n, x)
```

3.88.7 Maxima [F]

$$\int (a+b\cot(c+dx))^n\,dx = \int (b\cot(dx+c)+a)^n\,dx$$

input
```
integrate((a+b*cot(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*cot(d*x + c) + a)^n, x)
```

3.88.8 Giac [F]

$$\int (a+b\cot(c+dx))^n\,dx = \int (b\cot(dx+c)+a)^n\,dx$$

input
```
integrate((a+b*cot(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^n, x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cot(c+dx))^n\,dx = \int (a+b\cot(c+d\,x))^n\,dx$$

input
```
int((a + b*cot(c + d*x))^n,x)
```

output
```
int((a + b*cot(c + d*x))^n, x)
```

3.89 $\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx$

3.89.1 Optimal result

Integrand size = 23, antiderivative size = 193

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx =$$

$$-\frac{\text{AppellF1}\left(1-n,-m,1,2-n,-\frac{b\cot(e+fx)}{a},-i\cot(e+fx)\right)\cot(e+fx)(a+b\cot(e+fx))^m\left(1+\frac{b\cot}{\ }\right.}{2f(1-n)}$$

$$-\frac{\text{AppellF1}\left(1-n,-m,1,2-n,-\frac{b\cot(e+fx)}{a},i\cot(e+fx)\right)\cot(e+fx)(a+b\cot(e+fx))^m\left(1+\frac{b\cot(e}{a}\right.}{2f(1-n)}$$

output
```
-1/2*AppellF1(1-n,1,-m,2-n,-I*cot(f*x+e),-b*cot(f*x+e)/a)*cot(f*x+e)*(a+b*
cot(f*x+e))^m*(d*tan(f*x+e))^n/f/(1-n)/((1+b*cot(f*x+e)/a)^m)-1/2*AppellF1
(1-n,1,-m,2-n,I*cot(f*x+e),-b*cot(f*x+e)/a)*cot(f*x+e)*(a+b*cot(f*x+e))^m*
(d*tan(f*x+e))^n/f/(1-n)/((1+b*cot(f*x+e)/a)^m)
```

3.89.2 Mathematica [F]

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = \int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx$$

input
```
Integrate[(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n,x]
```

output
```
Integrate[(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n, x]
```

3.89.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4730, 3042, 4058, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^n(a+b\cot(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\tan(e+fx))^n(a+b\cot(e+fx))^m dx$$

$$\Big\downarrow \text{4730}$$

$$(d\tan(e+fx))^n(d\cot(e+fx))^n\int (d\cot(e+fx))^{-n}(a+b\cot(e+fx))^m dx$$

$$\Big\downarrow \text{3042}$$

$$(d\tan(e+fx))^n(d\cot(e+fx))^n\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a-b\tan\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{4058}$$

$$-\frac{(d\tan(e+fx))^n(d\cot(e+fx))^n\int \frac{(d\cot(e+fx))^{-n}(a+b\cot(e+fx))^m}{\cot^2(e+fx)+1}d\cot(e+fx)}{f}$$

$$\Big\downarrow \text{615}$$

$$-\frac{(d\tan(e+fx))^n(d\cot(e+fx))^n\int \left(\frac{i(d\cot(e+fx))^{-n}(a+b\cot(e+fx))^m}{2(i-\cot(e+fx))}+\frac{i(d\cot(e+fx))^{-n}(a+b\cot(e+fx))^m}{2(\cot(e+fx)+i)}\right)d\cot(e+fx}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(d\tan(e+fx))^n(d\cot(e+fx))^n\left(\frac{(d\cot(e+fx))^{1-n}(a+b\cot(e+fx))^m\left(\frac{b\cot(e+fx)}{a}+1\right)^{-m}\text{AppellF1}\left(1-n,-m,1,2-n,-\frac{b\cot(e+fx)}{a}\right.}{2d(1-n)}}{f}$$

input
```
Int[(a + b*Cot[e + f*x])^m*(d*Tan[e + f*x])^n,x]
```

output
```
-(((d*Cot[e + f*x])^n*((AppellF1[1 - n, -m, 1, 2 - n, -((b*Cot[e + f*x])/a
), (-I)*Cot[e + f*x]]*(d*Cot[e + f*x])^(1 - n)*(a + b*Cot[e + f*x])^m)/(2*
d*(1 - n)*(1 + (b*Cot[e + f*x])/a)^m) + (AppellF1[1 - n, -m, 1, 2 - n, -((
b*Cot[e + f*x])/a), I*Cot[e + f*x]]*(d*Cot[e + f*x])^(1 - n)*(a + b*Cot[e
+ f*x])^m)/(2*d*(1 - n)*(1 + (b*Cot[e + f*x])/a)^m))*(d*Tan[e + f*x])^n)/f
)
```

3.89.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4058
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
 Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4730
```
Int[(u_)*((c_.)*tan[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Cot[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownCotangentIntegrandQ[u
, x]
```

3.89.4 Maple [F]

$$\int (a+b\cot (fx+e))^{m} (d\tan (fx+e))^{n} dx$$

input
```
int((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x)
```

output
```
int((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x)
```

3.89.5 Fricas [F]

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = \int (b\cot (fx+e)+a)^{m}(d\tan (fx+e))^{n}\,dx$$

input
```
integrate((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*cot(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

3.89.6 Sympy [F]

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = \int (d\tan (e+fx))^{n} (a+b\cot (e+fx))^{m}\,dx$$

input
```
integrate((a+b*cot(f*x+e))**m*(d*tan(f*x+e))**n,x)
```

output
```
Integral((d*tan(e + f*x))**n*(a + b*cot(e + f*x))**m, x)
```

3.89.7 Maxima [F(-2)]

Exception generated.

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.89.8 Giac [F]

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = \int (b\cot(fx+e)+a)^m(d\tan(fx+e))^n\,dx$$

input
```
integrate((a+b*cot(f*x+e))^m*(d*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*cot(f*x + e) + a)^m*(d*tan(f*x + e))^n, x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cot(e+fx))^m(d\tan(e+fx))^n\,dx = \int (d\tan(e+f\,x))^n\,(a+b\cot(e+f\,x))^m\,dx$$

input
```
int((d*tan(e + f*x))^n*(a + b*cot(e + f*x))^m,x)
```

output
```
int((d*tan(e + f*x))^n*(a + b*cot(e + f*x))^m, x)
```

3.90 $\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

3.90.1 Optimal result

Integrand size = 27, antiderivative size = 45

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{2i\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d}$$

output
```
2*I*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{2i\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d}$$

input
```
Integrate[(1 + I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

output
```
((2*I)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)
```

3.90.3 Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-i\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4020}$$

$$-\frac{i\int -\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int \frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2\int \frac{1}{\frac{i\cot^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\arctan\left(\frac{\cot(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

input
```
Int[(1 + I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

output
```
(-2*ArcTan[Cot[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)
```

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

3.90.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 733 vs. 2(36) = 72.

Time = 0.42 (sec) , antiderivative size = 734, normalized size of antiderivative = 16.31

method	result
derivativedivides	$-\frac{\frac{\left(2i\sqrt{a^2+b^2}\,a^2+i\sqrt{a^2+b^2}\,b^2+2ia^3+2ia\,b^2-\sqrt{a^2+b^2}\,ab-a^2b-b^3\right)\ln\left(b\cot(dx+c)+a-\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right.}{2}}{}$
default	$-\frac{\frac{\left(2i\sqrt{a^2+b^2}\,a^2+i\sqrt{a^2+b^2}\,b^2+2ia^3+2ia\,b^2-\sqrt{a^2+b^2}\,ab-a^2b-b^3\right)\ln\left(b\cot(dx+c)+a-\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right.}{2}}{}$
parts	Expression too large to display

input
```
int((1+I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a
^2+b^2)*(1/2*(2*I*(a^2+b^2)^(1/2)*a^2+I*(a^2+b^2)^(1/2)*b^2+2*I*a^3+2*I*a*
b^2-(a^2+b^2)^(1/2)*a*b-a^2*b-b^3)*ln(b*cot(d*x+c)+a-(a+b*cot(d*x+c))^(1/2
)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-I*(a^2+b^2)^(1/2)*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-I*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-I*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2+(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*a*b+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b
^3+1/2*(2*I*(a^2+b^2)^(1/2)*a^2+I*(a^2+b^2)^(1/2)*b^2+2*I*a^3+2*I*a*b^2-(a
^2+b^2)^(1/2)*a*b-a^2*b-b^3)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(
1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(
1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))-1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2
+b^2)^(1/2)*(1/2*(-I*(a^2+b^2)^(1/2)-I*a+b)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x
+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-I*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b-1/2*(-I*(a^2+b^2)^(1
/2)-I*a+b)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*ar
ctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)
^(1/2)-2*a)^(1/2))))
```

3.90. $\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

3.90.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(33) = 66$.

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.53

$$\int \frac{1 + i\cot(c + dx)}{\sqrt{a + b\cot(c + dx)}}\,dx = -\frac{1}{2}\sqrt{-\frac{4i}{(i\,a+b)d^2}}\log\left(\frac{1}{2}(i\,a+b)d\sqrt{-\frac{4i}{(i\,a+b)d^2}} + \sqrt{\frac{(a+i\,b)e^{(2i\,dx+2i\,c)} - a + i\,b}{e^{(2i\,dx+2i\,c)} - 1}}\right) + \frac{1}{2}\sqrt{-\frac{4i}{(i\,a+b)d^2}}\log\left(\frac{1}{2}(-i\,a-b)d\sqrt{-\frac{4i}{(i\,a+b)d^2}} + \sqrt{\frac{(a+i\,b)e^{(2i\,dx+2i\,c)} - a + i\,b}{e^{(2i\,dx+2i\,c)} - 1}}\right)$$

input
```
integrate((1+I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-4*I/((I*a + b)*d^2))*log(1/2*(I*a + b)*d*sqrt(-4*I/((I*a + b)*d
^2)) + sqrt(((a + I*b)*e^(2*I*d*x + 2*I*c) - a + I*b)/(e^(2*I*d*x + 2*I*c)
 - 1))) + 1/2*sqrt(-4*I/((I*a + b)*d^2))*log(1/2*(-I*a - b)*d*sqrt(-4*I/((
I*a + b)*d^2)) + sqrt(((a + I*b)*e^(2*I*d*x + 2*I*c) - a + I*b)/(e^(2*I*d*
x + 2*I*c) - 1)))
```

3.90.6 Sympy [F]

$$\int \frac{1 + i\cot(c + dx)}{\sqrt{a + b\cot(c + dx)}}\,dx = i\left(\int \left(-\frac{i}{\sqrt{a + b\cot(c + dx)}}\right)dx + \int \frac{\cot(c + dx)}{\sqrt{a + b\cot(c + dx)}}\,dx\right)$$

input
```
integrate((1+I*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
```

output
```
I*(Integral(-I/sqrt(a + b*cot(c + d*x)), x) + Integral(cot(c + d*x)/sqrt(a
 + b*cot(c + d*x)), x))
```

3.90.7 Maxima [F]

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \int \frac{i\,\cot\left(dx+c\right)+1}{\sqrt{b\cot\left(dx+c\right)+a}}\,dx$$

input
```
integrate((1+I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

3.90.8 Giac [F]

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \int \frac{i\,\cot\left(dx+c\right)+1}{\sqrt{b\cot\left(dx+c\right)+a}}\,dx$$

input
```
integrate((1+I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 15.16 (sec) , antiderivative size = 1410, normalized size of antiderivative = 31.33

$$\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)*1i + 1)/(a + b*cot(c + d*x))^(1/2),x)
```

output
```
(log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*cot(c + d*x))^(1/2) + 1i)*(-1/(a
*d^2 - b*d^2*1i))^(1/2))/2 - log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*cot(
c + d*x))^(1/2)*1i + 1)*(-1/(4*(a*d^2 - b*d^2*1i)))^(1/2) + (log(16*b^3*d*
(-1/(d^2*(a - b*1i)))^(1/2) - 16*b^2*(a + b*cot(c + d*x))^(1/2) + (16*a*b^
2*(a + b*cot(c + d*x))^(1/2))/(a - b*1i))*(-1/(a*d^2 - b*d^2*1i))^(1/2))/2
 - log(16*b^2*(a + b*cot(c + d*x))^(1/2) + 16*b^3*d*(-1/(d^2*(a - b*1i)))^
(1/2) - (16*a*b^2*(a + b*cot(c + d*x))^(1/2))/(a - b*1i))*(-1/(4*(a*d^2 -
b*d^2*1i)))^(1/2) - 2*atanh((32*b^2*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*
a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((b^4*d^2*64i)/(4
*a^2*d^3 + 4*b^2*d^3) - (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^3*(
a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 +
 4*b^2*d^2))^(1/2)*128i)/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^
4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2
*d^3) - (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*(a + b*cot
(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^
2))^(1/2))/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4
*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a
*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^
(1/2) - 2*atanh((32*b^2*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*
b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((a^2*b^2*d^2*64i)/(4*a^2*...
```

3.90. $\int \frac{1+i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

3.91 $\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

3.91.1 Optimal result

Integrand size = 27, antiderivative size = 45

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=-\frac{2i\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}$$

output
```
-2*I*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=-\frac{2i\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}\,d}$$

input
```
Integrate[(1 - I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

output
```
((-2*I)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)
```

3.91.3 Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1+i\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4020}$$

$$\frac{i\int -\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i\int \frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2\int \frac{1}{-\frac{i\cot^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\arctan\left(\frac{\cot(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

input
```
Int[(1 - I*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

output
```
(-2*ArcTan[Cot[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

3.91.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 739, normalized size of antiderivative = 16.42

method	result
derivativedivides	$-\frac{\frac{\left(-2i\sqrt{a^2+b^2}\,a^2-i\sqrt{a^2+b^2}\,b^2-2ia^3-2ia\,b^2-\sqrt{a^2+b^2}\,ab-a^2b-b^3\right)\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\cot(dx+c)-\sqrt{a^2+b^2}-\right.}{2}}{}$
default	$-\frac{\frac{\left(-2i\sqrt{a^2+b^2}\,a^2-i\sqrt{a^2+b^2}\,b^2-2ia^3-2ia\,b^2-\sqrt{a^2+b^2}\,ab-a^2b-b^3\right)\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\cot(dx+c)-\sqrt{a^2+b^2}-\right.}{2}}{}$
parts	Expression too large to display

input
```
int((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a^
2+b^2)*(-1/2*(-2*I*(a^2+b^2)^(1/2)*a^2-I*(a^2+b^2)^(1/2)*b^2-2*I*a^3-2*I*a
*b^2-(a^2+b^2)^(1/2)*a*b-a^2*b-b^3)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)+2*(I*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+I*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+I*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)*a*b^2+(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*a*b+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2*b+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b
^3+1/2*(-2*I*(a^2+b^2)^(1/2)*a^2-I*(a^2+b^2)^(1/2)*b^2-2*I*a^3-2*I*a*b^2-(
a^2+b^2)^(1/2)*a*b-a^2*b-b^3)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^
(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)
^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a
^2+b^2)^(1/2)*(1/2*(-I*(a^2+b^2)^(1/2)-I*a-b)*ln(b*cot(d*x+c)+a+(a+b*cot(d
*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-I*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*a-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b-1/2*(-I*(a^2+b^2)^
(1/2)-I*a-b)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*
arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2))))
```

3.91. $\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

3.91.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(33) = 66$.

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.53

$$\int \frac{1 - i\cot(c + dx)}{\sqrt{a + b\cot(c + dx)}}\,dx = \frac{1}{2}\sqrt{\frac{4i}{(-ia+b)d^2}}\log\left(\frac{1}{2}(ia-b)d\sqrt{\frac{4i}{(-ia+b)d^2}} + \sqrt{\frac{(a+ib)e^{(2i\,dx+2i\,c)} - a + ib}{e^{(2i\,dx+2i\,c)} - 1}}\right) - \frac{1}{2}\sqrt{\frac{4i}{(-ia+b)d^2}}\log\left(\frac{1}{2}(-ia+b)d\sqrt{\frac{4i}{(-ia+b)d^2}} + \sqrt{\frac{(a+ib)e^{(2i\,dx+2i\,c)} - a + ib}{e^{(2i\,dx+2i\,c)} - 1}}\right)$$

input
```
integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(4*I/((-I*a + b)*d^2))*log(1/2*(I*a - b)*d*sqrt(4*I/((-I*a + b)*d^
2)) + sqrt(((a + I*b)*e^(2*I*d*x + 2*I*c) - a + I*b)/(e^(2*I*d*x + 2*I*c)
- 1))) - 1/2*sqrt(4*I/((-I*a + b)*d^2))*log(1/2*(-I*a + b)*d*sqrt(4*I/((-I
*a + b)*d^2)) + sqrt(((a + I*b)*e^(2*I*d*x + 2*I*c) - a + I*b)/(e^(2*I*d*x
 + 2*I*c) - 1)))
```

3.91.6 Sympy [F]

$$\int \frac{1 - i\cot(c + dx)}{\sqrt{a + b\cot(c + dx)}}\,dx = -i\left(\int \frac{i}{\sqrt{a + b\cot(c + dx)}}\,dx + \int \frac{\cot(c + dx)}{\sqrt{a + b\cot(c + dx)}}\,dx\right)$$

input
```
integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
```

output
```
-I*(Integral(I/sqrt(a + b*cot(c + d*x)), x) + Integral(cot(c + d*x)/sqrt(a
 + b*cot(c + d*x)), x))
```

3.91.7 Maxima [F]

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\int \frac{-i\,\cot\left(dx+c\right)+1}{\sqrt{b\cot\left(dx+c\right)+a}}\,dx$$

input
```
integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

3.91.8 Giac [F]

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\int \frac{-i\,\cot\left(dx+c\right)+1}{\sqrt{b\cot\left(dx+c\right)+a}}\,dx$$

input
```
integrate((1-I*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((-I*cot(d*x + c) + 1)/sqrt(b*cot(d*x + c) + a), x)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 14.07 (sec) , antiderivative size = 1410, normalized size of antiderivative = 31.33

$$\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\text{Too large to display}$$

input
```
int(-(cot(c + d*x)*1i - 1)/(a + b*cot(c + d*x))^(1/2),x)
```

output
```
(log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*1i + 1)*(-1/
(a*d^2 - b*d^2*1i))^(1/2))/2 - log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*co
t(c + d*x))^(1/2) + 1i)*(-1/(4*(a*d^2 - b*d^2*1i)))^(1/2) + (log(16*b^3*d*
(-1/(d^2*(a - b*1i)))^(1/2) - 16*b^2*(a + b*cot(c + d*x))^(1/2) + (16*a*b^
2*(a + b*cot(c + d*x))^(1/2))/(a - b*1i))*(-1/(a*d^2 - b*d^2*1i))^(1/2))/2
 - log(16*b^2*(a + b*cot(c + d*x))^(1/2) + 16*b^3*d*(-1/(d^2*(a - b*1i)))^
(1/2) - (16*a*b^2*(a + b*cot(c + d*x))^(1/2))/(a - b*1i))*(-1/(4*(a*d^2 -
b*d^2*1i)))^(1/2) - 2*atanh((32*b^2*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*
a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((b^4*d^2*64i)/(4
*a^2*d^3 + 4*b^2*d^3) - (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^3*(
a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 +
 4*b^2*d^2))^(1/2)*128i)/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^
4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2
*d^3) - (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b^2*(a + b*cot
(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^
2))^(1/2))/((b^6*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (a^2*b^4*d^2*256i)/(4
*a^2*d^3 + 4*b^2*d^3) - (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) - (256*a
*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^
(1/2) + 2*atanh((32*b^2*(a + b*cot(c + d*x))^(1/2)*((b*1i)/(4*a^2*d^2 + 4*
b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2))/((a^2*b^2*d^2*64i)/(4*a^2*...
```

3.91. $\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

3.92 $\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx$

3.92.1 Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx = \frac{(aA+bB)x}{a^2+b^2} - \frac{(Ab-aB)\log(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)\,d}$$

output
```
(A*a+B*b)*x/(a^2+b^2)-(A*b-B*a)*ln(b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d
```

3.92.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx = -\frac{2(aA+bB)\arctan(\cot(c+dx)) + (Ab-aB)\left(2\log(a+b\cot(c+dx)) - \log\left(\csc^2(c+dx)\right)\right)}{2\left(a^2+b^2\right)d}$$

input
```
Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x]),x]
```

output
```
-1/2*(2*(a*A + b*B)*ArcTan[Cot[c + d*x]] + (A*b - a*B)*(2*Log[a + b*Cot[c
+ d*x]] - Log[Csc[c + d*x]^2]))/((a^2 + b^2)*d)
```

3.92.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4014, 25, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-B\tan\left(c+dx+\frac{\pi}{2}\right)}{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4014}$$

$$\frac{x(aA+bB)}{a^2+b^2}-\frac{(Ab-aB)\int -\frac{b-a\cot(c+dx)}{a+b\cot(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(Ab-aB)\int \frac{b-a\cot(c+dx)}{a+b\cot(c+dx)}dx}{a^2+b^2}+\frac{x(aA+bB)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{b+a\tan\left(c+dx+\frac{\pi}{2}\right)}{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2+b^2}+\frac{x(aA+bB)}{a^2+b^2}$$

$$\Big\downarrow \text{4013}$$

$$\frac{x(aA+bB)}{a^2+b^2}-\frac{(Ab-aB)\log(a\sin(c+dx)+b\cos(c+dx))}{d\left(a^2+b^2\right)}$$

input
```
Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x]),x]
```

output
```
((a*A + b*B)*x)/(a^2 + b^2) - ((A*b - a*B)*Log[b*Cos[c + d*x] + a*Sin[c +
d*x]])/((a^2 + b^2)*d)
```

3.92.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.92.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{(-2Ab+2Ba)\ln(a\tan(dx+c)+b)+(Ab-Ba)\ln\left(\sec(dx+c)^2\right)+2dx(Aa+Bb)}{2d(a^2+b^2)}$
norman	$\frac{(Aa+Bb)x}{a^2+b^2}+\frac{(Ab-Ba)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}-\frac{(Ab-Ba)\ln(a\tan(dx+c)+b)}{d(a^2+b^2)}$
derivativedivides	$\frac{\frac{\frac{(Ab-Ba)\ln\left(\cot(dx+c)^2+1\right)}{2}+(-Aa-Bb)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{a^2+b^2}-\frac{(Ab-Ba)\ln(a+b\cot(dx+c))}{a^2+b^2}}{d}$
default	$\frac{\frac{\frac{(Ab-Ba)\ln\left(\cot(dx+c)^2+1\right)}{2}+(-Aa-Bb)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{a^2+b^2}-\frac{(Ab-Ba)\ln(a+b\cot(dx+c))}{a^2+b^2}}{d}$
risch	$\frac{ixB}{ib+a}+\frac{xA}{ib+a}+\frac{2iAbx}{a^2+b^2}-\frac{2iBax}{a^2+b^2}+\frac{2iAbc}{d(a^2+b^2)}-\frac{2iBac}{d(a^2+b^2)}-\frac{\ln\left(e^{2i(dx+c)}+\frac{ib-a}{ib+a}\right)Ab}{d(a^2+b^2)}+\frac{\ln\left(e^{2i(dx+c)}+\frac{ib-a}{ib+a}\right)}{d(a^2+b^2)}$

input
```
int((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*((-2*A*b+2*B*a)*ln(a*tan(d*x+c)+b)+(A*b-B*a)*ln(sec(d*x+c)^2)+2*d*x*(A
*a+B*b))/d/(a^2+b^2)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx = \frac{2\,(Aa+Bb)dx+(Ba-Ab)\log\left(ab\sin\left(2\,dx+2\,c\right)+\frac{1}{2}\,a^2+\frac{1}{2}\,b^2-\frac{1}{2}\,(a^2-b^2)\cos\left(2\,dx+2\,c\right)\right)}{2\,(a^2+b^2)d}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*(A*a + B*b)*d*x + (B*a - A*b)*log(a*b*sin(2*d*x + 2*c) + 1/2*a^2 +
1/2*b^2 - 1/2*(a^2 - b^2)*cos(2*d*x + 2*c)))/((a^2 + b^2)*d)
```

3.92.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 524, normalized size of antiderivative = 8.88

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x(A+B\cot(c))}{\cot(c)} \\ \frac{\frac{A\log\left(\tan^2(c+dx)+1\right)}{2d}+Bx}{b} \\ \frac{iAdx\cot(c+dx)}{2bd\cot(c+dx)-2ibd}+\frac{Adx}{2bd\cot(c+dx)-2ibd}-\frac{iA}{2bd\cot(c+dx)-2ibd}+\frac{Bdx\cot(c+dx)}{2bd\cot(c+dx)-2ibd}-\frac{iBdx}{2bd\cot(c+dx)-2ibd}+\frac{B}{2bd\cot(c+dx)-} \\ -\frac{iAdx\cot(c+dx)}{2bd\cot(c+dx)+2ibd}+\frac{Adx}{2bd\cot(c+dx)+2ibd}+\frac{iA}{2bd\cot(c+dx)+2ibd}+\frac{Bdx\cot(c+dx)}{2bd\cot(c+dx)+2ibd}+\frac{iBdx}{2bd\cot(c+dx)+2ibd}+\frac{B}{2bd\cot(c+dx} \\ \frac{x(A+B\cot(c))}{a+b\cot(c)} \\ \frac{2Aadx}{2a^2d+2b^2d}-\frac{2Ab\log\left(\tan(c+dx)+\frac{b}{a}\right)}{2a^2d+2b^2d}+\frac{Ab\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d}+\frac{2Ba\log\left(\tan(c+dx)+\frac{b}{a}\right)}{2a^2d+2b^2d}-\frac{Ba\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d}+\frac{2Bbdx}{2a^2d+2b^2} \end{cases}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(A + B*cot(c))/cot(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (
(A*log(tan(c + d*x)**2 + 1)/(2*d) + B*x)/b, Eq(a, 0)), (I*A*d*x*cot(c + d*
x)/(2*b*d*cot(c + d*x) - 2*I*b*d) + A*d*x/(2*b*d*cot(c + d*x) - 2*I*b*d) -
 I*A/(2*b*d*cot(c + d*x) - 2*I*b*d) + B*d*x*cot(c + d*x)/(2*b*d*cot(c + d*
x) - 2*I*b*d) - I*B*d*x/(2*b*d*cot(c + d*x) - 2*I*b*d) + B/(2*b*d*cot(c +
d*x) - 2*I*b*d), Eq(a, -I*b)), (-I*A*d*x*cot(c + d*x)/(2*b*d*cot(c + d*x)
+ 2*I*b*d) + A*d*x/(2*b*d*cot(c + d*x) + 2*I*b*d) + I*A/(2*b*d*cot(c + d*x
) + 2*I*b*d) + B*d*x*cot(c + d*x)/(2*b*d*cot(c + d*x) + 2*I*b*d) + I*B*d*x
/(2*b*d*cot(c + d*x) + 2*I*b*d) + B/(2*b*d*cot(c + d*x) + 2*I*b*d), Eq(a,
I*b)), (x*(A + B*cot(c))/(a + b*cot(c)), Eq(d, 0)), (2*A*a*d*x/(2*a**2*d +
 2*b**2*d) - 2*A*b*log(tan(c + d*x) + b/a)/(2*a**2*d + 2*b**2*d) + A*b*log
(tan(c + d*x)**2 + 1)/(2*a**2*d + 2*b**2*d) + 2*B*a*log(tan(c + d*x) + b/a
)/(2*a**2*d + 2*b**2*d) - B*a*log(tan(c + d*x)**2 + 1)/(2*a**2*d + 2*b**2*
d) + 2*B*b*d*x/(2*a**2*d + 2*b**2*d), True))
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.51

$$\int \frac{A + B\cot(c + dx)}{a + b\cot(c + dx)}\,dx = \frac{\frac{2\,(Aa+Bb)(dx+c)}{a^2+b^2} + \frac{2\,(Ba-Ab)\log(a\tan(dx+c)+b)}{a^2+b^2} - \frac{(Ba-Ab)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) + 2*(B*a - A*b)*log(a*tan(d*x + c
) + b)/(a^2 + b^2) - (B*a - A*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.61

$$\int \frac{A + B\cot(c + dx)}{a + b\cot(c + dx)}\,dx = \frac{\frac{2\,(Aa+Bb)(dx+c)}{a^2+b^2} - \frac{(Ba-Ab)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2} + \frac{2\,(Ba^2-Aab)\log(|a\tan(dx+c)+b|)}{a^3+ab^2}}{2\,d}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) - (B*a - A*b)*log(tan(d*x + c)^2
+ 1)/(a^2 + b^2) + 2*(B*a^2 - A*a*b)*log(abs(a*tan(d*x + c) + b))/(a^3 + a
*b^2))/d
```

3.92.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.63

$$\int \frac{A+B\cot(c+dx)}{a+b\cot(c+dx)}\,dx = \frac{A\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)}{2\,\left(b\,d+a\,d\,1\mathrm{i}\right)} - \frac{B\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)}{2\,\left(a\,d-b\,d\,1\mathrm{i}\right)} - \frac{A\,b\,\ln\left(a+b\cot(c+d\,x)\right)}{d\,\left(a^2+b^2\right)} + \frac{B\,a\,\ln\left(a+b\cot(c+d\,x)\right)}{d\,\left(a^2+b^2\right)} + \frac{A\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(a\,d+b\,d\,1\mathrm{i}\right)} - \frac{B\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(-b\,d+a\,d\,1\mathrm{i}\right)}$$

input
```
int((A + B*cot(c + d*x))/(a + b*cot(c + d*x)),x)
```

output
```
(A*log(cot(c + d*x) - 1i)*1i)/(2*(a*d + b*d*1i)) + (A*log(cot(c + d*x) + 1
i))/(2*(a*d*1i + b*d)) - (B*log(cot(c + d*x) + 1i))/(2*(a*d - b*d*1i)) - (
B*log(cot(c + d*x) - 1i)*1i)/(2*(a*d*1i - b*d)) - (A*b*log(a + b*cot(c + d
*x)))/(d*(a^2 + b^2)) + (B*a*log(a + b*cot(c + d*x)))/(d*(a^2 + b^2))
```

3.93 $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx$

3.93.1 Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx=\frac{(a^2A-Ab^2+2abB)\,x}{(a^2+b^2)^2}+\frac{Ab-aB}{(a^2+b^2)\,d(a+b\cot(c+dx))}-\frac{(2aAb-a^2B+b^2B)\log(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)^2\,d}$$

output
```
(A*a^2-A*b^2+2*B*a*b)*x/(a^2+b^2)^2+(A*b-B*a)/(a^2+b^2)/d/(a+b*cot(d*x+c))
-(2*A*a*b-B*a^2+B*b^2)*ln(b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)^2/d
```

3.93.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.30

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx=\frac{-\frac{(iA+B)\log(i-\tan(c+dx))}{(a-ib)^2}+\frac{i(A+iB)\log(i+\tan(c+dx))}{(a+ib)^2}+\frac{2(-2aAb+a^2B-b^2B)\log(b+a\tan(c+dx))}{(a^2+b^2)^2}+\frac{2b(-Ab+aB)}{a(a^2+b^2)(b+a\tan(c+dx))}}{2d}$$

input
```
Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^2,x]
```

output
```
(-(((I*A + B)*Log[I - Tan[c + d*x]])/(a - I*b)^2) + (I*(A + I*B)*Log[I + T
an[c + d*x]])/(a + I*b)^2 + (2*(-2*a*A*b + a^2*B - b^2*B)*Log[b + a*Tan[c
+ d*x]])/(a^2 + b^2)^2 + (2*b*(-(A*b) + a*B))/(a*(a^2 + b^2)*(b + a*Tan[c
+ d*x]))))/(2*d)
```

3.93.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4012, 3042, 4014, 25, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-B\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int \frac{aA+bB-(Ab-aB)\cot(c+dx)}{a+b\cot(c+dx)}dx}{a^2+b^2}+\frac{Ab-aB}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{aA+bB-(aB-Ab)\tan\left(c+dx+\frac{\pi}{2}\right)}{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2+b^2}+\frac{Ab-aB}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{4014}$$

$$\frac{\frac{x\left(a^2A+2abB-Ab^2\right)}{a^2+b^2}-\frac{\left(a^2(-B)+2aAb+b^2B\right)\int -\frac{b-a\cot(c+dx)}{a+b\cot(c+dx)}dx}{a^2+b^2}}{a^2+b^2}+\frac{Ab-aB}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\left(a^2(-B)+2aAb+b^2B\right)\int \frac{b-a\cot(c+dx)}{a+b\cot(c+dx)}dx}{a^2+b^2}+\frac{x\left(a^2A+2abB-Ab^2\right)}{a^2+b^2}}{a^2+b^2}+\frac{Ab-aB}{d\left(a^2+b^2\right)(a+b\cot(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(a^2(-B)+2aAb+b^2B\right)\int \frac{b+a\tan\left(c+dx+\frac{\pi}{2}\right)}{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2+b^2}+\frac{x\left(a^2A+2abB-Ab^2\right)}{a^2+b^2}}{a^2+b^2}+\frac{Ab-aB}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}$$

$$\Big\downarrow \text{4013}$$

$$\frac{Ab-aB}{d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)}+\frac{\frac{x\left(a^2A+2abB-Ab^2\right)}{a^2+b^2}-\frac{\left(a^2(-B)+2aAb+b^2B\right)\log(a\sin(c+dx)+b\cos(c+dx))}{d(a^2+b^2)}}{a^2+b^2}$$

input
```
Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^2,x]
```

output
```
(A*b - a*B)/((a^2 + b^2)*d*(a + b*Cot[c + d*x])) + (((a^2*A - A*b^2 + 2*a*
b*B)*x)/(a^2 + b^2) - ((2*a*A*b - a^2*B + b^2*B)*Log[b*Cos[c + d*x] + a*Si
n[c + d*x]])/((a^2 + b^2)*d))/(a^2 + b^2)
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.93.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{\frac{(2Aab-Ba^2+Bb^2)\ln(\cot(dx+c)^2+1)}{2}+(-Aa^2+Ab^2-2Bab)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{(a^2+b^2)^2}+\frac{Ab-Ba}{(a^2+b^2)(a+b\cot(dx+c))}-\frac{(2Aab-Ba^2 \cdots}{}}{d}$
default	$\frac{\frac{\frac{(2Aab-Ba^2+Bb^2)\ln(\cot(dx+c)^2+1)}{2}+(-Aa^2+Ab^2-2Bab)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{(a^2+b^2)^2}+\frac{Ab-Ba}{(a^2+b^2)(a+b\cot(dx+c))}-\frac{(2Aab-Ba^2 \cdots}{}}{d}$
parallelrisch	$\frac{-2(Aab-\frac{1}{2}Ba^2+\frac{1}{2}Bb^2)a(a\tan(dx+c)+b)\ln(a\tan(dx+c)+b)+(Aab-\frac{1}{2}Ba^2+\frac{1}{2}Bb^2)a(a\tan(dx+c)+b)\ln(\sec(dx+c) \cdots}{(a\tan(dx+c)+b)d(a^2+b^2 \cdots}$
norman	$\frac{\frac{b(Aa^2-Ab^2+2Bab)x}{a^4+2a^2b^2+b^4}+\frac{a(Aa^2-Ab^2+2Bab)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{(Ab-Ba)b}{ad(a^2+b^2)}}{a\tan(dx+c)+b}+\frac{(2Aab-Ba^2+Bb^2)\ln(1+\tan(dx+c)^2)}{2d(a^4+2a^2b^2+b^4)}-\frac{(2A \cdots}{}$
risch	$\frac{ixB}{2iab+a^2-b^2}+\frac{xA}{2iab+a^2-b^2}+\frac{4iAabx}{a^4+2a^2b^2+b^4}-\frac{2iBa^2x}{a^4+2a^2b^2+b^4}+\frac{2iBb^2x}{a^4+2a^2b^2+b^4}+\frac{4iAabc}{d(a^4+2a^2b^2+b^4)}-\frac{2i \cdots}{d(a^4+2 \cdots}$

input
```
int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^2*(1/2*(2*A*a*b-B*a^2+B*b^2)*ln(cot(d*x+c)^2+1)+(-A*a^2+A
*b^2-2*B*a*b)*(1/2*Pi-arccot(cot(d*x+c))))+(A*b-B*a)/(a^2+b^2)/(a+b*cot(d*
x+c))-(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2*ln(a+b*cot(d*x+c)))
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(111) = 222$.

Time = 0.30 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.06

$$\int \frac{A + B\cot(c + dx)}{(a + b\cot(c + dx))^2}\,dx$$
$$= \frac{2\,Ba^2b - 2\,Aab^2 + 2\,(Aa^2b + 2\,Bab^2 - Ab^3)dx + 2\,(Ba^2b - Aab^2 + (Aa^2b + 2\,Bab^2 - Ab^3)dx)\cos\left(2\,dx +\right.}{}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(2*B*a^2*b - 2*A*a*b^2 + 2*(A*a^2*b + 2*B*a*b^2 - A*b^3)*d*x + 2*(B*a^
2*b - A*a*b^2 + (A*a^2*b + 2*B*a*b^2 - A*b^3)*d*x)*cos(2*d*x + 2*c) + (B*a
^2*b - 2*A*a*b^2 - B*b^3 + (B*a^2*b - 2*A*a*b^2 - B*b^3)*cos(2*d*x + 2*c)
+ (B*a^3 - 2*A*a^2*b - B*a*b^2)*sin(2*d*x + 2*c))*log(a*b*sin(2*d*x + 2*c)
 + 1/2*a^2 + 1/2*b^2 - 1/2*(a^2 - b^2)*cos(2*d*x + 2*c)) - 2*(B*a*b^2 - A*
b^3 - (A*a^3 + 2*B*a^2*b - A*a*b^2)*d*x)*sin(2*d*x + 2*c))/((a^4*b + 2*a^2
*b^3 + b^5)*d*cos(2*d*x + 2*c) + (a^5 + 2*a^3*b^2 + a*b^4)*d*sin(2*d*x + 2
*c) + (a^4*b + 2*a^2*b^3 + b^5)*d)
```

3.93.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 3964, normalized size of antiderivative = 35.71

$$\int \frac{A + B\cot(c + dx)}{(a + b\cot(c + dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*(A + B*cot(c))/cot(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0))
, ((-A*x + A*tan(c + d*x)/d + B*log(tan(c + d*x)**2 + 1)/(2*d))/b**2, Eq(a
, 0)), (A*d*x*cot(c + d*x)**2/(4*a**2*d*cot(c + d*x)**2 + 8*I*a**2*d*cot(c
 + d*x) - 4*a**2*d) + 2*I*A*d*x*cot(c + d*x)/(4*a**2*d*cot(c + d*x)**2 + 8
*I*a**2*d*cot(c + d*x) - 4*a**2*d) - A*d*x/(4*a**2*d*cot(c + d*x)**2 + 8*I
*a**2*d*cot(c + d*x) - 4*a**2*d) - A*cot(c + d*x)/(4*a**2*d*cot(c + d*x)**
2 + 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) - 2*I*A/(4*a**2*d*cot(c + d*x)**2
+ 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) + I*B*d*x*cot(c + d*x)**2/(4*a**2*d*
cot(c + d*x)**2 + 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) - 2*B*d*x*cot(c + d*
x)/(4*a**2*d*cot(c + d*x)**2 + 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) - I*B*d
*x/(4*a**2*d*cot(c + d*x)**2 + 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) - I*B*c
ot(c + d*x)/(4*a**2*d*cot(c + d*x)**2 + 8*I*a**2*d*cot(c + d*x) - 4*a**2*d
), Eq(b, -I*a)), (A*d*x*cot(c + d*x)**2/(4*a**2*d*cot(c + d*x)**2 - 8*I*a*
*2*d*cot(c + d*x) - 4*a**2*d) - 2*I*A*d*x*cot(c + d*x)/(4*a**2*d*cot(c + d
*x)**2 - 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) - A*d*x/(4*a**2*d*cot(c + d*x
)**2 - 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) - A*cot(c + d*x)/(4*a**2*d*cot(
c + d*x)**2 - 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) + 2*I*A/(4*a**2*d*cot(c
+ d*x)**2 - 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) - I*B*d*x*cot(c + d*x)**2/
(4*a**2*d*cot(c + d*x)**2 - 8*I*a**2*d*cot(c + d*x) - 4*a**2*d) - 2*B*d*x*
cot(c + d*x)/(4*a**2*d*cot(c + d*x)**2 - 8*I*a**2*d*cot(c + d*x) - 4*a*...
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.67

$$\int \frac{A + B\cot(c + dx)}{(a + b\cot(c + dx))^2}\,dx$$
$$= \frac{\frac{2\,(Aa^2+2\,Bab-Ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4} + \frac{2\,(Ba^2-2\,Aab-Bb^2)\log(a\tan(dx+c)+b)}{a^4+2\,a^2b^2+b^4} - \frac{(Ba^2-2\,Aab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{2\,(Bab-A}{a^3b+ab^3+(a^4+a^2b^2}}{2\,d}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(2*(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B*
a^2 - 2*A*a*b - B*b^2)*log(a*tan(d*x + c) + b)/(a^4 + 2*a^2*b^2 + b^4) - (
B*a^2 - 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) +
 2*(B*a*b - A*b^2)/(a^3*b + a*b^3 + (a^4 + a^2*b^2)*tan(d*x + c)))/d
```

3.93.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(111) = 222$.

Time = 0.35 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.17

$$\int \frac{A + B\cot(c + dx)}{(a + b\cot(c + dx))^2}\, dx = \frac{\frac{2\,(Aa^2+2\,Bab-Ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4} - \frac{(Ba^2-2\,Aab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{2\,(Ba^3-2\,Aa^2b-Bab^2)\log(|a\tan(dx+c)+b|)}{a^5+2\,a^3b^2+ab^4} - \frac{2\,(Ba^4\tan(dx+}{}}{2\,d}$$

input

```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^2,x, algorithm="giac")
```

output

```
1/2*(2*(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (B*a^
2 - 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(
B*a^3 - 2*A*a^2*b - B*a*b^2)*log(abs(a*tan(d*x + c) + b))/(a^5 + 2*a^3*b^2
 + a*b^4) - 2*(B*a^4*tan(d*x + c) - 2*A*a^3*b*tan(d*x + c) - B*a^2*b^2*tan
(d*x + c) - A*a^2*b^2 - 2*B*a*b^3 + A*b^4)/((a^5 + 2*a^3*b^2 + a*b^4)*(a*t
an(d*x + c) + b)))/d
```

3.93.9 Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.41

$$\begin{aligned}\int \frac{A + B\cot(c + dx)}{(a + b\cot(c + dx))^2}\, dx &= \ln\left(a + b\cot(c + d\,x)\right)\left(\frac{B}{d\,(a^2 + b^2)} - \frac{2\,B\,b^2}{d\,(a^2 + b^2)^2}\right) \\ &\quad + \frac{A\,\ln\left(\cot(c + d\,x) - \mathrm{i}\right)}{2\,(-1\mathrm{i}\,d\,a^2 + 2\,d\,a\,b + 1\mathrm{i}\,d\,b^2)} - \frac{B\,\ln\left(\cot(c + d\,x) - \mathrm{i}\right)}{2\,(d\,a^2 + 2\mathrm{i}\,d\,a\,b - d\,b^2)} \\ &\quad + \frac{A\,b}{(a\,d + b\,d\cot(c + d\,x))\,(a^2 + b^2)} \\ &\quad - \frac{B\,a}{(a\,d + b\,d\cot(c + d\,x))\,(a^2 + b^2)} \\ &\quad - \frac{2\,A\,a\,b\,\ln\left(a + b\cot(c + d\,x)\right)}{d\,(a^2 + b^2)^2} \\ &\quad + \frac{A\,\ln\left(\cot(c + d\,x) + 1\mathrm{i}\right)\,1\mathrm{i}}{2\,(-d\,a^2 + 2\mathrm{i}\,d\,a\,b + d\,b^2)} - \frac{B\,\ln\left(\cot(c + d\,x) + 1\mathrm{i}\right)\,1\mathrm{i}}{2\,(1\mathrm{i}\,d\,a^2 + 2\,d\,a\,b - 1\mathrm{i}\,d\,b^2)}\end{aligned}$$

input

```
int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^2,x)
```

output
```
log(a + b*cot(c + d*x))*(B/(d*(a^2 + b^2)) - (2*B*b^2)/(d*(a^2 + b^2)^2))
+ (A*log(cot(c + d*x) + 1i)*1i)/(2*(b^2*d - a^2*d + a*b*d*2i)) + (A*log(co
t(c + d*x) - 1i))/(2*(b^2*d*1i - a^2*d*1i + 2*a*b*d)) - (B*log(cot(c + d*x
) - 1i))/(2*(a^2*d - b^2*d + a*b*d*2i)) - (B*log(cot(c + d*x) + 1i)*1i)/(2
*(a^2*d*1i - b^2*d*1i + 2*a*b*d)) + (A*b)/((a*d + b*d*cot(c + d*x))*(a^2 +
 b^2)) - (B*a)/((a*d + b*d*cot(c + d*x))*(a^2 + b^2)) - (2*A*a*b*log(a + b
*cot(c + d*x)))/(d*(a^2 + b^2)^2)
```

3.93. $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^2}\,dx$

3.94 $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx$

3.94.1 Optimal result

Integrand size = 23, antiderivative size = 175

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx = \frac{(a^3A-3aAb^2+3a^2bB-b^3B)\,x}{(a^2+b^2)^3} + \frac{Ab-aB}{2(a^2+b^2)\,d(a+b\cot(c+dx))^2} + \frac{2aAb-a^2B+b^2B}{(a^2+b^2)^2\,d(a+b\cot(c+dx))} - \frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\log(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)^3\,d}$$

output
```
(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*x/(a^2+b^2)^3+1/2*(A*b-B*a)/(a^2+b^2)/d/
(a+b*cot(d*x+c))^2+(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2/d/(a+b*cot(d*x+c))-(3
*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*ln(b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)^3/
d
```

3.94.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.32 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx$$
$$= \frac{-\frac{i(A-iB)\log(i-\tan(c+dx))}{(a-ib)^3}+\frac{i(A+iB)\log(i+\tan(c+dx))}{(a+ib)^3}+\frac{2(-3a^2Ab+Ab^3+a^3B-3ab^2B)\log(b+a\tan(c+dx))-\frac{b(a^2+b^2)\left(b\left(5a^2Ab+Ab^3\right.\right.}{}}{(a^2+b^2)^3}}{2d}$$

input
```
Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^3,x]
```

output
```
(((-I)*(A - I*B)*Log[I - Tan[c + d*x]])/(a - I*b)^3 + (I*(A + I*B)*Log[I +
 Tan[c + d*x]])/(a + I*b)^3 + (2*(-3*a^2*A*b + A*b^3 + a^3*B - 3*a*b^2*B)*
Log[b + a*Tan[c + d*x]] - (b*(a^2 + b^2)*(b*(5*a^2*A*b + A*b^3 - 3*a^3*B +
 a*b^2*B) + (6*a^3*A*b + 2*a*A*b^3 - 4*a^4*B)*Tan[c + d*x]))/(a^2*(b + a*T
an[c + d*x])^2))/(a^2 + b^2)^3)/(2*d)
```

3.94.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4012, 3042, 4012, 3042, 4014, 25, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A-B\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{4012}$$
$$\frac{\int \frac{aA+bB-(Ab-aB)\cot(c+dx)}{(a+b\cot(c+dx))^2}dx}{a^2+b^2}+\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{aA+bB-(aB-Ab)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 4012$$

$$\frac{\frac{\int \frac{Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)}{a+b\cot(c+dx)}dx}{a^2+b^2}+\frac{a^2(-B)+2aAb+b^2B}{d(a^2+b^2)(a+b\cot(c+dx))}}{a^2+b^2}+\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{Aa^2+2bBa-Ab^2-\left(Ba^2-2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2+b^2}+\frac{a^2(-B)+2aAb+b^2B}{d(a^2+b^2)(a+b\cot(c+dx))}}{a^2+b^2}+\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 4014$$

$$\frac{\frac{\frac{x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}-\frac{\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int -\frac{b-a\cot(c+dx)}{a+b\cot(c+dx)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a^2(-B)+2aAb+b^2B}{d(a^2+b^2)(a+b\cot(c+dx))}}{a^2+b^2}+\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{b-a\cot(c+dx)}{a+b\cot(c+dx)}dx}{a^2+b^2}+\frac{x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}}{a^2+b^2}+\frac{a^2(-B)+2aAb+b^2B}{d(a^2+b^2)(a+b\cot(c+dx))}}{a^2+b^2}+\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{b+a\tan\left(c+dx+\frac{\pi}{2}\right)}{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2+b^2}+\frac{x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}}{a^2+b^2}+\frac{a^2(-B)+2aAb+b^2B}{d(a^2+b^2)(a+b\cot(c+dx))}}{a^2+b^2}+\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}$$

$$\Big\downarrow \; 4013$$

$$\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\cot(c+dx))^2}+\frac{\frac{a^2(-B)+2aAb+b^2B}{d(a^2+b^2)(a+b\cot(c+dx))}+\frac{\frac{x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}-\frac{\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\log(a\sin(c+dx)+b\cos(c+dx))}{d\left(a^2+b^2\right)}}{a^2+b^2}}{a^2+b^2}$$

3.94. $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx$

input
```
Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^3,x]
```

output
```
(A*b - a*B)/(2*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^2) + ((2*a*A*b - a^2*B +
 b^2*B)/((a^2 + b^2)*d*(a + b*Cot[c + d*x])) + (((a^3*A - 3*a*A*b^2 + 3*a^
2*b*B - b^3*B)*x)/(a^2 + b^2) - ((3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)*L
og[b*Cos[c + d*x] + a*Sin[c + d*x]])/((a^2 + b^2)*d))/(a^2 + b^2))/(a^2 +
b^2)
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.94.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{\frac{\left(3A\,a^{2}b-A\,b^{3}-B\,a^{3}+3B a\,b^{2}\right)\ln\left(\cot(dx+c)^{2}+1\right)}{2}+\left(-A\,a^{3}+3Aa\,b^{2}-3B\,a^{2}b+B\,b^{3}\right)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{\left(a^{2}+b^{2}\right)^{3}}-\frac{\left(3A\,a^{2}b-A\,b^{3}-B\,a^{3}\right.}{(}}{d}$
default	$\frac{\frac{\frac{\left(3A\,a^{2}b-A\,b^{3}-B\,a^{3}+3B a\,b^{2}\right)\ln\left(\cot(dx+c)^{2}+1\right)}{2}+\left(-A\,a^{3}+3Aa\,b^{2}-3B\,a^{2}b+B\,b^{3}\right)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{\left(a^{2}+b^{2}\right)^{3}}-\frac{\left(3A\,a^{2}b-A\,b^{3}-B\,a^{3}\right.}{(}}{d}$
parallelrisch	$-6a^{2}\left(A\,a^{2}b-\frac{1}{3}A\,b^{3}-\frac{1}{3}B\,a^{3}+Ba\,b^{2}\right)(a\tan(dx+c)+b)^{2}\ln(a\tan(dx+c)+b)+3a^{2}\left(A\,a^{2}b-\frac{1}{3}A\,b^{3}-\frac{1}{3}B\,a^{3}+Ba\,b^{2}\right)(a\tan(d$
norman	$\frac{\frac{b^{2}\left(A\,a^{3}-3Aa\,b^{2}+3B\,a^{2}b-B\,b^{3}\right)x}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}+\frac{\left(A\,a^{3}-3Aa\,b^{2}+3B\,a^{2}b-B\,b^{3}\right)a^{2}x\tan(dx+c)^{2}}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}-\frac{b^{2}\left(5A\,a^{2}b+A\,b^{3}-3B\,a^{3}+Ba\,b^{2}\right)}{2d\,a^{2}\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{b\left(3A\,a^{2}b+\right.}{da}}{(a\tan(dx+c)+b)^{2}}$
risch	$\frac{ixB}{3ia^{2}b-ib^{3}+a^{3}-3a\,b^{2}}+\frac{xA}{3ia^{2}b-ib^{3}+a^{3}-3a\,b^{2}}+\frac{6iA\,a^{2}bx}{a^{6}+3b^{2}a^{4}+3b^{4}a^{2}+b^{6}}-\frac{2iA\,b^{3}x}{a^{6}+3b^{2}a^{4}+3b^{4}a^{2}+b^{6}}-\frac{2iB\,a^{3}x}{a^{6}+3b^{2}a^{4}+3b^{4}}$

input
```
int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^3*(1/2*(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*ln(cot(d*x+c)^2+
1)+(-A*a^3+3*A*a*b^2-3*B*a^2*b+B*b^3)*(1/2*Pi-arccot(cot(d*x+c))))-(3*A*a^
2*b-A*b^3-B*a^3+3*B*a*b^2)/(a^2+b^2)^3*ln(a+b*cot(d*x+c))+1/2*(A*b-B*a)/(a
^2+b^2)/(a+b*cot(d*x+c))^2+(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2/(a+b*cot(d*x+
c)))
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(171) = 342$.

Time = 0.34 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.14

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx$$
$$=\frac{2\,Ba^{3}b^{2}-2\,Aa^{2}b^{3}+2\,Bab^{4}-2\,Ab^{5}-2\,(Aa^{5}+3\,Ba^{4}b-2\,Aa^{3}b^{2}+2\,Ba^{2}b^{3}-3\,Aab^{4}-Bb^{5})dx-2\,(4\,B}{}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(2*B*a^3*b^2 - 2*A*a^2*b^3 + 2*B*a*b^4 - 2*A*b^5 - 2*(A*a^5 + 3*B*a^4*
b - 2*A*a^3*b^2 + 2*B*a^2*b^3 - 3*A*a*b^4 - B*b^5)*d*x - 2*(4*B*a^3*b^2 -
6*A*a^2*b^3 - 2*B*a*b^4 - (A*a^5 + 3*B*a^4*b - 4*A*a^3*b^2 - 4*B*a^2*b^3 +
 3*A*a*b^4 + B*b^5)*d*x)*cos(2*d*x + 2*c) - (B*a^5 - 3*A*a^4*b - 2*B*a^3*b
^2 - 2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5 - (B*a^5 - 3*A*a^4*b - 4*B*a^3*b^2 +
4*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cos(2*d*x + 2*c) + 2*(B*a^4*b - 3*A*a^3*b
^2 - 3*B*a^2*b^3 + A*a*b^4)*sin(2*d*x + 2*c))*log(a*b*sin(2*d*x + 2*c) + 1
/2*a^2 + 1/2*b^2 - 1/2*(a^2 - b^2)*cos(2*d*x + 2*c)) - 2*(2*B*a^4*b - 3*A*
a^3*b^2 - 3*B*a^2*b^3 + 3*A*a*b^4 + B*b^5 + 2*(A*a^4*b + 3*B*a^3*b^2 - 3*A
*a^2*b^3 - B*a*b^4)*d*x)*sin(2*d*x + 2*c))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 -
 b^8)*d*cos(2*d*x + 2*c) - 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*sin
(2*d*x + 2*c) - (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*d)
```

3.94.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.93

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx = \frac{\frac{2\left(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\left(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3\right)\log(a\tan(dx+c)+b)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3\right)\log\left(\tan(dx+c)^2\right.}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{2\,d}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + 2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(a*tan(d
*x + c) + b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (B*a^3 - 3*A*a^2*b - 3*
B*a*b^2 + A*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) + (3*B*a^3*b^2 - 5*A*a^2*b^3 - B*a*b^4 - A*b^5 + 2*(2*B*a^4*b - 3*A*a^3
*b^2 - A*a*b^4)*tan(d*x + c))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6 + (a^8 + 2*a^
6*b^2 + a^4*b^4)*tan(d*x + c)^2 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*tan(d*x
+ c)))/d
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(171) = 342$.

Time = 0.43 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.35

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx$$
$$= \frac{\frac{2\left(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{\left(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\left(Ba^4-3\,Aa^3b-3\,Ba^2b^2+Aab^3\right)\log(|a\tan(dx}{a^7+3\,a^5b^2+3\,a^3b^4+ab^6}}{}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) - (B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(tan(d*x +
 c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(B*a^4 - 3*A*a^3*b - 3*
B*a^2*b^2 + A*a*b^3)*log(abs(a*tan(d*x + c) + b))/(a^7 + 3*a^5*b^2 + 3*a^3
*b^4 + a*b^6) - (3*B*a^7*tan(d*x + c)^2 - 9*A*a^6*b*tan(d*x + c)^2 - 9*B*a
^5*b^2*tan(d*x + c)^2 + 3*A*a^4*b^3*tan(d*x + c)^2 + 2*B*a^6*b*tan(d*x + c
) - 12*A*a^5*b^2*tan(d*x + c) - 22*B*a^4*b^3*tan(d*x + c) + 14*A*a^3*b^4*t
an(d*x + c) + 2*A*a*b^6*tan(d*x + c) - 4*A*a^4*b^3 - 11*B*a^3*b^4 + 9*A*a^
2*b^5 + B*a*b^6 + A*b^7)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*tan(d
*x + c) + b)^2))/d
```

3.94.9 Mupad [B] (verification not implemented)

Time = 15.12 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.75

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^3}\,dx = \frac{\frac{5\,A\,a^2\,b+A\,b^3}{2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{2\,A\,a\,b^2\cot(c+d\,x)}{a^4+2\,a^2\,b^2+b^4}}{d\,a^2+2\,d\,a\,b\cot\left(c+d\,x\right)+d\,b^2\cot\left(c+d\,x\right)^2} - \ln\left(a+b\cot(c+d\,x)\right)\left(\frac{3\,A\,b}{d\,(a^2+b^2)^2}-\frac{4\,A\,b^3}{d\,(a^2+b^2)^3}\right) - \frac{\frac{3\,B\,a^3-B\,a\,b^2}{2\,(a^4+2\,a^2\,b^2+b^4)}-\frac{\cot(c+d\,x)\,(B\,b^3-B\,a^2\,b)}{a^4+2\,a^2\,b^2+b^4}}{d\,a^2+2\,d\,a\,b\cot\left(c+d\,x\right)+d\,b^2\cot\left(c+d\,x\right)^2} + \ln\left(a+b\cot(c+d\,x)\right)\left(\frac{B\,a}{d\,(a^2+b^2)^2}-\frac{4\,B\,a\,b^2}{d\,(a^2+b^2)^3}\right) + \frac{A\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(d\,a^3+3\mathrm{i}\,d\,a^2\,b-3\,d\,a\,b^2-1\mathrm{i}\,d\,b^3\right)} + \frac{A\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)}{2\,\left(1\mathrm{i}\,d\,a^3+3\,d\,a^2\,b-3\mathrm{i}\,d\,a\,b^2-d\,b^3\right)} - \frac{B\,\ln\left(\cot(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,\left(1\mathrm{i}\,d\,a^3-3\,d\,a^2\,b-3\mathrm{i}\,d\,a\,b^2+d\,b^3\right)} - \frac{B\,\ln\left(\cot(c+d\,x)+1\mathrm{i}\right)}{2\,\left(d\,a^3-3\mathrm{i}\,d\,a^2\,b-3\,d\,a\,b^2+1\mathrm{i}\,d\,b^3\right)}$$

input
```
int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^3,x)
```

output
```
((A*b^3 + 5*A*a^2*b)/(2*(a^4 + b^4 + 2*a^2*b^2)) + (2*A*a*b^2*cot(c + d*x)
)/(a^4 + b^4 + 2*a^2*b^2))/(a^2*d + b^2*d*cot(c + d*x)^2 + 2*a*b*d*cot(c +
 d*x)) - log(a + b*cot(c + d*x))*((3*A*b)/(d*(a^2 + b^2)^2) - (4*A*b^3)/(d
*(a^2 + b^2)^3)) - ((3*B*a^3 - B*a*b^2)/(2*(a^4 + b^4 + 2*a^2*b^2)) - (cot
(c + d*x)*(B*b^3 - B*a^2*b))/(a^4 + b^4 + 2*a^2*b^2))/(a^2*d + b^2*d*cot(c
 + d*x)^2 + 2*a*b*d*cot(c + d*x)) + log(a + b*cot(c + d*x))*((B*a)/(d*(a^2
 + b^2)^2) - (4*B*a*b^2)/(d*(a^2 + b^2)^3)) + (A*log(cot(c + d*x) - 1i)*1i
)/(2*(a^3*d - b^3*d*1i - 3*a*b^2*d + a^2*b*d*3i)) + (A*log(cot(c + d*x) +
1i))/(2*(a^3*d*1i - b^3*d - a*b^2*d*3i + 3*a^2*b*d)) - (B*log(cot(c + d*x)
 - 1i)*1i)/(2*(a^3*d*1i + b^3*d - a*b^2*d*3i - 3*a^2*b*d)) - (B*log(cot(c
+ d*x) + 1i))/(2*(a^3*d + b^3*d*1i - 3*a*b^2*d - a^2*b*d*3i))
```

3.95 $\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx$

3.95.1 Optimal result

Integrand size = 25, antiderivative size = 188

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = \frac{(a-ib)^{5/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{5/2}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2(Ab+aB)(a+b\cot(c+dx))^{3/2}}{3d} - \frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

output
```
(a-I*b)^(5/2)*(I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I
*b)^(5/2)*(I*A-B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2/3*(A*b
+B*a)*(a+b*cot(d*x+c))^(3/2)/d-2/5*B*(a+b*cot(d*x+c))^(5/2)/d-2*(2*A*a*b+B
*a^2-B*b^2)*(a+b*cot(d*x+c))^(1/2)/d
```

3.95.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 379 vs. $2(188) = 376$.

Time = 1.91 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.02

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = -\frac{2\left(\frac{\sqrt{a-\sqrt{-b^2}}\left(-3a^2b\left(A\sqrt{-b^2}+bB\right)+b^3\left(A\sqrt{-b^2}+bB\right)+a^3\left(Ab-\sqrt{-b^2}B\right)+3ab^2\left(-Ab+\sqrt{-b^2}B\right)\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{2\left(b^2+a\sqrt{-b^2}\right)}+\frac{\left(b^3\left(A\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*Cot[c + d*x])^(5/2)*(A + B*Cot[c + d*x]),x]
```

output
```
(-2*((Sqrt[a - Sqrt[-b^2]]*(-3*a^2*b*(A*Sqrt[-b^2] + b*B) + b^3*(A*Sqrt[-b
^2] + b*B) + a^3*(A*b - Sqrt[-b^2]*B) + 3*a*b^2*(-(A*b) + Sqrt[-b^2]*B))*A
rcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(2*(b^2 + a*Sqrt[-b
^2])) + ((b^3*(A*Sqrt[-b^2] - b*B) + 3*a^2*b*(-(A*Sqrt[-b^2]) + b*B) - a^3
*(A*b + Sqrt[-b^2]*B) + 3*a*b^2*(A*b + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*C
ot[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/(2*Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) +
(2*a*A*b + a^2*B - b^2*B)*Sqrt[a + b*Cot[c + d*x]] + ((A*b + a*B)*(a + b*C
ot[c + d*x])^(3/2))/3 + (B*(a + b*Cot[c + d*x])^(5/2))/5))/d
```

3.95.3 Rubi [A] (warning: unable to verify)

Time = 1.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4011, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(A-B\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\cot(c+dx))^{3/2}(aA-bB+(Ab+aB)\cot(c+dx))dx-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

$\Big\downarrow$ 3042

$$\int \left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(aA-bB-(Ab+aB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

$\Big\downarrow$ 4011

$$\int \sqrt{a+b\cot(c+dx)}\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)\right)dx-\frac{2(aB+Ab)(a+b\cot(c+dx))^{3/2}}{3d}-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

$\Big\downarrow$ 3042

$$\int \sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}\left(Aa^2-2bBa-Ab^2-\left(Ba^2+2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2(aB+Ab)(a+b\cot(c+dx))^{3/2}}{3d}-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

$\Big\downarrow$ 4011

$$\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx-\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2(aB+Ab)(a+b\cot(c+dx))^{3/2}}{3d}-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

$\Big\downarrow$ 3042

$$\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B-\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2(aB+Ab)(a+b\cot(c+dx))^{3/2}}{3d}-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

$\Big\downarrow$ 4022

$$\frac{1}{2}(a-ib)^3(A-iB)\int \frac{i\cot(c+dx)+1}{\sqrt{a+b\cot(c+dx)}}dx+\frac{1}{2}(a+ib)^3(A+iB)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx-\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2(aB+Ab)(a+b\cot(c+dx))^{3/2}}{3d}-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

3.95. $\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(a-ib)^3(A-iB)\int \frac{1-i\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{1}{2}(a+ib)^3(A+iB)\int \frac{i\tan\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2(aB+Ab)(a+b\cot(c+dx))^{3/2}}{3d}-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{4020}$$

$$-\frac{i(a-ib)^3(A-iB)\int -\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}+\frac{i(a+ib)^3(A+iB)\int -\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}-\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2(aB+Ab)(a+b\cot(c+dx))^{3/2}}{3d}-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{i(a-ib)^3(A-iB)\int \frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}-\frac{i(a+ib)^3(A+iB)\int \frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}-\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2(aB+Ab)(a+b\cot(c+dx))^{3/2}}{3d}-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(a-ib)^3(A-iB)\int \frac{1}{\frac{i\cot^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}-\frac{(a+ib)^3(A+iB)\int \frac{1}{-\frac{i\cot^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}-\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2(aB+Ab)(a+b\cot(c+dx))^{3/2}}{3d}-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{221}$$

3.95. $\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx$

$$-\frac{2(a^2B+2aAb-b^2B)\sqrt{a+b\cot(c+dx)}}{d}-\frac{(a-ib)^{5/2}(A-iB)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{5/2}(A+iB)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a+ib}}\right)}{d}-\frac{2(aB+Ab)(a+b\cot(c+dx))^{3/2}}{3d}-\frac{2B(a+b\cot(c+dx))^{5/2}}{5d}$$

input
```
Int[(a + b*Cot[c + d*x])^(5/2)*(A + B*Cot[c + d*x]),x]
```

output
```
-(((a - I*b)^(5/2)*(A - I*B)*ArcTan[Cot[c + d*x]/Sqrt[a - I*b]])/d) - ((a
+ I*b)^(5/2)*(A + I*B)*ArcTan[Cot[c + d*x]/Sqrt[a + I*b]])/d - (2*(2*a*A*b
 + a^2*B - b^2*B)*Sqrt[a + b*Cot[c + d*x]])/d - (2*(A*b + a*B)*(a + b*Cot[
c + d*x])^(3/2))/(3*d) - (2*B*(a + b*Cot[c + d*x])^(5/2))/(5*d)
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

3.95.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2391 vs. $2(160) = 320$.

Time = 0.17 (sec) , antiderivative size = 2392, normalized size of antiderivative = 12.72

method	result	size
parts	Expression too large to display	2392
derivativedivides	Expression too large to display	2405
default	Expression too large to display	2405

input
```
int((a+b*cot(d*x+c))^(5/2)*(A+B*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
A*(-2/3*b*(a+b*cot(d*x+c))^(3/2)/d-4/d*b*(a+b*cot(d*x+c))^(1/2)*a+1/4/d/b*
ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^
2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-1/4/d*b*ln
(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+
b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/4/d/b*ln(b*cot
(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(
1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+3/4/d*b*ln(b*cot(d*x+c)+a+(a+b*cot
(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a+2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot
(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
))*(a^2+b^2)^(1/2)*a-3/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*co
t(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/
2))*a^2+1/d*b^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(
1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/4/d/b
*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2
+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+1/4/d*b*l
n((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b
^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+1/4/d/b*ln((a+b
*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1
/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-3/4/d*b*ln((a+b*cot(d*x+c))^(1...
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5073 vs. $2(154) = 308$.

Time = 1.22 (sec) , antiderivative size = 5073, normalized size of antiderivative = 26.98

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c))^(5/2)*(A+B*cot(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

3.95.6 Sympy [F]

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = \int (A+B\cot(c+dx))\,(a+b\cot(c+dx))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))**(5/2)*(A+B*cot(d*x+c)),x)
```

output
```
Integral((A + B*cot(c + d*x))*(a + b*cot(c + d*x))**(5/2), x)
```

3.95.7 Maxima [F]

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = \int (B\cot(dx+c)+A)(b\cot(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^(5/2)*(A+B*cot(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cot(d*x + c) + A)*(b*cot(d*x + c) + a)^(5/2), x)
```

3.95.8 Giac [F]

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = \int (B\cot(dx+c)+A)(b\cot(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^(5/2)*(A+B*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cot(d*x + c) + A)*(b*cot(d*x + c) + a)^(5/2), x)
```

3.95.9 Mupad [B] (verification not implemented)

Time = 45.24 (sec) , antiderivative size = 3864, normalized size of antiderivative = 20.55

$$\int (a+b\cot(c+dx))^{5/2}(A+B\cot(c+dx))\,dx = \text{Too large to display}$$

input
```
int((A + B*cot(c + d*x))*(a + b*cot(c + d*x))^(5/2),x)
```

output
```
log((8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - ((((((-B^4*b^2*d^4*(5*
a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^
2*a*b^4*d^2)/d^4)^(1/2)*(32*B*a^4*b^2 - 32*B*b^6 + 32*a*b^2*d*(((-B^4*b^2*
d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2
 + 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) - (16*B^
2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^
2)*(((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*
B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2))/2)*((20*B^4*a^2*b^8*d^4 - B
^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d
^4)^(1/2)/(4*d^4) + (B^2*a^5)/(4*d^2) - (5*B^2*a^3*b^2)/(2*d^2) + (5*B^2*a
*b^4)/(4*d^2))^(1/2) - log((((((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2
)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(
32*B*b^6 - 32*B*a^4*b^2 + 32*a*b^2*d*(((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2
*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^
(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) - (16*B^2*b^2*(a + b*cot(c + d*x)
)^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(((-B^4*b^2*d^4*(5*a^4
 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a
*b^4*d^2)/d^4)^(1/2))/2 + (8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*(
((20*B^4*a^2*b^8*d^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^
4*d^4 - 25*B^4*a^8*b^2*d^4)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + ...
```

3.96 $\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx$

3.96.1 Optimal result

Integrand size = 25, antiderivative size = 150

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx = \frac{(a-ib)^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{3/2}(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2(Ab+aB)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

output
```
(a-I*b)^(3/2)*(I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I
*b)^(3/2)*(I*A-B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2/3*B*(a
+b*cot(d*x+c))^(3/2)/d-2*(A*b+B*a)*(a+b*cot(d*x+c))^(1/2)/d
```

3.96.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.96

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx = -\frac{\frac{3\sqrt{a-\sqrt{-b^2}}\left(-2ab\left(A\sqrt{-b^2}+bB\right)+a^2\left(Ab-\sqrt{-b^2}B\right)+b^2\left(-Ab+\sqrt{-b^2}B\right)\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{b^2+a\sqrt{-b^2}} + \frac{3\left(2ab\left(-A\sqrt{-b^2}+bB\right)-a^2\left(Ab+\right.\right.}{}}{3}$$

input
```
Integrate[(a + b*Cot[c + d*x])^(3/2)*(A + B*Cot[c + d*x]),x]
```

output
```
-1/3*((3*Sqrt[a - Sqrt[-b^2]]*(-2*a*b*(A*Sqrt[-b^2] + b*B) + a^2*(A*b - Sq
rt[-b^2]*B) + b^2*(-(A*b) + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*Cot[c + d*x]
]/Sqrt[a - Sqrt[-b^2]]])/(b^2 + a*Sqrt[-b^2]) + (3*(2*a*b*(-(A*Sqrt[-b^2])
 + b*B) - a^2*(A*b + Sqrt[-b^2]*B) + b^2*(A*b + Sqrt[-b^2]*B))*ArcTanh[Sqr
t[a + b*Cot[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^
2]]) + 6*(A*b + a*B)*Sqrt[a + b*Cot[c + d*x]] + 2*B*(a + b*Cot[c + d*x])^(
3/2))/d
```

3.96.3 Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(A-B\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4011}$$

$$\int \sqrt{a+b\cot(c+dx)}(aA-bB+(Ab+aB)\cot(c+dx))dx-\frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}\left(aA-bB-(Ab+aB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx-\frac{2(aB+Ab)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Aa^2-2bBa-Ab^2-\left(Ba^2+2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(aB+Ab)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(a-ib)^2(A-iB)\int \frac{i\cot(c+dx)+1}{\sqrt{a+b\cot(c+dx)}}dx+\frac{1}{2}(a+ib)^2(A+iB)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx-\frac{2(aB+Ab)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(a-ib)^2(A-iB)\int \frac{1-i\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{1}{2}(a+ib)^2(A+iB)\int \frac{i\tan\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(aB+Ab)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4020}$$

$$-\frac{i(a-ib)^2(A-iB)\int -\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}+\frac{i(a+ib)^2(A+iB)\int -\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}-\frac{2(aB+Ab)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{i(a-ib)^2(A-iB)\int \frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}-\frac{i(a+ib)^2(A+iB)\int \frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}-\frac{2(aB+Ab)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(a-ib)^2(A-iB)\int \frac{1}{\frac{i\cot^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}-\frac{(a+ib)^2(A+iB)\int \frac{1}{-\frac{i\cot^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}-\frac{2(aB+Ab)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

3.96. $\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx$

$$\downarrow 221$$

$$-\frac{(a-ib)^{3/2}(A-iB)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{3/2}(A+iB)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a+ib}}\right)}{d}-\frac{2(aB+Ab)\sqrt{a+b\cot(c+dx)}}{d}-\frac{2B(a+b\cot(c+dx))^{3/2}}{3d}$$

input
```
Int[(a + b*Cot[c + d*x])^(3/2)*(A + B*Cot[c + d*x]),x]
```

output
```
-(((a - I*b)^(3/2)*(A - I*B)*ArcTan[Cot[c + d*x]/Sqrt[a - I*b]])/d) - ((a
+ I*b)^(3/2)*(A + I*B)*ArcTan[Cot[c + d*x]/Sqrt[a + I*b]])/d - (2*(A*b + a
*B)*Sqrt[a + b*Cot[c + d*x]])/d - (2*B*(a + b*Cot[c + d*x])^(3/2))/(3*d)
```

3.96.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

3.96.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1656 vs. $2(126) = 252$.

Time = 0.12 (sec) , antiderivative size = 1657, normalized size of antiderivative = 11.05

method	result	size
parts	Expression too large to display	1657
derivativedivides	Expression too large to display	1665
default	Expression too large to display	1665

input
```
int((a+b*cot(d*x+c))^(3/2)*(A+B*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
A*(-2*b*(a+b*cot(d*x+c))^(1/2)/d+1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c)
)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)*(a^2+b^2)^(1/2)*a-1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1
/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*a^2+1/4/d*b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d*b/(2*(a
^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/
2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)-2/d*b/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2
)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-1/4/d/b*ln((a+b*cot(d*x+c))
^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+1/4/d/b*ln((a+b*cot(d*x+c))^(1/
2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*a^2-1/4/d*b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/
2)+2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a)+B*(-2/3/d*
(a+b*cot(d*x+c))^(3/2)-2/d*(a+b*cot(d*x+c))^(1/2)*a-1/4/d*ln(b*cot(d*x+...
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3252 vs. $2(120) = 240$.

Time = 0.63 (sec) , antiderivative size = 3252, normalized size of antiderivative = 21.68

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c))^(3/2)*(A+B*cot(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(3*d*sqrt((6*A*B*a^2*b - 2*A*B*b^3 - (A^2 - B^2)*a^3 + 3*(A^2 - B^2)*
a*b^2 + d^2*sqrt(-(4*A^2*B^2*a^6 + 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 1
4*A^2*B^2 + 3*B^4)*a^4*b^2 - 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B
^2 + B^4)*a^2*b^4 + 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6
)/d^4))/d^2)*log((2*(A^3*B + A*B^3)*a^5 + 3*(A^4 - B^4)*a^4*b - 4*(A^3*B +
 A*B^3)*a^3*b^2 + 2*(A^4 - B^4)*a^2*b^3 - 6*(A^3*B + A*B^3)*a*b^4 - (A^4 -
 B^4)*b^5)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x +
2*c)) + ((A*a - B*b)*d^3*sqrt(-(4*A^2*B^2*a^6 + 12*(A^3*B - A*B^3)*a^5*b +
 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 - 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(
A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 + 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B
^2 + B^4)*b^6)/d^4) - (2*A*B^2*a^4 + (5*A^2*B - 3*B^3)*a^3*b + 3*(A^3 - 3*
A*B^2)*a^2*b^2 - (7*A^2*B - B^3)*a*b^3 - (A^3 - A*B^2)*b^4)*d)*sqrt((6*A*B
*a^2*b - 2*A*B*b^3 - (A^2 - B^2)*a^3 + 3*(A^2 - B^2)*a*b^2 + d^2*sqrt(-(4*
A^2*B^2*a^6 + 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^
4*b^2 - 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 + 1
2*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/d^4))/d^2))*sin(2*d
*x + 2*c) - 3*d*sqrt((6*A*B*a^2*b - 2*A*B*b^3 - (A^2 - B^2)*a^3 + 3*(A^2 -
 B^2)*a*b^2 + d^2*sqrt(-(4*A^2*B^2*a^6 + 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A
^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 - 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8
*A^2*B^2 + B^4)*a^2*b^4 + 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 +...
```

3.96.6 Sympy [F]

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx = \int (A+B\cot(c+dx))\,(a+b\cot(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))**(3/2)*(A+B*cot(d*x+c)),x)
```

output
```
Integral((A + B*cot(c + d*x))*(a + b*cot(c + d*x))**(3/2), x)
```

3.96.7 Maxima [F]

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx = \int (B\cot(dx+c)+A)(b\cot(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^(3/2)*(A+B*cot(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cot(d*x + c) + A)*(b*cot(d*x + c) + a)^(3/2), x)
```

3.96.8 Giac [F]

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx = \int (B\cot(dx+c)+A)(b\cot(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^(3/2)*(A+B*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cot(d*x + c) + A)*(b*cot(d*x + c) + a)^(3/2), x)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 26.83 (sec) , antiderivative size = 2823, normalized size of antiderivative = 18.82

$$\int (a+b\cot(c+dx))^{3/2}(A+B\cot(c+dx))\,dx = \text{Too large to display}$$

input
```
int((A + B*cot(c + d*x))*(a + b*cot(c + d*x))^(3/2),x)
```

output
```
log((16*A^3*a*b^3*(a^2 + b^2)^2)/d^3 - (((16*b^2*(((-A^4*b^2*d^4*(3*a^2 -
b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b
 + a*d*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*
d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d + (16*A^2*b^2*(a + b*cot(c
+ d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2)*(((-A^4*b^2*d^4*(3*a^2 - b^2)^
2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2))/2)*((6*A^4*a^2*b^4*d
^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^(1/2)/(4*d^4) - (A^2*a^3)/(4*d^2) +
(3*A^2*a*b^2)/(4*d^2))^(1/2) - log((16*A^3*a*b^3*(a^2 + b^2)^2)/d^3 - (((1
6*b^2*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d
^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b - a*d*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/
2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2))
)/d - (16*A^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2)
*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d
^4)^(1/2))/2)*(((6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^(1/2
) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/(4*d^4))^(1/2) - log((16*A^3*a*b^3*(a^2
 + b^2)^2)/d^3 - (((16*b^2*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a
^3*d^2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b - a*d*(-((-A^4*b^2*d
^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(a +
 b*cot(c + d*x))^(1/2)))/d - (16*A^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^4 +
 b^4 - 6*a^2*b^2))/d^2)*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a...
```

3.96. $\int (a + b\cot(c + dx))^{3/2}(A + B\cot(c + dx))\,dx$

3.97 $\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx$

3.97.1 Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx = \frac{\sqrt{a-ib}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{\sqrt{a+ib}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

output
```
(I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))*(a-I*b)^(1/2)/d-(I*A
-B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))*(a+I*b)^(1/2)/d-2*B*(a+b
*cot(d*x+c))^(1/2)/d
```

3.97.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.74

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx = -\frac{\frac{\left(aAb-Ab\sqrt{-b^2}-b^2B-a\sqrt{-b^2}B\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} - \frac{\left(aAb+Ab\sqrt{-b^2}-b^2B+a\sqrt{-b^2}B\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}} + 2B}{d}$$

input
```
Integrate[Sqrt[a + b*Cot[c + d*x]]*(A + B*Cot[c + d*x]),x]
```

output
```
-((((a*A*b - A*b*Sqrt[-b^2] - b^2*B - a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*C
ot[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - ((
a*A*b + A*b*Sqrt[-b^2] - b^2*B + a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Cot[c
+ d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + 2*B*Sqr
t[a + b*Cot[c + d*x]])/d)
```

3.97.3 Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}\left(A-B\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \; 4011$$

$$\int \frac{aA-bB+(Ab+aB)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx-\frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{aA-bB-(Ab+aB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

$$\Big\downarrow \; 4022$$

$$\frac{1}{2}(a+ib)(A+iB)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx+\frac{1}{2}(a-ib)(A-iB)\int \frac{i\cot(c+dx)+1}{\sqrt{a+b\cot(c+dx)}}dx-\frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}(a-ib)(A-iB)\int \frac{1-i\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{1}{2}(a+ib)(A+iB)\int \frac{i\tan\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

$\downarrow$ 4020

$$-\frac{i(a-ib)(A-iB)\int -\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}+\frac{i(a+ib)(A+iB)\int -\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}-\frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

$\downarrow$ 25

$$\frac{i(a-ib)(A-iB)\int \frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}-\frac{i(a+ib)(A+iB)\int \frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}-\frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

$\downarrow$ 73

$$-\frac{(a+ib)(A+iB)\int \frac{1}{-\frac{i\cot^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}-\frac{(a-ib)(A-iB)\int \frac{1}{\frac{i\cot^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}-\frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

$\downarrow$ 221

$$-\frac{\sqrt{a-ib}(A-iB)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{\sqrt{a+ib}(A+iB)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a+ib}}\right)}{d}-\frac{2B\sqrt{a+b\cot(c+dx)}}{d}$$

input
```
Int[Sqrt[a + b*Cot[c + d*x]]*(A + B*Cot[c + d*x]),x]
```

output
```
-((Sqrt[a - I*b]*(A - I*B)*ArcTan[Cot[c + d*x]/Sqrt[a - I*b]])/d) - (Sqrt[
a + I*b]*(A + I*B)*ArcTan[Cot[c + d*x]/Sqrt[a + I*b]])/d - (2*B*Sqrt[a + b
*Cot[c + d*x]])/d
```

3.97. $\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx$

3.97.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

3.97. $\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx$

3.97.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(102) = 204$.

Time = 0.15 (sec) , antiderivative size = 814, normalized size of antiderivative = 6.67

method	result
parts	$-\frac{\ln\left(b\cot(dx+c)+a+\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)A\sqrt{2\sqrt{a^2+b^2}+2a}\,a}{4db}+\frac{\ln\left(b\cot(dx+c)+a+\sqrt{a+b}\right.}{}$
derivativedivides	$-\frac{2B\sqrt{a+b\cot(dx+c)}}{d}-\frac{\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\cot(dx+c)-\sqrt{a^2+b^2}-a\right)A\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}}{4db}$
default	$-\frac{2B\sqrt{a+b\cot(dx+c)}}{d}-\frac{\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\cot(dx+c)-\sqrt{a^2+b^2}-a\right)A\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}}{4db}$

input
```
int((a+b*cot(d*x+c))^(1/2)*(A+B*cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/4/d/b*ln(b*cot(
d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1
/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/d*b/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1
/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A+1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*A*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a-1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*
(a^2+b^2)^(1/2)+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*
x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*
A+B/d*(-2*(a+b*cot(d*x+c))^(1/2)+1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*co
t(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^
(1/2))-(a-(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*co
t(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/
2))-1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)-((a^2+b^2)^(1/2)-a)/(2
*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)
^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1329 vs. $2(96) = 192$.

Time = 0.33 (sec) , antiderivative size = 1329, normalized size of antiderivative = 10.89

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cot(d*x+c))^(1/2)*(A+B*cot(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(d*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b +
(A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B + A
*B^3)*a + (A^4 - B^4)*b)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b
)/sin(2*d*x + 2*c)) + (A*d^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b
+ (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (2*A*B^2*a + (A^2*B - B^3)*b)*d)*sqr
t((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A
^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)) - d*sqrt((2*A*B*b + d^2*sqr
t(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d
^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(
(b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - (A*d^3*s
qrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)
/d^4) - (2*A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*
B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2
 - B^2)*a)/d^2)) - d*sqrt((2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B -
 A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(
-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*
d*x + 2*c) + b)/sin(2*d*x + 2*c)) + (A*d^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B
 - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) + (2*A*B^2*a + (A^2*B -
B^3)*b)*d)*sqrt((2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*
b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)) + d*sqrt((...
```

3.97.6 Sympy [F]

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx = \int (A+B\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx$$

input
```
integrate((a+b*cot(d*x+c))**(1/2)*(A+B*cot(d*x+c)),x)
```

output
```
Integral((A + B*cot(c + d*x))*sqrt(a + b*cot(c + d*x)), x)
```

3.97.7 Maxima [F]

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx = \int (B\cot(dx+c)+A)\sqrt{b\cot(dx+c)+a}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^(1/2)*(A+B*cot(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*cot(d*x + c) + A)*sqrt(b*cot(d*x + c) + a), x)
```

3.97.8 Giac [F]

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx = \int (B\cot(dx+c)+A)\sqrt{b\cot(dx+c)+a}\,dx$$

input
```
integrate((a+b*cot(d*x+c))^(1/2)*(A+B*cot(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*cot(d*x + c) + A)*sqrt(b*cot(d*x + c) + a), x)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 15.52 (sec) , antiderivative size = 843, normalized size of antiderivative = 6.91

$$\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx$$

$$= \operatorname{atanh}\left(\frac{d^3\left(\frac{16\,(A^2\,b^4-A^2\,a^2\,b^2)\,\sqrt{a+b\cot(c+d\,x)}}{d^2}+\frac{16\,a\,b^2\,\left(\sqrt{-A^4\,b^2\,d^4}+A^2\,a\,d^2\right)\,\sqrt{a+b\cot(c+d\,x)}}{d^4}\right)\,\sqrt{-\frac{\sqrt{-A^4\,b^2\,d^4}+A^2\,a\,d^2}{d^4}}}{16\,\left(A^3\,a^2\,b^3+A^3\,b^5\right)}\right)$$
$$+\operatorname{atanh}\left(\frac{d^3\left(\frac{16\,(A^2\,b^4-A^2\,a^2\,b^2)\,\sqrt{a+b\cot(c+d\,x)}}{d^2}-\frac{16\,a\,b^2\,\left(\sqrt{-A^4\,b^2\,d^4}-A^2\,a\,d^2\right)\,\sqrt{a+b\cot(c+d\,x)}}{d^4}\right)\,\sqrt{\frac{\sqrt{-A^4\,b^2\,d^4}-A^2\,a\,d^2}{d^4}}}{16\,\left(A^3\,a^2\,b^3+A^3\,b^5\right)}\right)$$
$$+2\operatorname{atanh}\left(\frac{32\,B^2\,b^4\,\sqrt{\frac{\sqrt{-B^4\,b^2\,d^4}}{4\,d^4}+\frac{B^2\,a}{4\,d^2}}\,\sqrt{a+b\cot\left(c+d\,x\right)}}{\frac{16\,B\,b^4\,\sqrt{-B^4\,b^2\,d^4}}{d^3}+\frac{16\,B\,a^2\,b^2\,\sqrt{-B^4\,b^2\,d^4}}{d^3}}+\frac{32\,a\,b^2\,\sqrt{\frac{\sqrt{-B^4\,b^2\,d^4}}{4\,d^4}+\frac{B^2\,a}{4\,d^2}}\,\sqrt{a+b\cot\left(c+d\,x\right)}\,\sqrt{-B^4\,b^2\,d^4}}{\frac{16\,B\,b^4\,\sqrt{-B^4\,b^2\,d^4}}{d}+\frac{16\,B\,a^2\,b^2\,\sqrt{-B^4\,b^2\,d^4}}{d}}\right)\sqrt{\frac{\sqrt{-B^4\,b^2\,d^4}+B^2\,a\,d^2}{4\,d^4}}$$
$$-2\operatorname{atanh}\left(\frac{32\,B^2\,b^4\,\sqrt{\frac{B^2\,a}{4\,d^2}-\frac{\sqrt{-B^4\,b^2\,d^4}}{4\,d^4}}\,\sqrt{a+b\cot\left(c+d\,x\right)}}{\frac{16\,B\,b^4\,\sqrt{-B^4\,b^2\,d^4}}{d^3}+\frac{16\,B\,a^2\,b^2\,\sqrt{-B^4\,b^2\,d^4}}{d^3}}-\frac{32\,a\,b^2\,\sqrt{\frac{B^2\,a}{4\,d^2}-\frac{\sqrt{-B^4\,b^2\,d^4}}{4\,d^4}}\,\sqrt{a+b\cot\left(c+d\,x\right)}\,\sqrt{-B^4\,b^2\,d^4}}{\frac{16\,B\,b^4\,\sqrt{-B^4\,b^2\,d^4}}{d}+\frac{16\,B\,a^2\,b^2\,\sqrt{-B^4\,b^2\,d^4}}{d}}\right)\sqrt{-\frac{\sqrt{-B^4\,b^2\,d^4}-B^2\,a\,d^2}{4\,d^4}}$$
$$-\frac{2\,B\,\sqrt{a+b\cot\left(c+d\,x\right)}}{d}$$

input
```
int((A + B*cot(c + d*x))*(a + b*cot(c + d*x))^(1/2),x)
```

output
```
atanh((d^3*((16*(A^2*b^4 - A^2*a^2*b^2)*(a + b*cot(c + d*x))^(1/2))/d^2 +
(16*a*b^2*((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)*(a + b*cot(c + d*x))^(1/2))/d
^4)*(-((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)/d^4)^(1/2))/(16*(A^3*b^5 + A^3*a^
2*b^3)))*(-((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)/d^4)^(1/2) + atanh((d^3*((16
*(A^2*b^4 - A^2*a^2*b^2)*(a + b*cot(c + d*x))^(1/2))/d^2 - (16*a*b^2*((-A^
4*b^2*d^4)^(1/2) - A^2*a*d^2)*(a + b*cot(c + d*x))^(1/2))/d^4)*(((-A^4*b^2
*d^4)^(1/2) - A^2*a*d^2)/d^4)^(1/2))/(16*(A^3*b^5 + A^3*a^2*b^3)))*(((-A^4
*b^2*d^4)^(1/2) - A^2*a*d^2)/d^4)^(1/2) + 2*atanh((32*B^2*b^4*((-B^4*b^2*d
^4)^(1/2)/(4*d^4) + (B^2*a)/(4*d^2))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((1
6*B*b^4*(-B^4*b^2*d^4)^(1/2))/d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d^
3) + (32*a*b^2*((-B^4*b^2*d^4)^(1/2)/(4*d^4) + (B^2*a)/(4*d^2))^(1/2)*(a +
 b*cot(c + d*x))^(1/2)*(-B^4*b^2*d^4)^(1/2))/((16*B*b^4*(-B^4*b^2*d^4)^(1/
2))/d + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d))*(((-B^4*b^2*d^4)^(1/2) + B
^2*a*d^2)/(4*d^4))^(1/2) - 2*atanh((32*B^2*b^4*((B^2*a)/(4*d^2) - (-B^4*b^
2*d^4)^(1/2)/(4*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*B*b^4*(-B^4*b
^2*d^4)^(1/2))/d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d^3) - (32*a*b^2*
((B^2*a)/(4*d^2) - (-B^4*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*(a + b*cot(c + d*x)
)^(1/2)*(-B^4*b^2*d^4)^(1/2))/((16*B*b^4*(-B^4*b^2*d^4)^(1/2))/d + (16*B*a
^2*b^2*(-B^4*b^2*d^4)^(1/2))/d))*(-((-B^4*b^2*d^4)^(1/2) - B^2*a*d^2)/(4*d
^4))^(1/2) - (2*B*(a + b*cot(c + d*x))^(1/2))/d
```

3.97. $\int \sqrt{a+b\cot(c+dx)}(A+B\cot(c+dx))\,dx$

3.98 $\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx$

3.98.1 Optimal result

Integrand size = 27, antiderivative size = 151

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx = -\frac{(ia-b)(a-ib)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{5/2}(ia+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2b(a^2+b^2)\sqrt{a+b\cot(c+dx)}}{d} - \frac{2b(a+b\cot(c+dx))^{5/2}}{5d}$$

output
```
-(I*a-b)*(a-I*b)^(5/2)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+
I*b)^(5/2)*(I*a+b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2/5*b*(
a+b*cot(d*x+c))^(5/2)/d+2*b*(a^2+b^2)*(a+b*cot(d*x+c))^(1/2)/d
```

3.98.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.47

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx = \frac{b\left(\frac{5(a^2+b^2)\left(a^2-b^2-2a\sqrt{-b^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} - \frac{5(a^2+b^2)\left(a^2-b^2+2a\sqrt{-b^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}}\right.}{5d}$$

input
```
Integrate[(-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(5/2),x]
```

output
```
(b*((5*(a^2 + b^2)*(a^2 - b^2 - 2*a*Sqrt[-b^2])*ArcTanh[Sqrt[a + b*Cot[c +
 d*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - (5*(a^2
+ b^2)*(a^2 - b^2 + 2*a*Sqrt[-b^2])*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[
a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + 10*(a^2 + b^2)*Sqrt[
a + b*Cot[c + d*x]] - 2*(a + b*Cot[c + d*x])^(5/2)))/(5*d)
```

3.98.3 Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4011, 27, 3042, 3963, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cot(c+dx)-a)(a+b\cot(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4011}$$

$$\int \left(-a^2-b^2\right)(a+b\cot(c+dx))^{3/2}dx-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$-\left(a^2+b^2\right)\int (a+b\cot(c+dx))^{3/2}dx-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\left(a^2+b^2\right)\int \left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3963}$$

$$-\left(a^2+b^2\right)\left(\int \frac{a^2+2b\cot(c+dx)a-b^2}{\sqrt{a+b\cot(c+dx)}}dx-\frac{2b\sqrt{a+b\cot(c+dx)}}{d}\right)-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-(a^2+b^2)\left(\int \frac{a^2-2b\tan\left(c+dx+\frac{\pi}{2}\right)a-b^2}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b\cot(c+dx)}}{d}\right)-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}$$

$\downarrow$ 4022

$$-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}-(a^2+b^2)\left(\frac{1}{2}(a-ib)^2\int \frac{i\cot(c+dx)+1}{\sqrt{a+b\cot(c+dx)}}dx+\frac{1}{2}(a+ib)^2\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx-\frac{2b\sqrt{a+b\cot(c+dx)}}{d}\right)$$

$\downarrow$ 3042

$$-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}-(a^2+b^2)\left(\frac{1}{2}(a-ib)^2\int \frac{1-i\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{1}{2}(a+ib)^2\int \frac{i\tan\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b\cot}}{d}\right.$$

$\downarrow$ 4020

$$-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}-(a^2+b^2)\left(-\frac{i(a-ib)^2\int -\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}+\frac{i(a+ib)^2\int -\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}}{2d}\right.$$

$\downarrow$ 25

$$-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}-(a^2+b^2)\left(\frac{i(a-ib)^2\int \frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}-\frac{i(a+ib)^2\int \frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i}{2d}\right.$$

$\downarrow$ 73

$$-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}-(a^2+b^2)\left(-\frac{(a-ib)^2\int \frac{1}{\frac{i\cot^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}-\frac{(a+ib)^2\int \frac{1}{-\frac{i\cot^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}\right.$$

$\downarrow$ 221

3.98. $\int(-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx$

$$-\frac{2b(a+b\cot(c+dx))^{5/2}}{5d}-$$
$$(a^2+b^2)\left(-\frac{(a-ib)^{3/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{3/2}\arctan\left(\frac{\cot(c+dx)}{\sqrt{a+ib}}\right)}{d}-\frac{2b\sqrt{a+b\cot(c+dx)}}{d}\right)$$

input
```
Int[(-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(5/2),x]
```

output
```
(-2*b*(a + b*Cot[c + d*x])^(5/2))/(5*d) - (a^2 + b^2)*(-(((a - I*b)^(3/2)*
ArcTan[Cot[c + d*x]/Sqrt[a - I*b]])/d) - ((a + I*b)^(3/2)*ArcTan[Cot[c + d
*x]/Sqrt[a + I*b]])/d - (2*b*Sqrt[a + b*Cot[c + d*x]])/d)
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3963
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

3.98.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. $2(127) = 254$.

Time = 0.08 (sec) , antiderivative size = 1375, normalized size of antiderivative = 9.11

method	result	size
derivativedivides	Expression too large to display	1375
default	Expression too large to display	1375
parts	Expression too large to display	2386

input
```
int((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*b*(a+b*cot(d*x+c))^(5/2)/d+2/d*b*(a+b*cot(d*x+c))^(1/2)*a^2+2/d*b^3*(
a+b*cot(d*x+c))^(1/2)+1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*
(a^2+b^2)^(1/2)*a^3+1/4/d*b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a
^2+b^2)^(1/2)*a-1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+1/
4/d*b^3*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+
c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d*b/(2*(a^2+b^2)^(1/
2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1
/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)*a^2+1/d*b^3/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2
*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)-2/d*b/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2
*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-2/d*b^3/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c
))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^3-1/4/d*b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))
^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2...
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1684 vs. 2(118) = 236.

Time = 0.31 (sec) , antiderivative size = 1684, normalized size of antiderivative = 11.15

$$\int (-a + b\cot(c+dx))(a + b\cot(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/10*(5*(d*cos(2*d*x + 2*c) - d)*sqrt(-(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^
6 + d^2*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b
^10 - 2*a^2*b^12 + b^14)/d^4))/d^2)*log(-(3*a^10*b + 11*a^8*b^3 + 14*a^6*b
^5 + 6*a^4*b^7 - a^2*b^9 - b^11)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x +
2*c) + b)/sin(2*d*x + 2*c)) + (a*d^3*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*
a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4) + (3*a^6*b^2 +
5*a^4*b^4 + a^2*b^6 - b^8)*d)*sqrt(-(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6 +
 d^2*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10
 - 2*a^2*b^12 + b^14)/d^4))/d^2)) - 5*(d*cos(2*d*x + 2*c) - d)*sqrt(-(a^7
- a^5*b^2 - 5*a^3*b^4 - 3*a*b^6 + d^2*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31
*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4))/d^2)*log(-(3*
a^10*b + 11*a^8*b^3 + 14*a^6*b^5 + 6*a^4*b^7 - a^2*b^9 - b^11)*sqrt((b*cos
(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - (a*d^3*sqrt(-(
9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^1
2 + b^14)/d^4) + (3*a^6*b^2 + 5*a^4*b^4 + a^2*b^6 - b^8)*d)*sqrt(-(a^7 - a
^5*b^2 - 5*a^3*b^4 - 3*a*b^6 + d^2*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^
8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4))/d^2)) - 5*(d*cos
(2*d*x + 2*c) - d)*sqrt(-(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6 - d^2*sqrt(-
(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^
12 + b^14)/d^4))/d^2)*log(-(3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5 + 6*a^4*...
```

3.98.6 Sympy [F]

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx = -\int a^3\sqrt{a+b\cot(c+dx)}\,dx - \int\left(-b^3\sqrt{a+b\cot(c+dx)}\cot^3(c+dx)\right)dx - \int\left(-ab^2\sqrt{a+b\cot(c+dx)}\cot^2(c+dx)\right)dx - \int a^2b\sqrt{a+b\cot(c+dx)}\cot(c+dx)\,dx$$

input
```
integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))**(5/2),x)
```

output
```
-Integral(a**3*sqrt(a + b*cot(c + d*x)), x) - Integral(-b**3*sqrt(a + b*co
t(c + d*x))*cot(c + d*x)**3, x) - Integral(-a*b**2*sqrt(a + b*cot(c + d*x)
)*cot(c + d*x)**2, x) - Integral(a**2*b*sqrt(a + b*cot(c + d*x))*cot(c + d
*x), x)
```

3.98.7 Maxima [F]

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx = \int (b\cot(dx+c)+a)^{\frac{5}{2}}(b\cot(dx+c)-a)\,dx$$

input
```
integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cot(d*x + c) + a)^(5/2)*(b*cot(d*x + c) - a), x)
```

3.98.8 Giac [F]

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx = \int (b\cot(dx+c)+a)^{\frac{5}{2}}(b\cot(dx+c)-a)\,dx$$

input
```
integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^(5/2)*(b*cot(d*x + c) - a), x)
```

3.98.9 Mupad [B] (verification not implemented)

Time = 40.09 (sec) , antiderivative size = 3442, normalized size of antiderivative = 22.79

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
int(-(a + b*cot(c + d*x))^(5/2)*(a - b*cot(c + d*x)),x)
```

output
```
log(((((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3
*b^4*d^2 + 10*a^5*b^2*d^2)/d^4)^(1/2)*(((((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*
a^2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d^2 + 10*a^5*b^2*d^2)/d^4)^(1/2)*(
64*a^2*b^5 + 64*a^4*b^3 - 32*a*b^2*d*(((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2
*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d^2 + 10*a^5*b^2*d^2)/d^4)^(1/2)*(a +
 b*cot(c + d*x))^(1/2)))/(2*d) - (16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a
^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2))/2 - (8*a^3*b^3*(3*a^2 - b^2)*(a
^2 + b^2)^3)/d^3)*((20*a^6*b^8*d^4 - a^4*b^10*d^4 - 110*a^8*b^6*d^4 + 100*
a^10*b^4*d^4 - 25*a^12*b^2*d^4)^(1/2)/(4*d^4) - a^7/(4*d^2) - (5*a^3*b^4)/
(4*d^2) + (5*a^5*b^2)/(2*d^2))^(1/2) - log(((-((-a^4*b^2*d^4*(5*a^4 + b^4
- 10*a^2*b^2)^2)^(1/2) + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1
/2)*(((-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*d^2 + 5*a
^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(64*a^2*b^5 + 64*a^4*b^3 + 32*a*b^
2*d*(-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*d^2 + 5*a^3
*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/(2*d) +
 (16*a^2*b^2*(a + b*cot(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b
^2))/d^2))/2 - (8*a^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3)*(-(a^7*d^2 + (
20*a^6*b^8*d^4 - a^4*b^10*d^4 - 110*a^8*b^6*d^4 + 100*a^10*b^4*d^4 - 25*a^
12*b^2*d^4)^(1/2) + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/(4*d^4))^(1/2) + log((
(-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7*d^2 + 5*a^3*...
```

3.98. $\int (-a + b\cot(c + dx))(a + b\cot(c + dx))^{5/2}\, dx$

3.99 $\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx$

3.99.1 Optimal result

Integrand size = 27, antiderivative size = 408

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = \frac{b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} - \frac{b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} - \frac{2b(a+b\cot(c+dx))^{3/2}}{3d} + \frac{b(a^2+b^2)\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} - \frac{b(a^2+b^2)\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d}$$

output
```
-2/3*b*(a+b*cot(d*x+c))^(3/2)/d+1/2*b*(a^2+b^2)*arctanh((-2^(1/2)*(a+b*cot
(d*x+c))^(1/2)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))/d*2^(
1/2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/2*b*(a^2+b^2)*arctanh((2^(1/2)*(a+b*cot(d
*x+c))^(1/2)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))/d*2^(1/
2)/(a-(a^2+b^2)^(1/2))^(1/2)+1/4*b*(a^2+b^2)*ln(a+b*cot(d*x+c)+(a^2+b^2)^(
1/2)-2^(1/2)*(a+b*cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(
a+(a^2+b^2)^(1/2))^(1/2)-1/4*b*(a^2+b^2)*ln(a+b*cot(d*x+c)+(a^2+b^2)^(1/2)
+2^(1/2)*(a+b*cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))/d*2^(1/2)/(a+(a
^2+b^2)^(1/2))^(1/2)
```

3.99.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.62 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.44

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = \frac{(-a+b\cot(c+dx))(a+b\cot(c+dx))\left(3i\sqrt{a-ib}(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)-3i\sqrt{a}\right.}{-3b^2d\cos^2(c+dx)+3a^2d\,s}$$

input
```
Integrate[(-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(3/2),x]
```

output
```
((-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])*((3*I)*Sqrt[a - I*b]*(a^2 + b^
2)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]] - (3*I)*Sqrt[a + I*b]*(
a^2 + b^2)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b]] + 2*b*(a + b*Co
t[c + d*x])^(3/2))*Sin[c + d*x]^2)/(-3*b^2*d*Cos[c + d*x]^2 + 3*a^2*d*Sin[
c + d*x]^2)
```

3.99.3 Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {3042, 4011, 27, 3042, 3966, 483, 1449, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cot(c+dx)-a)(a+b\cot(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4011}$$

$$\int \left(-a^2-b^2\right)\sqrt{a+b\cot(c+dx)}dx-\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$-\left(a^2+b^2\right)\int \sqrt{a+b\cot(c+dx)}dx-\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 3042$$

$$-\left(a^2+b^2\right)\int \sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 3966$$

$$\frac{b\left(a^2+b^2\right)\int \frac{\sqrt{a+b\cot(c+dx)}}{\cot^2(c+dx)b^2+b^2}d(b\cot(c+dx))}{d}-\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 483$$

$$\frac{2b\left(a^2+b^2\right)\int \frac{b^2\cot^2(c+dx)}{b^4\cot^4(c+dx)-2ab^2\cot^2(c+dx)+a^2+b^2}d\sqrt{a+b\cot(c+dx)}}{d}-\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 1449$$

$$\frac{2b\left(a^2+b^2\right)\left(\frac{\int \frac{\sqrt{a+b\cot(c+dx)}}{b^2\cot^2(c+dx)-\sqrt{2}b\sqrt{a+\sqrt{a^2+b^2}}\cot(c+dx)+\sqrt{a^2+b^2}}d\sqrt{a+b\cot(c+dx)}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}-\frac{\int \frac{\sqrt{a+b\cot(c+dx)}}{b^2\cot^2(c+dx)+\sqrt{2}b\sqrt{a+\sqrt{a^2+b^2}}\cot(c+dx)+\sqrt{a^2+b^2}}d\sqrt{\cdots}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\cdots}{d}$$

$$\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 1142$$

$$2b\left(a^2+b^2\right)\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}}d\sqrt{a+b\cot(c+dx)}}{\sqrt{2}}+\frac{1}{2}\int -\frac{\sqrt{2}\left(\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cdots}\right.}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2\cdots}}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\cdots\right.$$

$$\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 25$$

$$2b\left(a^2+b^2\right)\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}}d\sqrt{a+b\cot(c+dx)}}{\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot\cdots}\right.}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+\cdots}}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\cdots\right.$$

$$\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 27$$

3.99. $\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx$

$$\frac{2b(a^2+b^2)\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}}d\sqrt{a+b\cot(c+dx)}}{\sqrt{2}}-\frac{\int \frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+}}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}}}{\sqrt{2}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.}{}$$

$$\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

$\downarrow$ 1083

$$\frac{2b(a^2+b^2)\left(\frac{-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{2\left(a-\sqrt{a^2+b^2}\right)-b^2\cot^2(c+dx)}d\left(2\sqrt{a+b\cot(c+dx)}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\right)-\frac{\int \frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b}}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{}}}}{}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.}{}$$

$$\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

$\downarrow$ 219

$$\frac{2b(a^2+b^2)\left(\frac{-\frac{\int \frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}}d\sqrt{a+b\cot(c+dx)}}{\sqrt{2}}-\frac{\sqrt{\sqrt{a^2+b^2}+a}\,\text{arctanh}\left(\frac{2\sqrt{a+b\cot(c+dx)}-\sqrt{2}\sqrt{\sqrt{a^2+}}}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{a-\sqrt{a^2+b^2}}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.}{}$$

$$\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

$\downarrow$ 1103

$$\frac{2b(a^2+b^2)\left(\frac{\frac{1}{2}\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\cot(c+dx)}+\sqrt{a^2+b^2}+b^2\cot^2(c+dx)\right)-\frac{\sqrt{\sqrt{a^2+b^2}+a}\,\text{arctanh}\left(\frac{2\sqrt{a+b\cot(c+dx)}-\sqrt{2}\sqrt{\sqrt{a^2+b^2+}}}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{a-\sqrt{a^2+b^2}}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.}{d}$$

$$\frac{2b(a+b\cot(c+dx))^{3/2}}{3d}$$

input
```
Int[(-a + b*Cot[c + d*x])*(a + b*Cot[c + d*x])^(3/2),x]
```

3.99. $\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx$

output
```
(-2*b*(a + b*Cot[c + d*x])^(3/2))/(3*d) + (2*b*(a^2 + b^2)*((-((Sqrt[a + S
qrt[a^2 + b^2]]*ArcTanh[(-(Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]) + 2*Sqrt[a +
 b*Cot[c + d*x]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]])])/Sqrt[a - Sqrt[a^2
+ b^2]]) + Log[Sqrt[a^2 + b^2] + b^2*Cot[c + d*x]^2 - Sqrt[2]*Sqrt[a + Sqr
t[a^2 + b^2]]*Sqrt[a + b*Cot[c + d*x]]]/2)/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 +
b^2]]) - ((Sqrt[a + Sqrt[a^2 + b^2]]*ArcTanh[(Sqrt[2]*Sqrt[a + Sqrt[a^2 +
b^2]] + 2*Sqrt[a + b*Cot[c + d*x]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]])])/
Sqrt[a - Sqrt[a^2 + b^2]] + Log[Sqrt[a^2 + b^2] + b^2*Cot[c + d*x]^2 + Sqr
t[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Cot[c + d*x]]]/2)/(2*Sqrt[2]*Sqr
t[a + Sqrt[a^2 + b^2]])))/d
```

3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1449
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Simp[1/(2*c*r)   Int[x^(m - 1)/(q + r*x + x^2), x],
 x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m,
3] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

3.99.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. $2(333) = 666$.

Time = 0.07 (sec) , antiderivative size = 986, normalized size of antiderivative = 2.42

method	result
derivativedivides	$-\frac{2b(a+b\cot(dx+c))^{\frac{3}{2}}}{3d}+\frac{\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\cot(dx+c)-\sqrt{a^2+b^2}-a\right)\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}}{4db}$
default	$-\frac{2b(a+b\cot(dx+c))^{\frac{3}{2}}}{3d}+\frac{\ln\left(\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\cot(dx+c)-\sqrt{a^2+b^2}-a\right)\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}}{4db}$
parts	Expression too large to display

input
```
int((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*b*(a+b*cot(d*x+c))^(3/2)/d+1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+1/4/d*b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*(a^2+b^2)^(1/2)-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+
b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2))*a^2-1/d*b^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+
c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/
4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)
-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/4/d*b*ln((a+b*cot(
d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a
)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c
))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-1/4/d*b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))
^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*(a^2+b^2)^(1/2)+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a
+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a
)^(1/2))*a^2+1/d*b^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+
c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/
4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^...
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(335) = 670$.

Time = 0.29 (sec) , antiderivative size = 1148, normalized size of antiderivative = 2.81

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(3*d*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 + d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4
+ 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*log(d^3*sqrt(-(a^5 + 2*a^3*b^2
+ a*b^4 + d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d
^4))/d^2)*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4)
+ (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2
*d*x + 2*c) + b)/sin(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 3*d*sqrt(-(a^5 + 2*
a^3*b^2 + a*b^4 + d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 +
 b^10)/d^4))/d^2)*log(-d^3*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 + d^2*sqrt(-(a^8
*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*sqrt(-(a^8*b^2
 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4) + (a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d
*x + 2*c)))*sin(2*d*x + 2*c) - 3*d*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 - d^2*sq
rt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*log(d^
3*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 - d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*
b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6
+ 4*a^2*b^8 + b^10)/d^4) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sqrt((b*c
os(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)))*sin(2*d*x + 2
*c) + 3*d*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 - d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4
+ 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*log(-d^3*sqrt(-(a^5 + 2*a^3*b^2
 + a*b^4 - d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^1...
```

3.99.6 Sympy [F]

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = -\int a^2\sqrt{a+b\cot(c+dx)}\,dx - \int \left(-b^2\sqrt{a+b\cot(c+dx)}\cot^2(c+dx)\right)\,dx$$

input
```
integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))**(3/2),x)
```

output
```
-Integral(a**2*sqrt(a + b*cot(c + d*x)), x) - Integral(-b**2*sqrt(a + b*co
t(c + d*x))*cot(c + d*x)**2, x)
```

3.99.7 Maxima [F]

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = \int (b\cot(dx+c)+a)^{\frac{3}{2}}(b\cot(dx+c)-a)\,dx$$

input
```
integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cot(d*x + c) + a)^(3/2)*(b*cot(d*x + c) - a), x)
```

3.99.8 Giac [F]

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = \int (b\cot(dx+c)+a)^{\frac{3}{2}}(b\cot(dx+c)-a)\,dx$$

input
```
integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) + a)^(3/2)*(b*cot(d*x + c) - a), x)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 25.17 (sec) , antiderivative size = 2529, normalized size of antiderivative = 6.20

$$\int (-a+b\cot(c+dx))(a+b\cot(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
int(-(a + b*cot(c + d*x))^(3/2)*(a - b*cot(c + d*x)),x)
```

output
```
log((((16*b^4*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (1
6*a*b^2*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^
4)^(1/2)*(a^2*b + b^3 + d*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2
 + a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*(((-b^6*d^4*(3*
a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(1/2))/2 + (8*b^5*(a
^2 - b^2)*(a^2 + b^2)^2)/d^3)*((6*a^2*b^8*d^4 - b^10*d^4 - 9*a^4*b^6*d^4)^
(1/2)/(4*d^4) - (3*a*b^4)/(4*d^2) + (a^3*b^2)/(4*d^2))^(1/2) - log((8*b^5*
(a^2 - b^2)*(a^2 + b^2)^2)/d^3 - (((16*b^4*(a + b*cot(c + d*x))^(1/2)*(a^4
 + b^4 - 6*a^2*b^2))/d^2 + (16*a*b^2*(-((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) +
 3*a*b^4*d^2 - a^3*b^2*d^2)/d^4)^(1/2)*(a^2*b + b^3 - d*(-((-b^6*d^4*(3*a^
2 - b^2)^2)^(1/2) + 3*a*b^4*d^2 - a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d
*x))^(1/2)))/d)*(-((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4*d^2 - a^3*b^
2*d^2)/d^4)^(1/2))/2)*(-((6*a^2*b^8*d^4 - b^10*d^4 - 9*a^4*b^6*d^4)^(1/2)
+ 3*a*b^4*d^2 - a^3*b^2*d^2)/(4*d^4))^(1/2) - log((8*b^5*(a^2 - b^2)*(a^2
+ b^2)^2)/d^3 - (((16*b^4*(a + b*cot(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^
2))/d^2 + (16*a*b^2*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3
*b^2*d^2)/d^4)^(1/2)*(a^2*b + b^3 - d*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) -
 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(1/2)*(a + b*cot(c + d*x))^(1/2)))/d)*(((
-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(1/2))/2
)*(((6*a^2*b^8*d^4 - b^10*d^4 - 9*a^4*b^6*d^4)^(1/2) - 3*a*b^4*d^2 + a^...
```

3.99. $\int (-a + b\cot(c + dx))(a + b\cot(c + dx))^{3/2}\, dx$

3.100 $\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx$

3.100.1 Optimal result

Integrand size = 27, antiderivative size = 422

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx = \frac{b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} - \frac{b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} - \frac{2b\sqrt{a+b\cot(c+dx)}}{d} - \frac{b\sqrt{a^2+b^2}\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} + \frac{b\sqrt{a^2+b^2}\log\left(a+\sqrt{a^2+b^2}+b\cot(c+dx)+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}\right)}{2\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d}$$

output
```
-2*b*(a+b*cot(d*x+c))^(1/2)/d+1/2*b*arctanh((-2^(1/2)*(a+b*cot(d*x+c))^(1/
2)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)^(1/2)/d
*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/2*b*arctanh((2^(1/2)*(a+b*cot(d*x+c))
^(1/2)+(a+(a^2+b^2)^(1/2))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)^(1/
2)/d*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)-1/4*b*ln(a+b*cot(d*x+c)+(a^2+b^2)^(
1/2)-2^(1/2)*(a+b*cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)^(
1/2)/d*2^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)+1/4*b*ln(a+b*cot(d*x+c)+(a^2+b^2)
^(1/2)+2^(1/2)*(a+b*cot(d*x+c))^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2))*(a^2+b^2)
^(1/2)/d*2^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)
```

3.100.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.36

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx = \frac{b\left(\frac{(a^2+b^2)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}}-\frac{(a^2+b^2)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}}-2\sqrt{a+b\cot(c+dx)}\right)}{d}$$

input
```
Integrate[(-a + b*Cot[c + d*x])*Sqrt[a + b*Cot[c + d*x]],x]
```

output
```
(b*(((a^2 + b^2)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(
Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - ((a^2 + b^2)*ArcTanh[Sqrt[a + b*Cot[c +
 d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) - 2*Sqrt[a
 + b*Cot[c + d*x]]))/d
```

3.100.3 Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {3042, 4011, 27, 3042, 3966, 484, 1407, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cot(c+dx)-a)\sqrt{a+b\cot(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{-a^2-b^2}{\sqrt{a+b\cot(c+dx)}}dx-\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$-(a^2+b^2)\int \frac{1}{\sqrt{a+b\cot(c+dx)}}dx-\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

$\Big\downarrow$ 3042

$$-\left(a^2+b^2\right)\int \frac{1}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

$\Big\downarrow$ 3966

$$\frac{b\left(a^2+b^2\right)\int \frac{1}{\sqrt{a+b\cot(c+dx)}\left(\cot^2(c+dx)b^2+b^2\right)}d(b\cot(c+dx))}{d}-\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

$\Big\downarrow$ 484

$$\frac{2b\left(a^2+b^2\right)\int \frac{1}{b^4\cot^4(c+dx)-2ab^2\cot^2(c+dx)+a^2+b^2}d\sqrt{a+b\cot(c+dx)}}{d}-\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

$\Big\downarrow$ 1407

$$\frac{2b\left(a^2+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{a+b\cot(c+dx)}}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}}d\sqrt{a+b\cot(c+dx)}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}+\frac{\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{a+b\cot(c+dx)}}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+}}\right.}{d}$$

$$\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

$\Big\downarrow$ 1142

$$2b\left(a^2+b^2\right)\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}}d\sqrt{a+b\cot(c+dx)}}{\sqrt{2}}-\frac{1}{2}\int -\frac{\sqrt{2}\left(\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b}\right.}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2}}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$$\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

$\Big\downarrow$ 25

$$2b\left(a^2+b^2\right)\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}}d\sqrt{a+b\cot(c+dx)}}{\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot}\right.}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+}}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$$\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

$\Big\downarrow$ 27

3.100. $\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx$

$$2b\left(a^2+b^2\right)\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}}d\sqrt{a+b\cot(c+dx)}}{\sqrt{2}}+\frac{\int \frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+}}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}}}{\sqrt{2}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$$\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

$\downarrow$ 1083

$$2b\left(a^2+b^2\right)\left(\frac{\frac{\int \frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}}d\sqrt{a+b\cot(c+dx)}}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{2\left(a-\sqrt{a^2+b^2}\right)-b^2\cot^2(c+dx)}d\left(2\sqrt{\right.}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$$\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

$\downarrow$ 219

$$2b\left(a^2+b^2\right)\left(\frac{\frac{\int \frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\cot(c+dx)}}{b^2\cot^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cot(c+dx)}}d\sqrt{a+b\cot(c+dx)}}{\sqrt{2}}-\frac{\sqrt{\sqrt{a^2+b^2}+a}\,\text{arctanh}\left(\frac{2\sqrt{a+b\cot(c+dx)}-\sqrt{2}\sqrt{\sqrt{a^2+b^2}}}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{a-\sqrt{a^2+b^2}}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$$\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

$\downarrow$ 1103

$$2b\left(a^2+b^2\right)\left(\frac{-\frac{\sqrt{\sqrt{a^2+b^2}+a}\,\text{arctanh}\left(\frac{2\sqrt{a+b\cot(c+dx)}-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{a-\sqrt{a^2+b^2}}}-\frac{1}{2}\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\cot(c+dx)}+\sqrt{a^2+b^2}+b^2\cot^2(c+\right.}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$$\frac{2b\sqrt{a+b\cot(c+dx)}}{d}$$

input
```
Int[(-a + b*Cot[c + d*x])*Sqrt[a + b*Cot[c + d*x]],x]
```

3.100. $\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx$

output
```
(-2*b*Sqrt[a + b*Cot[c + d*x]])/d + (2*b*(a^2 + b^2)*((-((Sqrt[a + Sqrt[a^
2 + b^2]]*ArcTanh[(-(Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]) + 2*Sqrt[a + b*Cot
[c + d*x]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]])])/Sqrt[a - Sqrt[a^2 + b^2]
]) - Log[Sqrt[a^2 + b^2] + b^2*Cot[c + d*x]^2 - Sqrt[2]*Sqrt[a + Sqrt[a^2
+ b^2]]*Sqrt[a + b*Cot[c + d*x]]]/2)/(2*Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a + S
qrt[a^2 + b^2]]) + (-((Sqrt[a + Sqrt[a^2 + b^2]]*ArcTanh[(Sqrt[2]*Sqrt[a +
 Sqrt[a^2 + b^2]] + 2*Sqrt[a + b*Cot[c + d*x]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2
 + b^2]])])/Sqrt[a - Sqrt[a^2 + b^2]]) + Log[Sqrt[a^2 + b^2] + b^2*Cot[c +
 d*x]^2 + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Cot[c + d*x]]]/2)/(
2*Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a + Sqrt[a^2 + b^2]])))/d
```

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 484
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[2*
d   Subst[Int[1/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.100. $\int(-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

3.100.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(341) = 682$.

Time = 0.06 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.93

method	result
parts	$b\left(-2\sqrt{a+b\cot(dx+c)}+\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(b\cot(dx+c)+a+\sqrt{a+b\cot(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)}{4}-\frac{\left(a-\sqrt{a^2+b^2}\right)\text{arctan}}{}\right.$
derivativedivides	Expression too large to display
default	Expression too large to display

3.100. $\int(-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx$

input
```
int((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
b/d*(-2*(a+b*cot(d*x+c))^(1/2)+1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*cot(
d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1
/2))-(a-(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(
d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
)-1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)-((a^2+b^2)^(1/2)-a)/(2*(
a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(
1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/d/b*ln(b*cot(d*x+c)+a
+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/d/b*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^
(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*(a^2+b^2)^(1/2)*a+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(
a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2))*a-1/4/d/b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+1/4/d/
b*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^
2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-1/d*b/(2*(
a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(
1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 849 vs. $2(343) = 686$.

Time = 0.31 (sec) , antiderivative size = 849, normalized size of antiderivative = 2.01

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx = \frac{d\sqrt{-\frac{a^3+ab^2+d^2\sqrt{-\frac{a^4b^2+2a^2b^4+b^6}{d^4}}}{d^2}}\log\left(\left(a^4b+2a^2b^3+b^5\right)\sqrt{\frac{b\cos(2dx+2c)+a\sin(2dx+2c)+b}{\sin(2dx+2c)}}+\left(ad^3\sqrt{-\frac{a^4b^2+2a^2b^4+b}{d^4}}\right.\right.}{}$$

input
```
integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(d*sqrt(-(a^3 + a*b^2 + d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^
2)*log((a^4*b + 2*a^2*b^3 + b^5)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x +
2*c) + b)/sin(2*d*x + 2*c)) + (a*d^3*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4
) + (a^2*b^2 + b^4)*d)*sqrt(-(a^3 + a*b^2 + d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4
 + b^6)/d^4))/d^2)) - d*sqrt(-(a^3 + a*b^2 + d^2*sqrt(-(a^4*b^2 + 2*a^2*b^
4 + b^6)/d^4))/d^2)*log((a^4*b + 2*a^2*b^3 + b^5)*sqrt((b*cos(2*d*x + 2*c)
 + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - (a*d^3*sqrt(-(a^4*b^2 + 2*a
^2*b^4 + b^6)/d^4) + (a^2*b^2 + b^4)*d)*sqrt(-(a^3 + a*b^2 + d^2*sqrt(-(a^
4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)) - d*sqrt(-(a^3 + a*b^2 - d^2*sqrt(-(a
^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)*log((a^4*b + 2*a^2*b^3 + b^5)*sqrt((b
*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + (a*d^3*sqr
t(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4) - (a^2*b^2 + b^4)*d)*sqrt(-(a^3 + a*b^
2 - d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)) + d*sqrt(-(a^3 + a*b
^2 - d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)*log((a^4*b + 2*a^2*b
^3 + b^5)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2
*c)) - (a*d^3*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4) - (a^2*b^2 + b^4)*d)*
sqrt(-(a^3 + a*b^2 - d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)) - 4
*b*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)))/d
```

3.100.6 Sympy [F]

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx$$
$$=-\int a\sqrt{a+b\cot{(c+dx)}}\,dx-\int\left(-b\sqrt{a+b\cot{(c+dx)}}\cot{(c+dx)}\right)\,dx$$

input
```
integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))**(1/2),x)
```

output
```
-Integral(a*sqrt(a + b*cot(c + d*x)), x) - Integral(-b*sqrt(a + b*cot(c +
d*x))*cot(c + d*x), x)
```

3.100.7 Maxima [F]

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx = \int \sqrt{b\cot(dx+c)+a}(b\cot(dx+c)-a)\,dx$$

input
```
integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cot(d*x + c) + a)*(b*cot(d*x + c) - a), x)
```

3.100.8 Giac [F]

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx = \int \sqrt{b\cot(dx+c)+a}(b\cot(dx+c)-a)\,dx$$

input
```
integrate((-a+b*cot(d*x+c))*(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cot(d*x + c) + a)*(b*cot(d*x + c) - a), x)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.38

$$\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx =$$
$$-\operatorname{atanh}\left(\frac{d^3\left(\frac{16\,(a^2\,b^4-a^4\,b^2)\,\sqrt{a+b\cot(c+d\,x)}}{d^2}+\frac{16\,a\,b^2\,(a^3+1\mathrm{i}\,b\,a^2)\,\sqrt{a+b\cot(c+d\,x)}}{d^2}\right)\sqrt{-\frac{a^3+1\mathrm{i}\,b\,a^2}{d^2}}}{16\,(a^5\,b^3+a^3\,b^5)}\right)\sqrt{-\frac{a^3+1\mathrm{i}\,b\,a^2}{d^2}}$$
$$-\operatorname{atanh}\left(\frac{d^3\,\sqrt{\frac{-a^3+a^2\,b\,1\mathrm{i}}{d^2}}\left(\frac{16\,(a^2\,b^4-a^4\,b^2)\,\sqrt{a+b\cot(c+d\,x)}}{d^2}-\frac{16\,a\,b^2\,(-a^3+a^2\,b\,1\mathrm{i})\,\sqrt{a+b\cot(c+d\,x)}}{d^2}\right)}{16\,(a^5\,b^3+a^3\,b^5)}\right)\sqrt{\frac{-a^3+a^2\,b\,1\mathrm{i}}{d^2}}$$
$$-\frac{2\,b\,\sqrt{a+b\cot\left(c+d\,x\right)}}{d}+\operatorname{atan}\left(\frac{b^6\,\sqrt{\frac{a\,b^2}{4\,d^2}-\frac{b^3\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\cot\left(c+d\,x\right)}\,32\mathrm{i}}{\frac{b^8\,16\mathrm{i}}{d}+\frac{a^2\,b^6\,16\mathrm{i}}{d}}\right.$$
$$\left.+\frac{32\,a\,b^5\,\sqrt{\frac{a\,b^2}{4\,d^2}-\frac{b^3\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\cot\left(c+d\,x\right)}}{\frac{b^8\,16\mathrm{i}}{d}+\frac{a^2\,b^6\,16\mathrm{i}}{d}}\right)\sqrt{\frac{a\,b^2-b^3\,1\mathrm{i}}{4\,d^2}}\,2\mathrm{i}$$
$$-\operatorname{atan}\left(\frac{b^6\,\sqrt{\frac{a\,b^2}{4\,d^2}+\frac{b^3\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\cot\left(c+d\,x\right)}\,32\mathrm{i}}{\frac{b^8\,16\mathrm{i}}{d}+\frac{a^2\,b^6\,16\mathrm{i}}{d}}\right.$$
$$\left.-\frac{32\,a\,b^5\,\sqrt{\frac{a\,b^2}{4\,d^2}+\frac{b^3\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\cot\left(c+d\,x\right)}}{\frac{b^8\,16\mathrm{i}}{d}+\frac{a^2\,b^6\,16\mathrm{i}}{d}}\right)\sqrt{\frac{b^3\,1\mathrm{i}+a\,b^2}{4\,d^2}}\,2\mathrm{i}$$

input
```
int(-(a + b*cot(c + d*x))^(1/2)*(a - b*cot(c + d*x)),x)
```

output
```
atan((b^6*((a*b^2)/(4*d^2) - (b^3*1i)/(4*d^2))^(1/2)*(a + b*cot(c + d*x))^
(1/2)*32i)/((b^8*16i)/d + (a^2*b^6*16i)/d) + (32*a*b^5*((a*b^2)/(4*d^2) -
(b^3*1i)/(4*d^2))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((b^8*16i)/d + (a^2*b^
6*16i)/d))*((a*b^2 - b^3*1i)/(4*d^2))^(1/2)*2i - atan((b^6*((b^3*1i)/(4*d^
2) + (a*b^2)/(4*d^2))^(1/2)*(a + b*cot(c + d*x))^(1/2)*32i)/((b^8*16i)/d +
 (a^2*b^6*16i)/d) - (32*a*b^5*((b^3*1i)/(4*d^2) + (a*b^2)/(4*d^2))^(1/2)*(
a + b*cot(c + d*x))^(1/2))/((b^8*16i)/d + (a^2*b^6*16i)/d))*((a*b^2 + b^3*
1i)/(4*d^2))^(1/2)*2i - atanh((d^3*((16*(a^2*b^4 - a^4*b^2)*(a + b*cot(c +
 d*x))^(1/2))/d^2 + (16*a*b^2*(a^2*b*1i + a^3)*(a + b*cot(c + d*x))^(1/2))
/d^2)*(-(a^2*b*1i + a^3)/d^2)^(1/2))/(16*(a^3*b^5 + a^5*b^3)))*(-(a^2*b*1i
 + a^3)/d^2)^(1/2) - atanh((d^3*((a^2*b*1i - a^3)/d^2)^(1/2)*((16*(a^2*b^4
 - a^4*b^2)*(a + b*cot(c + d*x))^(1/2))/d^2 - (16*a*b^2*(a^2*b*1i - a^3)*(
a + b*cot(c + d*x))^(1/2))/d^2))/(16*(a^3*b^5 + a^5*b^3)))*((a^2*b*1i - a^
3)/d^2)^(1/2) - (2*b*(a + b*cot(c + d*x))^(1/2))/d
```

3.100. $\int (-a+b\cot(c+dx))\sqrt{a+b\cot(c+dx)}\,dx$

3.101 $\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

3.101.1 Optimal result

Integrand size = 25, antiderivative size = 102

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} - \frac{(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

output
```
(I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)-(I*A
-B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.51

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \frac{\left(\sqrt{a+ib}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)+\sqrt{a-ib}(-iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)\right)(A+B\cot(c+}{\sqrt{a-ib}\sqrt{a+ib}d(B\cos(c+dx)+A\sin(c+dx))}$$

input
```
Integrate[(A + B*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

output
```
((Sqrt[a + I*b]*(I*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - I*b]]
+ Sqrt[a - I*b]*((-I)*A + B)*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + I*b
]])*(A + B*Cot[c + d*x])*Sin[c + d*x])/(Sqrt[a - I*b]*Sqrt[a + I*b]*d*(B*C
os[c + d*x] + A*Sin[c + d*x]))
```

3.101.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-B\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(A+iB)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{i\cot(c+dx)+1}{\sqrt{a+b\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A-iB)\int \frac{1-i\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{1}{2}(A+iB)\int \frac{i\tan\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(A+iB)\int -\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}-\frac{i(A-iB)\int -\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{i(A-iB)\int \frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}-\frac{i(A+iB)\int \frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(A+iB)\int \frac{1}{-\frac{i\cot^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}-\frac{(A-iB)\int \frac{1}{\frac{i\cot^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(A-iB)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{(A+iB)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

input
```
Int[(A + B*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

output
```
-(((A - I*B)*ArcTan[Cot[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)) - ((A
+ I*B)*ArcTan[Cot[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)
```

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

3.101.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1889 vs. $2(84) = 168$.

Time = 0.09 (sec) , antiderivative size = 1890, normalized size of antiderivative = 18.53

method	result	size
parts	Expression too large to display	1890
derivativedivides	Expression too large to display	3976
default	Expression too large to display	3976

input
```
int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/4/d/b/(a^2+b^2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/
d*b/(a^2+b^2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d/b/(a^2+b^2
)^(3/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/4/d*b/(a^2+b^2)^
(3/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d/b/(a^2+b^2)^(1/2)/
(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2
)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/d*b/(a^2+b^2)^(1/
2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+
b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/d/b/(a^2+b^2)^(3/2
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b
^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4-3/d*b/(a^2+b^2)^(
3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^
2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-2/d*b^3/(a^2+b
^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/4/d/b/(a^2+
b^2)*ln(b*cot(d*x+c)+a-(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+1/4/d*b/(a^2+b^2)*...
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. $2(78) = 156$.

Time = 0.32 (sec) , antiderivative size = 1773, normalized size of antiderivative = 17.38

$$\int \frac{A + B\cot(c + dx)}{\sqrt{a + b\cot(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b +
(A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A
^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*a - (A^4 - B^4)*b)*
sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + ((A
*a^3 + B*a^2*b + A*a*b^2 + B*b^3)*d^3*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*
B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (
2*A*B^2*a^2 - (3*A^2*B - B^3)*a*b + (A^3 - A*B^2)*b^2)*d)*sqrt(-((a^2 + b^
2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B
^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2 - B^2)*a)/((a^2 +
 b^2)*d^2))) - 1/2*sqrt(-((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B
- A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4))
 + 2*A*B*b + (A^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*a -
(A^4 - B^4)*b)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*
x + 2*c)) - ((A*a^3 + B*a^2*b + A*a*b^2 + B*b^3)*d^3*sqrt(-(4*A^2*B^2*a^2
- 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 +
 b^4)*d^4)) + (2*A*B^2*a^2 - (3*A^2*B - B^3)*a*b + (A^3 - A*B^2)*b^2)*d)*s
qrt(-((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4
- 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2 -
B^2)*a)/((a^2 + b^2)*d^2))) - 1/2*sqrt(((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a
^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2...
```

3.101.6 Sympy [F]

$$\int \frac{A + B\cot(c+dx)}{\sqrt{a + b\cot(c+dx)}}\,dx = \int \frac{A + B\cot{(c+dx)}}{\sqrt{a + b\cot{(c+dx)}}}\,dx$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*cot(c + d*x))/sqrt(a + b*cot(c + d*x)), x)
```

3.101.7 Maxima [F]

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\int \frac{B\cot\left(dx+c\right)+A}{\sqrt{b\cot\left(dx+c\right)+a}}\,dx$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*cot(d*x + c) + A)/sqrt(b*cot(d*x + c) + a), x)
```

3.101.8 Giac [F]

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\int \frac{B\cot\left(dx+c\right)+A}{\sqrt{b\cot\left(dx+c\right)+a}}\,dx$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((B*cot(d*x + c) + A)/sqrt(b*cot(d*x + c) + a), x)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 2909, normalized size of antiderivative = 28.52

$$\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\text{Too large to display}$$

input
```
int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^(1/2),x)
```

output
```
2*atanh((32*B^2*b^2*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^
4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*
B^3*b^2)/d - (16*B^3*a^2*b^2*d^3)/(a^2*d^4 + b^2*d^4) + (4*B*a*b^2*d^2*(-1
6*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*((B^2*a*d^2)/(4*(a^2
*d^4 + b^2*d^4)) - (-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2)
*(a + b*cot(c + d*x))^(1/2)*(-16*B^4*b^2*d^4)^(1/2))/(16*B^3*b^4*d + 16*B^
3*a^2*b^2*d - (16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - (16*B^3*a^4*b^2*d
^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d
^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5
)) - (32*B^2*a^2*b^2*d^2*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b
^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/
(16*B^3*b^4*d + 16*B^3*a^2*b^2*d - (16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^2*d^4
) - (16*B^3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b
^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^(1/2
))/(a^2*d^5 + b^2*d^5)))*((B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*B^4*b
^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2) + 2*atanh((8*a*b^2*((-16*B^4
*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d
^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-16*B^4*b^2*d^4)^(1/2))/((16*B^3*a
^2*b^4*d^5)/(a^2*d^4 + b^2*d^4) - 16*B^3*a^2*b^2*d - 16*B^3*b^4*d + (16*B^
3*a^4*b^2*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)...
```

3.101. $\int \frac{A+B\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

3.102 $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx$

3.102.1 Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx=\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}-\frac{(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}+\frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}$$

output
```
(I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d-(I*A
-B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+2*(A*b-B
*a)/(a^2+b^2)/d/(a+b*cot(d*x+c))^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.64

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx=-\frac{\frac{\left(aAb+Ab\sqrt{-b^2}+b^2B-a\sqrt{-b^2}B\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}}-\frac{\left(aAb-Ab\sqrt{-b^2}+b^2B+a\sqrt{-b^2}B\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}}+\frac{2(-A}{\sqrt{a+b}}}{(a^2+b^2)\,d}$$

input
```
Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^(3/2),x]
```

output
```
-((((a*A*b + A*b*Sqrt[-b^2] + b^2*B - a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*C
ot[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - ((
a*A*b - A*b*Sqrt[-b^2] + b^2*B + a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Cot[c
+ d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + (2*(-(A
*b) + a*B))/Sqrt[a + b*Cot[c + d*x]])/((a^2 + b^2)*d))
```

3.102.3 Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4012, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-B\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int \frac{aA+bB-(Ab-aB)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx}{a^2+b^2}+\frac{2(Ab-aB)}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{aA+bB-(aB-Ab)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}+\frac{2(Ab-aB)}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}$$

$$\Big\downarrow \text{4022}$$

$$\frac{2(Ab-aB)}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}+\frac{\frac{1}{2}(a-ib)(A+iB)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx+\frac{1}{2}(a+ib)(A-iB)\int \frac{i\cot(c+dx)+1}{\sqrt{a+b\cot(c+dx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2(Ab-aB)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{\frac{1}{2}(a+ib)(A-iB)\int\frac{1-i\tan(c+dx+\frac{\pi}{2})}{\sqrt{a-b\tan(c+dx+\frac{\pi}{2})}}dx+\frac{1}{2}(a-ib)(A+iB)\int\frac{i\tan(c+dx+\frac{\pi}{2})+1}{\sqrt{a-b\tan(c+dx+\frac{\pi}{2})}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{4020}$$

$$\frac{2(Ab-aB)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{\frac{i(a-ib)(A+iB)\int-\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}-\frac{i(a+ib)(A-iB)\int-\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2(Ab-aB)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{\frac{i(a+ib)(A-iB)\int\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}-\frac{i(a-ib)(A+iB)\int\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}}{a^2+b^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{2(Ab-aB)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{-\frac{(a-ib)(A+iB)\int\frac{1}{-\frac{i\cot^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}-\frac{(a+ib)(A-iB)\int\frac{1}{\frac{i\cot^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}}{a^2+b^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(Ab-aB)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{-\frac{(a+ib)(A-iB)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{(a-ib)(A+iB)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}}{a^2+b^2}$$

input
```
Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^(3/2),x]
```

output
```
(-(((a + I*b)*(A - I*B)*ArcTan[Cot[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]
*d)) - ((a - I*b)*(A + I*B)*ArcTan[Cot[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a +
I*b]*d))/(a^2 + b^2) + (2*(A*b - a*B))/((a^2 + b^2)*d*Sqrt[a + b*Cot[c + d
*x]])
```

3.102. $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx$

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3682 vs. $2(118) = 236$.

Time = 0.15 (sec) , antiderivative size = 3683, normalized size of antiderivative = 26.69

method	result	size
parts	Expression too large to display	3683
derivativedivides	Expression too large to display	7951
default	Expression too large to display	7951

input
```
int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/4/d/b/(a^2+b^2)^2*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/
4/d*b/(a^2+b^2)^2*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/4/d/b/(a
^2+b^2)^(5/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-1/4/d*b^3/(a
^2+b^2)^(5/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d/b/(a^2+b^2)^
(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+1/d*b/(a^2+b^
2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2
*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a+1/d*b/(a^2+b
^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-1/d/b/(a^2+b^
2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2
*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^5+1/d*b^3/(a
^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-3/d*b^3/(a^2+
b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+
(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-4/d*b/(...
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4572 vs. $2(112) = 224$.

Time = 0.80 (sec) , antiderivative size = 4572, normalized size of antiderivative = 33.13

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(((a^2*b + b^3)*d*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*sin(2*d*x + 2*c)
 + (a^2*b + b^3)*d)*sqrt(-(6*A*B*a^2*b - 2*A*B*b^3 + (A^2 - B^2)*a^3 - 3*(
A^2 - B^2)*a*b^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(4*A^2*B^
2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2
+ 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*a^2*b^4 - 12*(A^3
*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 + 6*a^10*b^2 + 15*
a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*d^2))*log(-(2*(A^3*B + A*B^3)*a^3 - 3*(A^4 - B^4)
*a^2*b - 6*(A^3*B + A*B^3)*a*b^2 + (A^4 - B^4)*b^3)*sqrt((b*cos(2*d*x + 2*
c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + ((A*a^8 + 2*B*a^7*b + 2*A
*a^6*b^2 + 6*B*a^5*b^3 + 6*B*a^3*b^5 - 2*A*a^2*b^6 + 2*B*a*b^7 - A*b^8)*d^
3*sqrt(-(4*A^2*B^2*a^6 - 12*(A^3*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2
+ 3*B^4)*a^4*b^2 + 40*(A^3*B - A*B^3)*a^3*b^3 - 6*(A^4 - 8*A^2*B^2 + B^4)*
a^2*b^4 - 12*(A^3*B - A*B^3)*a*b^5 + (A^4 - 2*A^2*B^2 + B^4)*b^6)/((a^12 +
 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^
4)) + (2*A*B^2*a^5 - (7*A^2*B - 3*B^3)*a^4*b + 2*(3*A^3 - 7*A*B^2)*a^3*b^2
 + 4*(4*A^2*B - B^3)*a^2*b^3 - 2*(A^3 - 4*A*B^2)*a*b^4 - (A^2*B - B^3)*b^5
)*d)*sqrt(-(6*A*B*a^2*b - 2*A*B*b^3 + (A^2 - B^2)*a^3 - 3*(A^2 - B^2)*a*b^
2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(4*A^2*B^2*a^6 - 12*(A^3
*B - A*B^3)*a^5*b + 3*(3*A^4 - 14*A^2*B^2 + 3*B^4)*a^4*b^2 + 40*(A^3*B ...
```

3.102.6 Sympy [F]

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \int \frac{A+B\cot{(c+dx)}}{(a+b\cot{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*cot(c + d*x))/(a + b*cot(c + d*x))**(3/2), x)
```

3.102.7 Maxima [F]

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \int \frac{B\cot(dx+c)+A}{(b\cot(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*cot(d*x + c) + A)/(b*cot(d*x + c) + a)^(3/2), x)
```

3.102.8 Giac [F]

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \int \frac{B\cot(dx+c)+A}{(b\cot(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((B*cot(d*x + c) + A)/(b*cot(d*x + c) + a)^(3/2), x)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 19.29 (sec) , antiderivative size = 5737, normalized size of antiderivative = 41.57

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^(3/2),x)
```

output
```
(log((((a + b*cot(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 -
32*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) + ((((96*A^4*a^2*b^4*d^4 - 16*A^4
*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/
(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(64*A*a*b^11*d^
4 - ((((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) -
 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*
a^4*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^
10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2
*d^5))/4 + 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^4 + 256*A*a^7*b^5*d^4 + 64*
A*a^9*b^3*d^4))/4)*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^
2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^
2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 - 8*A^3*b^9*d^2 - 24*A^3*a^2*b^7*d^2
- 24*A^3*a^4*b^5*d^2 - 8*A^3*a^6*b^3*d^2)*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b
^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a
^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 + (log((((a +
b*cot(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b
^4*d^3 - 16*A^2*a^8*b^2*d^3) + ((-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 -
144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 +
b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(64*A*a*b^11*d^4 - ((-((96
*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*...
```

3.102. $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx$

3.103 $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx$

3.103.1 Optimal result

Integrand size = 25, antiderivative size = 185

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx = \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} + \frac{2(Ab-aB)}{3(a^2+b^2)\,d(a+b\cot(c+dx))^{3/2}} + \frac{2(2aAb-a^2B+b^2B)}{(a^2+b^2)^2\,d\sqrt{a+b\cot(c+dx)}}$$

output
```
(I*A+B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d-(I*A
-B)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d+2/3*(A*b
-B*a)/(a^2+b^2)/d/(a+b*cot(d*x+c))^(3/2)+2*(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)
^2/d/(a+b*cot(d*x+c))^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 3.76 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.72

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx = -\frac{\frac{3\left(2ab\left(A\sqrt{-b^2}+bB\right)+a^2\left(Ab-\sqrt{-b^2}B\right)+b^2\left(-Ab+\sqrt{-b^2}B\right)\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} + \frac{3\left(2ab\left(A\sqrt{-b^2}-bB\right)-a^2\left(Ab+\sqrt{-b^2}B\right)+b^2\left(\ldots\right.\right.}{\sqrt{-b^2}\sqrt{a\ldots}}}{3(a^2+b^2)^2\,d}$$

input
```
Integrate[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^(5/2),x]
```

output
```
-1/3*((3*(2*a*b*(A*Sqrt[-b^2] + b*B) + a^2*(A*b - Sqrt[-b^2]*B) + b^2*(-(A
*b) + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - Sqrt[-b^2]]
])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) + (3*(2*a*b*(A*Sqrt[-b^2] - b*B) - a^
2*(A*b + Sqrt[-b^2]*B) + b^2*(A*b + Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*Cot[
c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + (2*(a
^2 + b^2)*(-(A*b) + a*B))/(a + b*Cot[c + d*x])^(3/2) + (6*(-2*a*A*b + a^2*
B - b^2*B))/Sqrt[a + b*Cot[c + d*x]])/((a^2 + b^2)^2*d)
```

3.103.3 Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4012, 3042, 4012, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A-B\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\Big\downarrow \text{4012}$$
$$\frac{\int \frac{aA+bB-(Ab-aB)\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}dx}{a^2+b^2}+\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\cot(c+dx))^{3/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{aA+bB-(aB-Ab)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{a^2+b^2}+\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\cot(c+dx))^{3/2}}$$
$$\Big\downarrow \text{4012}$$
$$\frac{\frac{\int \frac{Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx}{a^2+b^2}+\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}}{a^2+b^2}+\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{Aa^2+2bBa-Ab^2-\left(Ba^2-2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}+\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}}{a^2+b^2}+\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}$$

$$\Big\downarrow \; 4022$$

$$\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}+\frac{\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{\frac{1}{2}(a-ib)^2(A+iB)\int\frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx+\frac{1}{2}(a+ib)^2(A-iB)\int\frac{i\cot(c+dx)+1}{\sqrt{a+b\cot(c+dx)}}dx}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}+\frac{\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{\frac{1}{2}(a-ib)^2(A+iB)\int\frac{i\tan\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{1}{2}(a+ib)^2(A-iB)\int\frac{1-i\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 4020$$

$$\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}+\frac{\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{\frac{i(a-ib)^2(A+iB)\int-\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}-\frac{i(a+ib)^2(A-iB)\int-\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d}{2d}}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}+\frac{\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{\frac{i(a+ib)^2(A-iB)\int\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}-\frac{i(a-ib)^2(A+iB)\int\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot}{2d}}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 73$$

$$\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}+\frac{\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{-\frac{(a-ib)^2(A+iB)\int\frac{1}{-\frac{i\cot^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}-\frac{(a+ib)^2(A-iB)\int\frac{1}{\frac{i\cot^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}}{a^2+b^2}}{a^2+b^2}$$

3.103. $\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx$

$$\Big\downarrow 221$$

$$\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}+\frac{\frac{2(a^2(-B)+2aAb+b^2B)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{-\frac{(a-ib)^2(A+iB)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}-\frac{(a+ib)^2(A-iB)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}}{a^2+b^2}}{a^2+b^2}$$

input
```
Int[(A + B*Cot[c + d*x])/(a + b*Cot[c + d*x])^(5/2),x]
```

output
```
(2*(A*b - a*B))/(3*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^(3/2)) + ((-(((a + I
*b)^2*(A - I*B)*ArcTan[Cot[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)) - (
(a - I*b)^2*(A + I*B)*ArcTan[Cot[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d
))/(a^2 + b^2) + (2*(2*a*A*b - a^2*B + b^2*B))/((a^2 + b^2)*d*Sqrt[a + b*C
ot[c + d*x]]))/(a^2 + b^2)
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

3.103.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4471 vs. $2(161) = 322$.

Time = 0.16 (sec) , antiderivative size = 4472, normalized size of antiderivative = 24.17

method	result	size
parts	Expression too large to display	4472
derivativedivides	Expression too large to display	12836
default	Expression too large to display	12836

input
```
int((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(2/3/d*b/(a^2+b^2)/(a+b*cot(d*x+c))^(3/2)+2/d*b/(a^2+b^2)^3/(2*(a^2+b^2)
^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)
^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-4/d*b/(a^2+b^2)^(7/2)/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2
*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4-4/d*b/(a^2+b^2)^(7/2)/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2
)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4+1/2/d*b/(a^2+b^2)^(7/2)*l
n(b*cot(d*x+c)+a-(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2
+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/2/d*b/(a^2+b^2)^(7/2)*ln(
b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b
^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/4/d/b/(a^2+b^2)^(7/2)*ln(b*
cot(d*x+c)+a-(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2
)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+3/4/d*b^3/(a^2+b^2)^(7/2)*ln(b*
cot(d*x+c)+a-(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2
)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+2/d*b^3/(a^2+b^2)^3/(2*(a^2+b^2)^
(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^
(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a+1/4/d/b/(a^2+b^2)^3*ln(b*cot(d*x+c
)+a-(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+1/d/b/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)
-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1...
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7422 vs. $2(155) = 310$.

Time = 3.60 (sec) , antiderivative size = 7422, normalized size of antiderivative = 40.12

$$\int \frac{A + B\cot(c + dx)}{(a + b\cot(c + dx))^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.103.6 Sympy [F]

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx=\int \frac{A+B\cot{(c+dx)}}{(a+b\cot{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*cot(c + d*x))/(a + b*cot(c + d*x))**(5/2), x)
```

3.103.7 Maxima [F]

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx=\int \frac{B\cot{(dx+c)}+A}{(b\cot{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*cot(d*x + c) + A)/(b*cot(d*x + c) + a)^(5/2), x)
```

3.103.8 Giac [F]

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx=\int \frac{B\cot{(dx+c)}+A}{(b\cot{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((B*cot(d*x + c) + A)/(b*cot(d*x + c) + a)^(5/2), x)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 31.32 (sec) , antiderivative size = 9453, normalized size of antiderivative = 51.10

$$\int \frac{A+B\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((A + B*cot(c + d*x))/(a + b*cot(c + d*x))^(5/2),x)
```

output
```
(log((((a + b*cot(c + d*x))^(1/2)*(320*A^2*a^4*b^14*d^3 - 16*A^2*b^18*d^3
+ 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*a^10*b^8*d^3 +
320*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) + ((((320*A^4*a^2*b^8*d^4 - 16
*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*
b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a
^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a
^8*b^2*d^4))^(1/2)*(896*A*a^6*b^15*d^4 - ((((320*A^4*a^2*b^8*d^4 - 16*A^4*
b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d
^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d
^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^
2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5
 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11
*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 +
640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 - 160*A*a^2*b^19*d^4 - 128*A*a^4*b^
17*d^4 - 32*A*b^21*d^4 + 3136*A*a^8*b^13*d^4 + 4928*A*a^10*b^11*d^4 + 4480
*A*a^12*b^9*d^4 + 2432*A*a^14*b^7*d^4 + 736*A*a^16*b^5*d^4 + 96*A*a^18*b^3
*d^4))/4)*(((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4
+ 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A
^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 +
10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 - 96*A^3*a^3...
```

3.104 $\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

3.104.1 Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=-\frac{(ia-b)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d}+\frac{(ia+b)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

output
```
-(I*a-b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/d/(a-I*b)^(1/2)+(I*
a+b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/d/(a+I*b)^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.34

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\frac{\frac{\left(b^2-a\sqrt{-b^2}\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{a-\sqrt{-b^2}}}+\frac{\left(b^2+a\sqrt{-b^2}\right)\text{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{a+\sqrt{-b^2}}}}{bd}$$

input
```
Integrate[(-a + b*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

output
```
(((b^2 - a*Sqrt[-b^2])*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a - Sqrt[-b^2
]]])/Sqrt[a - Sqrt[-b^2]] + ((b^2 + a*Sqrt[-b^2])*ArcTanh[Sqrt[a + b*Cot[c
 + d*x]]/Sqrt[a + Sqrt[-b^2]]])/Sqrt[a + Sqrt[-b^2]])/(b*d)
```

3.104.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b\cot(c+dx)-a}{\sqrt{a+b\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-a-b\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4022}$$

$$-\frac{1}{2}(a-ib)\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx-\frac{1}{2}(a+ib)\int \frac{i\cot(c+dx)+1}{\sqrt{a+b\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}(a+ib)\int \frac{1-i\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{1}{2}(a-ib)\int \frac{i\tan\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(a+ib)\int -\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}-\frac{i(a-ib)\int -\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{i(a-ib)\int \frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}-\frac{i(a+ib)\int \frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a-ib)\int \frac{1}{-\frac{i\cot^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}+\frac{(a+ib)\int \frac{1}{\frac{i\cot^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}$$

$$\Big\downarrow \text{221}$$

$$\frac{(a+ib)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{(a-ib)\arctan\left(\frac{\cot(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

input
```
Int[(-a + b*Cot[c + d*x])/Sqrt[a + b*Cot[c + d*x]],x]
```

output
```
((a + I*b)*ArcTan[Cot[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) + ((a - I
*b)*ArcTan[Cot[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1890 vs. $2(84) = 168$.

Time = 0.08 (sec) , antiderivative size = 1891, normalized size of antiderivative = 18.54

method	result	size
parts	Expression too large to display	1891
derivativedivides	Expression too large to display	1905
default	Expression too large to display	1905

input
```
int((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
b/d*(-1/2/(a^2+b^2)^(1/2)*(-1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*cot(d*x
+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2)
)+2*((a^2+b^2)^(1/2)-a)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d
*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))
)-1/2/(a^2+b^2)^(1/2)*(1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*cot(d*x+c)+a
-(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(
(a^2+b^2)^(1/2)-a)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c)
)^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))-a*
(-1/4/d/b/(a^2+b^2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/d*
b/(a^2+b^2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d/b/(a^2+b^2)^
(3/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/4/d*b/(a^2+b^2)^(3
/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d/b/(a^2+b^2)^(1/2)/(2
*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^
(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/d*b/(a^2+b^2)^(1/2)
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^
2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/d/b/(a^2+b^2)^(3/...
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1219 vs. $2(75) = 150$.

Time = 0.31 (sec) , antiderivative size = 1219, normalized size of antiderivative = 11.95

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*
a^2*b^2 + b^4)*d^4)) + a^3 - 3*a*b^2)/((a^2 + b^2)*d^2))*log(-(3*a^4*b + 2
*a^2*b^3 - b^5)*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d
*x + 2*c)) + ((a^4 - b^4)*d^3*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 +
2*a^2*b^2 + b^4)*d^4)) + 2*(3*a^3*b^2 - a*b^4)*d)*sqrt(-((a^2 + b^2)*d^2*s
qrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + a^3 -
3*a*b^2)/((a^2 + b^2)*d^2))) + 1/2*sqrt(-((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2
 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + a^3 - 3*a*b^2)/((a^2
+ b^2)*d^2))*log(-(3*a^4*b + 2*a^2*b^3 - b^5)*sqrt((b*cos(2*d*x + 2*c) + a
*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) - ((a^4 - b^4)*d^3*sqrt(-(9*a^4*b
^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*(3*a^3*b^2 - a*b^
4)*d)*sqrt(-((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2
*a^2*b^2 + b^4)*d^4)) + a^3 - 3*a*b^2)/((a^2 + b^2)*d^2))) + 1/2*sqrt(((a^
2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*
d^4)) - a^3 + 3*a*b^2)/((a^2 + b^2)*d^2))*log(-(3*a^4*b + 2*a^2*b^3 - b^5)
*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + ((
a^4 - b^4)*d^3*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4
)*d^4)) - 2*(3*a^3*b^2 - a*b^4)*d)*sqrt(((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2
- 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - a^3 + 3*a*b^2)/((a^2 +
 b^2)*d^2))) - 1/2*sqrt(((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 +...
```

3.104.6 Sympy [F]

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx = -\int \frac{a}{\sqrt{a+b\cot(c+dx)}}\,dx - \int \left(-\frac{b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\right)dx$$

input
```
integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))**(1/2),x)
```

output
```
-Integral(a/sqrt(a + b*cot(c + d*x)), x) - Integral(-b*cot(c + d*x)/sqrt(a
 + b*cot(c + d*x)), x)
```

3.104.7 Maxima [F]

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\int \frac{b\cot\left(dx+c\right)-a}{\sqrt{b\cot\left(dx+c\right)+a}}\,dx$$

input
```
integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cot(d*x + c) - a)/sqrt(b*cot(d*x + c) + a), x)
```

3.104.8 Giac [F]

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\int \frac{b\cot\left(dx+c\right)-a}{\sqrt{b\cot\left(dx+c\right)+a}}\,dx$$

input
```
integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) - a)/sqrt(b*cot(d*x + c) + a), x)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 2731, normalized size of antiderivative = 26.77

$$\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx=\text{Too large to display}$$

input
```
int(-(a - b*cot(c + d*x))/(a + b*cot(c + d*x))^(1/2),x)
```

output
```
2*atanh((32*a^4*b^2*d^2*(- (-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)
) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/(
(16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4
) + (4*a^3*b^3*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*a*b^5
*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*a^2*b^2*(- (-16*a
^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d
^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16*a^4*b^3*d^3)/(a^2*d^4 + b^2*d
^4) + (4*a*b^3*d^2*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (8*a*b^
2*(- (-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*
d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(-16*a^4*b^2*d^4)^(1/2))
/((16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d
^4) + (4*a^3*b^3*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*a*b
^5*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*(- (-16*a^4*b^2*d^4)
^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)
 - 2*atanh((32*a^2*b^2*((-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) -
 (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d*x))^(1/2))/((16
*a^4*b^3*d^3)/(a^2*d^4 + b^2*d^4) - (4*a*b^3*d^2*(-16*a^4*b^2*d^4)^(1/2))/
(a^2*d^5 + b^2*d^5)) - (32*a^4*b^2*d^2*((-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d
^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*cot(c + d
*x))^(1/2))/((16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a...
```

3.104. $\int \frac{-a+b\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}\,dx$

3.105 $\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx$

3.105.1 Optimal result

Integrand size = 27, antiderivative size = 132

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx=-\frac{(ia-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}+\frac{(ia+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}-\frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\cot(c+dx)}}$$

output
```
-(I*a-b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d+(I*
a+b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d-4*a*b/(
a^2+b^2)/d/(a+b*cot(d*x+c))^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.44

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx=\frac{b\left(\frac{\left(a^2-b^2+2a\sqrt{-b^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}}+\frac{\left(-a^2+b^2+2a\sqrt{-b^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+d}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a+\sqrt{-b^2}}}\right.}{(a^2+b^2)\,d}$$

input
```
Integrate[(-a + b*Cot[c + d*x])/(a + b*Cot[c + d*x])^(3/2),x]
```

output
```
(b*(((a^2 - b^2 + 2*a*Sqrt[-b^2])*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a
- Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) + ((-a^2 + b^2 + 2*a*Sqr
t[-b^2])*ArcTanh[Sqrt[a + b*Cot[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^
2]*Sqrt[a + Sqrt[-b^2]]) - (4*a)/Sqrt[a + b*Cot[c + d*x]]))/((a^2 + b^2)*d
)
```

3.105.3 Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {3042, 4012, 25, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b\cot(c+dx)-a}{(a+b\cot(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-a-b\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int -\frac{a^2-2b\cot(c+dx)a-b^2}{\sqrt{a+b\cot(c+dx)}}dx}{a^2+b^2}-\frac{4ab}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{a^2-2b\cot(c+dx)a-b^2}{\sqrt{a+b\cot(c+dx)}}dx}{a^2+b^2}-\frac{4ab}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{a^2+2b\tan\left(c+dx+\frac{\pi}{2}\right)a-b^2}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}-\frac{4ab}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}$$

$$\Big\downarrow \text{4022}$$

$$-\frac{4ab}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}-\frac{\frac{1}{2}(a-ib)^2\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx+\frac{1}{2}(a+ib)^2\int \frac{i\cot(c+dx)+1}{\sqrt{a+b\cot(c+dx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4ab}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}-\frac{\frac{1}{2}(a-ib)^2\int\frac{i\tan\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{1}{2}(a+ib)^2\int\frac{1-i\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}$$

$$\Big\downarrow \; 4020$$

$$-\frac{4ab}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}-\frac{\frac{i(a-ib)^2\int-\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}-\frac{i(a+ib)^2\int-\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}}{a^2+b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{4ab}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}-\frac{\frac{i(a+ib)^2\int\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}-\frac{i(a-ib)^2\int\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}}{a^2+b^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{4ab}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}-\frac{-\frac{(a-ib)^2\int\frac{1}{-\frac{i\cot^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}-\frac{(a+ib)^2\int\frac{1}{\frac{i\cot^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}}{a^2+b^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{4ab}{d\left(a^2+b^2\right)\sqrt{a+b\cot(c+dx)}}-\frac{-\frac{(a-ib)^2\arctan\left(\frac{\cot(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}-\frac{(a+ib)^2\arctan\left(\frac{\cot(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}}{a^2+b^2}$$

input
```
Int[(-a + b*Cot[c + d*x])/(a + b*Cot[c + d*x])^(3/2),x]
```

output
```
-((-(((a + I*b)^2*ArcTan[Cot[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)) -
 ((a - I*b)^2*ArcTan[Cot[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d))/(a^2
+ b^2)) - (4*a*b)/((a^2 + b^2)*d*Sqrt[a + b*Cot[c + d*x]])
```

3.105. $\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx$

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

3.105. $\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx$

3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2290 vs. 2(112) = 224.

Time = 0.07 (sec) , antiderivative size = 2291, normalized size of antiderivative = 17.36

method	result	size
derivativedivides	Expression too large to display	2291
default	Expression too large to display	2291
parts	Expression too large to display	3684

input
```
int((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*b/(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c)
)^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+
1/4/d/b/(a^2+b^2)^(5/2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+1/
d/b/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c
))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^6
+1/4/d/b/(a^2+b^2)^2*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-2/d*b
^3/(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1
/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-3/4/d*
b^3/(a^2+b^2)^(5/2)*ln((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)-b*cot(d*x+c)-(a^2+b^2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d*b^3/
(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^
(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/
d/b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+
c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^
4-1/d/b/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(
d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
)*a^6+3/4/d*b^3/(a^2+b^2)^(5/2)*ln(b*cot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(...
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2574 vs. $2(103) = 206$.

Time = 0.36 (sec) , antiderivative size = 2574, normalized size of antiderivative = 19.50

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(8*a*b*sqrt((b*cos(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x +
 2*c))*sin(2*d*x + 2*c) - ((a^2*b + b^3)*d*cos(2*d*x + 2*c) + (a^3 + a*b^2
)*d*sin(2*d*x + 2*c) + (a^2*b + b^3)*d)*sqrt(-(a^5 - 10*a^3*b^2 + 5*a*b^4
+ (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(25*a^8*b^2 - 100*a^6*b^4
+ 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a
^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*d^2))*log((5*a^6*b - 5*a^4*b^3 - 9*a^2*b^5 + b^7)*sqrt((b*cos(2
*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + ((a^9 - 6*a^5*b^
4 - 8*a^3*b^6 - 3*a*b^8)*d^3*sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6
 - 20*a^2*b^8 + b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a
^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + (15*a^6*b^2 - 35*a^4*b^4 + 13*a^2*b^6
- b^8)*d)*sqrt(-(a^5 - 10*a^3*b^2 + 5*a*b^4 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*d^2*sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b
^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^
10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))) + ((a^2*b +
b^3)*d*cos(2*d*x + 2*c) + (a^3 + a*b^2)*d*sin(2*d*x + 2*c) + (a^2*b + b^3)
*d)*sqrt(-(a^5 - 10*a^3*b^2 + 5*a*b^4 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*d^2*sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/(
(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b
^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))*log((5*a^6*b - 5...
```

3.105.6 Sympy [F]

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx = -\int \frac{a}{a\sqrt{a+b\cot(c+dx)}+b\sqrt{a+b\cot(c+dx)}\cot(c+dx)}\,dx - \int \left(-\frac{b\cot(c+dx)}{a\sqrt{a+b\cot(c+dx)}+b\sqrt{a+b\cot(c+dx)}\cot(c+dx)}\right)dx$$

input
```
integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))**(3/2),x)
```

output
```
-Integral(a/(a*sqrt(a + b*cot(c + d*x)) + b*sqrt(a + b*cot(c + d*x))*cot(c
 + d*x)), x) - Integral(-b*cot(c + d*x)/(a*sqrt(a + b*cot(c + d*x)) + b*sq
rt(a + b*cot(c + d*x))*cot(c + d*x)), x)
```

3.105.7 Maxima [F]

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx=\int \frac{b\cot\left(dx+c\right)-a}{\left(b\cot\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cot(d*x + c) - a)/(b*cot(d*x + c) + a)^(3/2), x)
```

3.105.8 Giac [F]

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx=\int \frac{b\cot\left(dx+c\right)-a}{\left(b\cot\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) - a)/(b*cot(d*x + c) + a)^(3/2), x)
```

3.105.9 Mupad [B] (verification not implemented)

Time = 19.01 (sec) , antiderivative size = 5475, normalized size of antiderivative = 41.48

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(-(a - b*cot(c + d*x))/(a + b*cot(c + d*x))^(3/2),x)
```

output
```
log(8*a*b^11*d^2 - (((((24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^
4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 12*a*b^4*d^2 +
4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(
1/2)*(64*a^6*b^7*d^4 - 96*a^2*b^11*d^4 - 64*a^4*b^9*d^4 - 32*b^13*d^4 + 96
*a^8*b^5*d^4 + 32*a^10*b^3*d^4 + (a + b*cot(c + d*x))^(1/2)*((((24*a*b^4*d
^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 +
48*a^4*b^2*d^4))^(1/2) - 12*a*b^4*d^2 + 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*
d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10
*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d
^5)) + (a + b*cot(c + d*x))^(1/2)*(16*b^12*d^3 + 32*a^2*b^10*d^3 - 32*a^6*
b^6*d^3 - 16*a^8*b^4*d^3))*((((24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16
*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 12*a*b^4
*d^2 + 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d
^4)))^(1/2) + 24*a^3*b^9*d^2 + 24*a^5*b^7*d^2 + 8*a^7*b^5*d^2)*((((24*a*b^
4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4
 + 48*a^4*b^2*d^4))^(1/2) - 12*a*b^4*d^2 + 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b
^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + log(8*a*b^11*d^2 - ((-((
(24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2
*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) + 12*a*b^4*d^2 - 4*a^3*b^2*d^2)/(16*(a^6
*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(64*a^6*b^7*d^4...
```

3.105. $\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{3/2}}\,dx$

3.106 $\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx$

3.106.1 Optimal result

Integrand size = 27, antiderivative size = 174

$$\begin{aligned}\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx = &-\frac{(ia-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} \\ &+\frac{(ia+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} \\ &-\frac{4ab}{3(a^2+b^2)\,d(a+b\cot(c+dx))^{3/2}}-\frac{2b(3a^2-b^2)}{(a^2+b^2)^2\,d\sqrt{a+b\cot(c+dx)}}\end{aligned}$$

output
```
-(I*a-b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d+(I*
a+b)*arctanh((a+b*cot(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d-4/3*a*b
/(a^2+b^2)/d/(a+b*cot(d*x+c))^(3/2)-2*b*(3*a^2-b^2)/(a^2+b^2)^2/d/(a+b*cot
(d*x+c))^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.45

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx = \frac{b\left(\frac{3\left(a^3-3ab^2+3a^2\sqrt{-b^2}+(-b^2)^{3/2}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\cot(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}}+\frac{3\left(-a^3+3ab^2+3a^2\sqrt{-b^2}+(-b^2\right.}{\sqrt{-b^2}}\right.}{3(a^2+b^2)^2\,d}$$

input
```
Integrate[(-a + b*Cot[c + d*x])/(a + b*Cot[c + d*x])^(5/2),x]
```

output
```
(b*((3*(a^3 - 3*a*b^2 + 3*a^2*Sqrt[-b^2] + (-b^2)^(3/2))*ArcTanh[Sqrt[a +
b*Cot[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) +
 (3*(-a^3 + 3*a*b^2 + 3*a^2*Sqrt[-b^2] + (-b^2)^(3/2))*ArcTanh[Sqrt[a + b*
Cot[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) - (
4*a*(a^2 + b^2))/(a + b*Cot[c + d*x])^(3/2) + (6*(-3*a^2 + b^2))/Sqrt[a +
b*Cot[c + d*x]]))/(3*(a^2 + b^2)^2*d)
```

3.106.3 Rubi [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4012, 25, 3042, 4012, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b\cot(c+dx)-a}{(a+b\cot(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-a-b\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int -\frac{a^2-2b\cot(c+dx)a-b^2}{(a+b\cot(c+dx))^{3/2}}dx}{a^2+b^2}-\frac{4ab}{3d\left(a^2+b^2\right)(a+b\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{a^2-2b\cot(c+dx)a-b^2}{(a+b\cot(c+dx))^{3/2}}dx}{a^2+b^2}-\frac{4ab}{3d\left(a^2+b^2\right)(a+b\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{a^2+2b\tan\left(c+dx+\frac{\pi}{2}\right)a-b^2}{\left(a-b\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{a^2+b^2}-\frac{4ab}{3d\left(a^2+b^2\right)(a+b\cot(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4012}$$

$$-\frac{\frac{\int \frac{a\left(a^2-3b^2\right)-b\left(3a^2-b^2\right)\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx}{a^2+b^2}+\frac{2b(3a^2-b^2)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}}{a^2+b^2}-\frac{4ab}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{a\left(a^2-3b^2\right)+b\left(3a^2-b^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}+\frac{2b(3a^2-b^2)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}}{a^2+b^2}-\frac{4ab}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}$$

$\Big\downarrow$ 4022

$$-\frac{4ab}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}-\frac{\frac{2b(3a^2-b^2)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{\frac{1}{2}(a-ib)^3\int \frac{1-i\cot(c+dx)}{\sqrt{a+b\cot(c+dx)}}dx+\frac{1}{2}(a+ib)^3\int \frac{i\cot(c+dx)+1}{\sqrt{a+b\cot(c+dx)}}dx}{a^2+b^2}}{a^2+b^2}$$

$\Big\downarrow$ 3042

$$-\frac{4ab}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}-\frac{\frac{2b(3a^2-b^2)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{\frac{1}{2}(a-ib)^3\int \frac{i\tan\left(c+dx+\frac{\pi}{2}\right)+1}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{1}{2}(a+ib)^3\int \frac{1-i\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-b\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{a^2+b^2}$$

$\Big\downarrow$ 4020

$$-\frac{4ab}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}-\frac{\frac{2b(3a^2-b^2)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{\frac{i(a-ib)^3\int -\frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}-\frac{i(a+ib)^3\int -\frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}}{a^2+b^2}}{a^2+b^2}$$

$\Big\downarrow$ 25

$$-\frac{4ab}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}-\frac{\frac{2b(3a^2-b^2)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{\frac{i(a+ib)^3\int \frac{1}{(1-i\cot(c+dx))\sqrt{a+b\cot(c+dx)}}d(i\cot(c+dx))}{2d}-\frac{i(a-ib)^3\int \frac{1}{(i\cot(c+dx)+1)\sqrt{a+b\cot(c+dx)}}d(-i\cot(c+dx))}{2d}}{a^2+b^2}}{a^2+b^2}$$

$\Big\downarrow$ 73

3.106. $\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx$

$$-\frac{4ab}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}-\frac{\frac{2b(3a^2-b^2)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{-\frac{(a-ib)^3\int\frac{1}{-\frac{i\cot^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}-\frac{(a+ib)^3\int\frac{1}{\frac{i\cot^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\cot(c+dx)}}{bd}}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{4ab}{3d\left(a^2+b^2\right)\left(a+b\cot(c+dx)\right)^{3/2}}-\frac{\frac{2b(3a^2-b^2)}{d(a^2+b^2)\sqrt{a+b\cot(c+dx)}}+\frac{-\frac{(a-ib)^3\arctan\left(\frac{\cot(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}-\frac{(a+ib)^3\arctan\left(\frac{\cot(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}}{a^2+b^2}}{a^2+b^2}$$

input
```
Int[(-a + b*Cot[c + d*x])/(a + b*Cot[c + d*x])^(5/2),x]
```

output
```
(-4*a*b)/(3*(a^2 + b^2)*d*(a + b*Cot[c + d*x])^(3/2)) - ((-(((a + I*b)^3*A
rcTan[Cot[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)) - ((a - I*b)^3*ArcTa
n[Cot[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d))/(a^2 + b^2) + (2*b*(3*a^
2 - b^2))/((a^2 + b^2)*d*Sqrt[a + b*Cot[c + d*x]]))/(a^2 + b^2)
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3054 vs. $2(150) = 300$.

Time = 0.12 (sec) , antiderivative size = 3055, normalized size of antiderivative = 17.56

method	result	size
derivativedivides	Expression too large to display	3055
default	Expression too large to display	3055
parts	Expression too large to display	4473

input
```
int((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-6/d*b/(a^2+b^2)^2/(a+b*cot(d*x+c))^(1/2)*a^2+1/4/d*b^5/(a^2+b^2)^(7/2)*ln
((a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^
2)^(1/2)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/4/d*b^5/(a^2+b^2)^(7/2)*ln(b*c
ot(d*x+c)+a+(a+b*cot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)
^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d*b^5/(a^2+b^2)^3/(2*(a^2+b^2)^(1/
2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/
2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/d*b^5/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/d/b/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^7-5/d*b^3/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(
1/2)-2*a)^(1/2)*arctan((2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(
1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+1/4/d/b/(a^2+b^2)^(7/2)*ln((a+b*c
ot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2
)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^6-5/4/d*b^3/(a^2+b^2)^(7/2)*ln((a+b*c
ot(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*cot(d*x+c)-(a^2+b^2)^(1/2
)-a)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+7/d*b^5/(a^2+b^2)^(7/2)/(2*(a^2+b^2
)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*
a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-1/d/b/(a^2+b^2)^(7/2)/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2)*arctan((-2*(a+b*cot(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/...
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3922 vs. $2(141) = 282$.

Time = 0.42 (sec) , antiderivative size = 3922, normalized size of antiderivative = 22.54

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(3*((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(2*d*x + 2*c) - 2*(a^5*b + 2
*a^3*b^3 + a*b^5)*d*sin(2*d*x + 2*c) - (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
*d)*sqrt(-(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6 + (a^10 + 5*a^8*b^2 + 1
0*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^2*sqrt(-(49*a^12*b^2 - 490*a^
10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4*b^10 - 42*a^2*b^12 + b^14)/
((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*b^6 + 210*a^12*b^8 + 252*a^1
0*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a^4*b^16 + 10*a^2*b^18 + b^20)*d
^4)))/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^2
))*log(-(7*a^8*b - 28*a^6*b^3 - 14*a^4*b^5 + 20*a^2*b^7 - b^9)*sqrt((b*cos
(2*d*x + 2*c) + a*sin(2*d*x + 2*c) + b)/sin(2*d*x + 2*c)) + ((a^14 - a^12*
b^2 - 19*a^10*b^4 - 45*a^8*b^6 - 45*a^6*b^8 - 19*a^4*b^10 - a^2*b^12 + b^1
4)*d^3*sqrt(-(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 5
11*a^4*b^10 - 42*a^2*b^12 + b^14)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120
*a^14*b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 4
5*a^4*b^16 + 10*a^2*b^18 + b^20)*d^4)) + 4*(7*a^9*b^2 - 42*a^7*b^4 + 56*a^
5*b^6 - 22*a^3*b^8 + a*b^10)*d)*sqrt(-(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a
*b^6 + (a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*d^2
*sqrt(-(49*a^12*b^2 - 490*a^10*b^4 + 1519*a^8*b^6 - 1484*a^6*b^8 + 511*a^4
*b^10 - 42*a^2*b^12 + b^14)/((a^20 + 10*a^18*b^2 + 45*a^16*b^4 + 120*a^14*
b^6 + 210*a^12*b^8 + 252*a^10*b^10 + 210*a^8*b^12 + 120*a^6*b^14 + 45*a...
```

3.106.6 Sympy [F]

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx =$$
$$-\int \frac{a}{a^2\sqrt{a+b\cot{(c+dx)}}+2ab\sqrt{a+b\cot{(c+dx)}}\cot{(c+dx)}+b^2\sqrt{a+b\cot{(c+dx)}}\cot^2{(c+dx)}}\,dx$$
$$-\int \left(-\frac{b\cot{(c+dx)}}{a^2\sqrt{a+b\cot{(c+dx)}}+2ab\sqrt{a+b\cot{(c+dx)}}\cot{(c+dx)}+b^2\sqrt{a+b\cot{(c+dx)}}\cot^2{(c+dx)}}\right.$$

input
```
integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))**(5/2),x)
```

output
```
-Integral(a/(a**2*sqrt(a + b*cot(c + d*x)) + 2*a*b*sqrt(a + b*cot(c + d*x)
)*cot(c + d*x) + b**2*sqrt(a + b*cot(c + d*x))*cot(c + d*x)**2), x) - Inte
gral(-b*cot(c + d*x)/(a**2*sqrt(a + b*cot(c + d*x)) + 2*a*b*sqrt(a + b*cot
(c + d*x))*cot(c + d*x) + b**2*sqrt(a + b*cot(c + d*x))*cot(c + d*x)**2),
x)
```

3.106.7 Maxima [F]

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx=\int \frac{b\cot(dx+c)-a}{(b\cot(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cot(d*x + c) - a)/(b*cot(d*x + c) + a)^(5/2), x)
```

3.106.8 Giac [F]

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx=\int \frac{b\cot(dx+c)-a}{(b\cot(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-a+b*cot(d*x+c))/(a+b*cot(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cot(d*x + c) - a)/(b*cot(d*x + c) + a)^(5/2), x)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 29.37 (sec) , antiderivative size = 8438, normalized size of antiderivative = 48.49

$$\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
int(-(a - b*cot(c + d*x))/(a + b*cot(c + d*x))^(5/2),x)
```

output

```
(log(((-(4*a^7*d^2 - (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4
 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b
^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4
*d^4 + 5*a^8*b^2*d^4))^(1/2)*((a + b*cot(c + d*x))^(1/2)*(320*a^6*b^14*d^3
 - 16*a^2*b^18*d^3 + 1024*a^8*b^12*d^3 + 1440*a^10*b^10*d^3 + 1024*a^12*b^
8*d^3 + 320*a^14*b^6*d^3 - 16*a^18*b^2*d^3) - ((-(4*a^7*d^2 - (320*a^6*b^8
*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4 - 400*a^12*b
^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*
a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(((-
(4*a^7*d^2 - (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*
a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/
(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5
*a^8*b^2*d^4))^(1/2)*(a + b*cot(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b
^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 161
28*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6
*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 - 32*a*b^21*d^4 - 160*a^3*b^
19*d^4 - 128*a^5*b^17*d^4 + 896*a^7*b^15*d^4 + 3136*a^9*b^13*d^4 + 4928*a^
11*b^11*d^4 + 4480*a^13*b^9*d^4 + 2432*a^15*b^7*d^4 + 736*a^17*b^5*d^4 + 9
6*a^19*b^3*d^4))/4))/4 + 16*a^4*b^15*d^2 + 96*a^6*b^13*d^2 + 240*a^8*b^11*
d^2 + 320*a^10*b^9*d^2 + 240*a^12*b^7*d^2 + 96*a^14*b^5*d^2 + 16*a^16*b...
```

3.106. $\int \frac{-a+b\cot(c+dx)}{(a+b\cot(c+dx))^{5/2}}\,dx$

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```